An Estimator for Compound-Gaussian Multilook SAR Clutter Amplitude with Inverse Gamma Texture

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***Abstract*—In the context of synthetic aperture radar (SAR) data analysis, formulation of accurate models for clutter statistics is a crucial task. In this paper, compound-Gaussian distribution with inverse gamma texture (IΓ-CG), is presented for multilook SAR amplitude data. An estimator based on method of log-cumulants (MoLC), which stems from the adoption of second kind statistics and Mellin transform is developed for estimating its parameters. The IΓ-CG model is validated by using multilook synthetic data and single look real clutter data of amplitude SAR images. Experimental results show that the IΓ-CG model outmatches the state-of-the-art pdfs that clearly demonstrates the applicability of the model.**

Keywords*—Clutter, compound-Gaussian model, synthetic aperture radar (SAR) image, method of log-cumulants, Mellin transform.

I. Introduction

Over the last couple of decades, synthetic aperture radar (SAR), has earned its importance due to its applicability in diverse fields such as surveillance, environmental science, hydrology, archeology etc. This imaging system is capable of producing high resolution SAR images of earth's surface in both range and azimuth while avoiding some limitation of other imaging system [1]. Specifically, SAR imaging systems overcome the night-time limitations of optical cameras and the cloud-cover limitations of infrared imaging.

However, the precise knowledge of the SAR clutter statistics plays an important role in image processing and applications. These statistical characterization may effectively improve the performance of denoising [2], target detection [3] in SAR images. Several statistical models have been reported in the literature in order to relate physical features and statistical properties of SAR data. Out of these models, the multiplicative model has been widely used in modeling SAR clutter amplitude statistics which states that the return results from the product between the texture and the speckle. Compound-Gaussian model is one special case of multiplicative model in which speckle is assumed to be Gaussian distributed. This model is found to be attractive in characterizing heavy-tailed clutter distributions in high-resolution radar [4]–[6]. The key problems in compound-Gaussian clutter modeling are choosing texture distribution and estimating its parameters. Many distributions such as gamma, generalized gamma, log-normal have been reported in the literature for the backscatter [5]–[9], aiming at modeling areas with varying degree of homogeneity and their parameters are estimated typically using the method of moments (MoM) [10], maximum likelihood estimation (MLE) [11] etc. In this paper, we extend the compound-Gaussian distribution with inverse gamma texture proposed for radar clutter amplitude in [11] to multilook cases and use it for statistical modeling of amplitude SAR clutter data. We have derived the closed form pdf of compound-Gaussian model with inverse gamma texture for multilook case. Also, an estimator based on method of log-cumulants (MoLC) is developed for the parameters of the same. We illustrate the applicability of IΓ-CG model to multilook synthetic data and single look real SAR clutter data obtained from Sandia National Laboratory. Principal characteristics of the SAR system and SAR image data files are given in detail in Section V.

This paper is organized as follows: Section II gives overview on multiplicative model and speckle noise, Section III describes the formulation of IΓ-CG model for multilook SAR clutter amplitude, Section IV presents the MoLC estimator for this model, Section V presents description of SAR datasets and experimental result analysis. Finally, concluding remarks are given in Section VI.

II. The Multiplicative Model and Speckle Noise

Speckle noise is a kind of multiplicative noise that is always associated with SAR clutter. It is caused due to constructive and destructive interference of coherent waves reflected by elementary scatterers contained within the imaged resolution cell. The presence of speckle noise degrades SAR image significantly which makes visual and automatic interpretation a difficult task. This leads to loss of crucial information. In order to remove the speckle, statistical properties of the same may need to be analyzed carefully. So, the multiplicative model (compound model) has become an important framework used to characterize SAR clutter statistics.

The multiplicative model assumes that the observed pixel-by-pixel variability within a SAR image is the result of the product between two components: the texture and the speckle [8]. Mathematically, clutter amplitude corresponding to (u,v) spatial coordinate of a pixel in the image can be given as

$$r\left(u,v\right)=\sqrt{\tau\left(u,v\right)}\cdot\left|x\left(u,v\right)\right| \tag{1}$$

where $\tau\left(u,v\right)$ and $\left|x\left(u,v\right)\right|$ represent texture and speckle amplitude, respectively.

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The texture takes into account the local power variation of the radar backscattering in the image. It is modeled as a non-negative real random process whose statistical characteristics depend on the structure of the illuminated area. On the other hand, the speckle is modeled as a complex Gaussian process $x(u,v) = x_I(u,v) + jx_Q(u,v)$, where $x_I(u,v)$ and $x_Q(u,v)$ are the in-phase and quadrature phase components, respectively. These independent components are normal distributed with zero mean i.e. $E\{x_I(u,v)\} = E\{x_Q(u,v)\} = 0$ and variance 1/2 i.e. var $\{x_I(u,v)\} =$ var $\{x_Q(u,v)\} = 1/2$. So, the amplitude speckle follows Rayleigh distribution for single look case which can be expressed as

$$p(x) = 2xe^{-x^2} \tag{2}$$

The square root of gamma distribution has been introduced in [12], [13] to characterize multilook amplitude speckle. This model generalizes the Rayleigh distribution given in (2) by averaging L single-look Rayleigh distributed amplitudes, and is given as

$$p(x) = \frac{2L^L}{\Gamma(L)} x^{2L-1} e^{-Lx^2} \tag{3}$$

where $\Gamma(\cdot)$ is Gamma function. Though L should be an integer in principle, it is usually not the case when estimated from the real data. Therefore, it will be more appropriate to call L as "equivalent number of looks".

III. Inverse Gamma-Compound Gaussian (IΓ-CG) Model

In this section, compound-Gaussian distribution with inverse gamma distributed texture is formulated for statistical characterization of multilook SAR clutter amplitude.

A. Formulation of pdf

Here, inverse gamma distribution is considered for texture τ [11], [14]and its pdf can be expressed as

$$p(\tau) = \frac{1}{\eta^\kappa \Gamma(\kappa)} \tau^{-(\kappa+1)} e^{-1/\eta\tau} \tag{4}$$

where κ and η denote shape and scale parameter, respectively. For a given τ, the observed clutter amplitude in an L-look image is square root of gamma distributed (Nakagami distributed) and its pdf is given by

$$p(r|\tau) = \frac{2L^L r^{2L-1}}{\Gamma(L)\tau^L} e^{-Lr^2/\tau} \tag{5}$$

From the compound model (multiplicative model) given in (1), the pdf of SAR clutter amplitude is derived using

$$p_R(r) = \int_{-\infty}^{\infty} p(r|\tau) p(\tau)\, d\tau \tag{6}$$

where r denotes a realization of the random process R. Substituting (4) and (5) in (6), the pdf of clutter amplitude is given by

$$p_R(r) = \int_0^\infty \frac{1}{\eta^\kappa \Gamma(\kappa)} \tau^{-(\kappa+1)} e^{-1/\eta\tau} \frac{2L^L r^{2L-1}}{\Gamma(L)\tau^L} e^{-Lr^2/\tau} d\tau \tag{7}$$

$$= \frac{2L^L r^{2L-1}}{\eta^\kappa \Gamma(\kappa)\Gamma(L)} \int_0^\infty \tau^{-(\kappa+L+1)} e^{-(\eta L r^2+1)/\eta\tau} d\tau \tag{8}$$

Substituting $(\eta L r^2 + 1)/\eta\tau$ in (8) with q and using the identity $\Gamma(t) = \int_0^\infty y^{t-1} e^{-y} dy$, the closed-form pdf of multilook SAR clutter amplitude may be given as

$$p_R(r) = \frac{2L^L \eta^L r^{2L-1}}{\Gamma(\kappa)\Gamma(L)(\eta L r^2+1)^{\kappa+L}} \int_0^\infty q^{\kappa+L-1} e^{-q} dq \tag{9}$$

$$= \frac{2L^L \eta^L \Gamma(\kappa+L) r^{2L-1}}{\Gamma(\kappa)\Gamma(L)(\eta L r^2+1)^{\kappa+L}} \tag{10}$$

IV. Parameter Estimation of IΓ-CG Model

In this section, MoLC estimator based on second-kind statistics to estimate the parameters of IΓ-CG model is formulated.

A. Mellin Transform and Second-kind Statistics

Statistical approaches classically used to analyze a pdf are based on Fourier transform. In [15], author proposed Mellin Transform (MT) to perform an effective analysis of distributions, used for characterizing SAR clutter statistics.

Let r be a random variable with pdf $p_R(r)$. Its second-kind first characteristic function $\phi(s)$ is defined by the MT of $p_R(r)$ as

$$\phi(s) = MT[p_R(r) : s] = \int_0^\infty r^{s-1} p_R(r) dr = E(R^{s-1}) \tag{11}$$

provided the integral converges. Particularly, for $s = 1, \phi(s)$ satisfies the property $\phi(s)|_{s=1} = \int_0^\infty p_R(r) dr = 1$. Then the second-kind second characteristic function can be expressed as

$$\varphi(s) = \log(\phi(s)) \tag{12}$$

and the n^{th}-order second-kind cumulants $\widetilde{k_n}$ is given by

$$\widetilde{k_n} = \left.\frac{d^n \varphi(s)}{ds^n}\right|_{s=1} \tag{13}$$

B. Formulation of MoLC Estimator

From the definition of the moment, the n^{th} order moment of R is

$$E[R^n] = \frac{2L^L \eta^L \Gamma(\kappa+L)}{\Gamma(\kappa)\Gamma(L)} \int_0^\infty \frac{r^{n+2L-1}}{(\eta L r^2+1)^{\kappa+L}} dr \tag{14}$$

where $E[\cdot]$ denotes expectation. Substituting $\eta L r^2$ in (14) with k and using the identity $\int_0^\infty t^{z-1} \big/ (1+t)^{z+w} dt =$

$\Gamma(z)\Gamma(w)/\Gamma(z+w)$, the closed form of (14) may be given as

$$E[R^n] = \frac{2L^L\eta^L\Gamma(\kappa+L)}{\Gamma(\kappa)\Gamma(L)}\left[\frac{1}{2(\eta L)^{\frac{n}{2}+L}}\int_0^\infty \frac{k^{\frac{n}{2}+L-1}}{(1+k)^{\kappa+L}}dk\right] \tag{15}$$

$$= \left(\frac{1}{\eta L}\right)^{n/2}\frac{\Gamma(n/2+L)\Gamma(\kappa-n/2)}{\Gamma(\kappa)\Gamma(L)} \tag{16}$$

Substituting (16) in (11), we have

$$\phi(s) = \left(\frac{1}{\eta L}\right)^{(s-1)/2}\frac{\Gamma\left(\frac{s-1}{2}+L\right)\Gamma\left(\kappa-\frac{s-1}{2}\right)}{\Gamma(\kappa)\Gamma(L)} \tag{17}$$

$$\begin{aligned}\varphi(s) &= \frac{s-1}{2}\log\left(\frac{1}{\eta L}\right)+\log\left(\Gamma\left(\frac{s-1}{2}+L\right)\right)\\ &+\log\left(\Gamma\left(\kappa-\frac{s-1}{2}\right)\right)-\log(\Gamma(\kappa))-\log(\Gamma(L))\end{aligned} \tag{18}$$

To estimate the unknown parameters κ, η and L of IΓ-CG distribution, the first, second and third order log-cumulants of (10) are computed from (13) using (18) as

$$\widetilde{k_1} = \frac{1}{2}\log\left(\frac{1}{\eta L}\right)+\frac{1}{2}\psi(L)-\frac{1}{2}\psi(\kappa) \tag{19}$$

$$\widetilde{k_2} = \frac{1}{4}\psi(1,L)+\frac{1}{4}\psi(1,\kappa) \tag{20}$$

$$\widetilde{k_3} = \frac{1}{8}\psi(2,L)-\frac{1}{8}\psi(2,\kappa) \tag{21}$$

where $\psi(t) = \frac{d}{dt}(\log\Gamma(t))$ is the Digamma function and $\psi(n,t) = \frac{d^{n+1}}{dt^{n+1}}(\log\Gamma(t))$ is the Polygamma function [16].

Now, the parameter estimates of IΓ-CG distribution based on MoLC can be obtained by solving (19)-(21) using numerical methods where the first three second-kind cumulants computed from measured data can be expressed as

$$\begin{aligned}\widehat{\widetilde{k_1}} &= \frac{1}{N}\sum_{i=1}^{N}[\log(r_i)]\\ \widehat{\widetilde{k_n}} &= \frac{1}{N}\sum_{i=1}^{N}\left[\left(\log(r_i)-\widehat{\widetilde{k_1}}\right)^n\right]\end{aligned} \tag{22}$$

where $n = 2,3$ and N is the number of independent data samples used for estimation of parameters.

V. Experimental Results

In this section, brief discussion is made on multilook synthetic data and single look real SAR data used for our experimental analysis. Experimental results are presented to show the effectiveness of IΓ-CG model over state-of-the-art distributions used for modeling clutter amplitude.

A. Description of Dataset

1) Synthetic Data: The multilook synthetic data shown in Fig.1(a) is generated by degrading original speckleless "westconcordorthophoto" image available in MATLAB Image Processing Toolbox with 3-look synthetic speckle in amplitude format. Here, "westconcordorthophoto" image is chosen due to its identical content with several real SAR images. Experiments are made on a region (marked with a rectangle) of the synthetic data shown in Fig.1(a).

2) Real SAR Data: The SAR data we used for our analysis have been obtained by the Sandia National Laboratory with Miniature SAR (Mini-SAR) platform [17]. Main characteristics of this SAR system are given below in Table I.

TABLE I. Miniature Synthetic Aperture Radar(Mini-SAR) Characteristics

Data Collectors	Sandia National Lab
Radar Platform name	Miniature SAR (Mini-SAR)
Sensor Name	Twin Otter
Range Resolution	0.1016 m
Cross range resolution	0.1016 m
Range pixel size	0.0847 m
Cross range pixel size	0.0847 m
Range Count	1638
Azimuth Count	2510
Operating frequency band	Ku-band
Central frequency	16.8 GHz
Bits per pixel	16
Number of Look	1

The dataset consists of 20 image files in .gff file format; each file contains a header followed by moduli and phases of the data. The header file presents information regarding radar characteristics such as resolution, frequency, squint etc. Experiments are conducted on four different SAR images from the dataset which are shown in Fig.1(b)-(e) named by "HarSitVeh","TijArGolCou","EubGatEnt","NatGuaArm", respectively. Among those, first image is related to area with minute texture, second and third images are related to areas with moderate texture and last image is related to area with extreme variation. This characterization is based on visual inspection only.

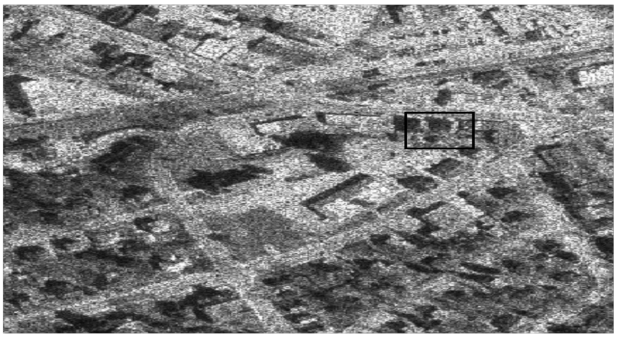
B. Comparison between estimated pdf and SAR data histogram

In order to demonstrate the effectiveness of IΓ-CG distribution compared to Weibull (WBL), log-normal (LGN), K distribution, experiment is conducted on aforementioned SAR images. Here, MoLC is employed to obtain parameter estimates of the above-mentioned distribution using the estimation formula given in Table II. Estimated parameter values of the IΓ-CG pdf for the experimental images are given in Table III.

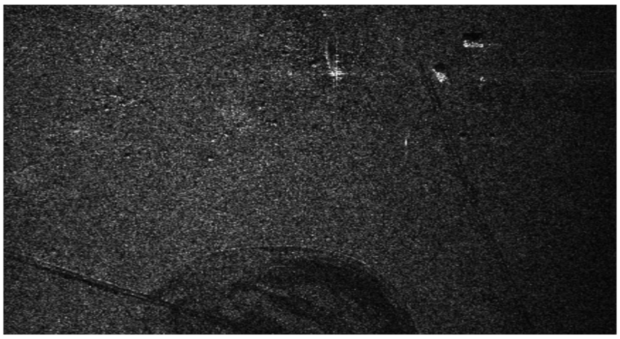
To evaluate the histogram matching accuracy quantitatively, the Kullback-Leibler (KL) distance D_{KL} is considered which quantifies how close an estimated pdf to the data histogram. Smaller is the value of D_{KL}, better is the goodness-of-fit of the estimated pdf to the real data histogram. For r^{th} gray level, if the estimated pdf and empirical data histogram are respectively

TABLE II. PDFs AND ESTIMATION FORMULA OF COMPARATIVE MODELS

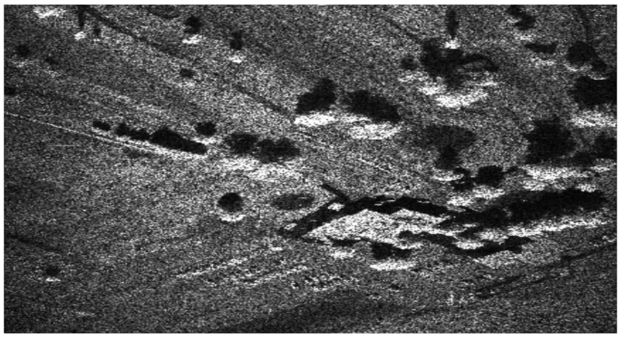
Model	Distribution	Estimation Formula
LGN [18]	$p(r;[\kappa,\eta]) = \frac{1}{r\kappa\sqrt{2\pi}} \exp\left(-\frac{(\log r-\eta)^2}{2\kappa^2}\right)$	$\begin{cases} \widehat{k_1} = \eta \\ \widehat{k_2} = \kappa^2 \end{cases}$
WBL [18]	$p(r;[\kappa,\eta]) = \frac{\kappa}{\eta^\kappa} r^{\kappa-1} \exp\left(-\left(\frac{r}{\eta}\right)^\kappa\right)$	$\begin{cases} \widehat{k_1} = \log\eta + \psi(1)\kappa^{-1} \\ \widehat{k_2} = \psi(1,1)\kappa^{-2} \end{cases}$
K [18]	$p(r;[\kappa,\eta,L]) = \frac{4}{\Gamma(L)\Gamma(\kappa)} \left(\frac{L\kappa}{\eta}\right)^{\frac{L+\kappa}{2}} r^{L+\kappa-1} K_{\kappa-L}\left(2x\sqrt{\frac{L\kappa}{\eta}}\right)$	$\begin{cases} 2\widetilde{k_1} = \log\frac{\eta}{L\kappa} + \psi(L) + \psi(\kappa) \\ 4\widetilde{k_2} = \psi(1,L) + \psi(1,\kappa) \\ 8\widetilde{k_3} = \psi(2,L) + \psi(2,\kappa) \end{cases}$



(a) Synthetic Image

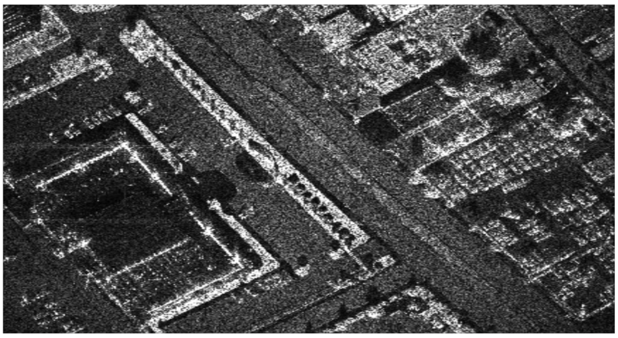


(b) "HarSitVeh" Image



(c) "TijArGolCou" Image

(d) "EubGatEnt" Image



(e) "NatGuaArm" Image

Fig. 1. Synthetic image and real SAR images used for analysis

be represented as $h_e(r)$ and $h_d(r)$, the D_{KL} between them can be expressed as

$$D_{KL} = \sum_r \log\left(\frac{h_d(r)}{h_e(r)}\right) h_d(r) \tag{23}$$

In order to compute the goodness-of-fit, KL distance is evaluated using Weibull, log-normal and K distribution for the considered SAR images. The obtained results are given in Table IV.

Also, for visual comparison, the normalized histogram of clutter amplitude data and estimated pdfs are shown in Fig.2. It is found from Fig.2 that IΓ-CG pdf outmatches the considered parametric models. Furthermore, Table IV shows that the Weibull and K sistributions cannot fit well to the

TABLE III. Estimated Parameters of IΓ-CG Distribution for Five Experimental Images

Image	L	κ	η	ENL
Synthetic	3	5.7389	1.4462e-05	2.8386
HarSitVeh	1	1.6580	4.2840e-04	1.0207
TijArGolCou	1	1.3316	1.6406e-04	0.9010
EubGatEnt	1	1.2199	4.2987e-04	1.0158
NatGuaArm	1	0.9785	3.8362e-04	1.0431

TABLE IV. Values of KL Distance Obtained Using Weibull, Lognormal, K and IΓ-CG Distributions for Five Experimental Images

Image	**KL Distance**			
	WBL	**LGN**	**K**	**IΓ-CG**
Synthetic	0.1781	0.0907	–	**0.0855**
HarSitVeh	0.0644	0.0191	0.0995	**0.0110**
TijArGolCou	0.1134	0.0186	0.1236	**0.0066**
EubGatEnt	0.2183	0.0077	0.1581	**0.0054**
NatGuaArm	0.3850	0.0084	0.2309	**0.0025**

clutter amplitude data histogram due to their large D_{KL} values. On the contrary, IΓ-CG distribution achieves best matching in terms of minimal KL distance followed by log-normal distribution for all experimental images with varying degree of roughness.

VI. Conclusion

In this paper, a compound model with inverse gamma texture for statistical modeling of multilook SAR clutter amplitude is presented. It is found that, IΓ-CG model has closed form expression for its pdf. Here, an estimation technique based on MoLC is developed for estimating its parameters. Experimental results illustrate the superiority of IΓ-CG distribution over the log-normal, Weibull and K pdfs in terms of histogram matching accuracy in multilook and single look clutter. Furthermore, IΓ-CG distribution is found to be attractive in modeling amplitude SAR images with varying degree of roughness. Future work may address the applicability of the model in MAP based despeckling of amplitude SAR images.

Acknowledgment

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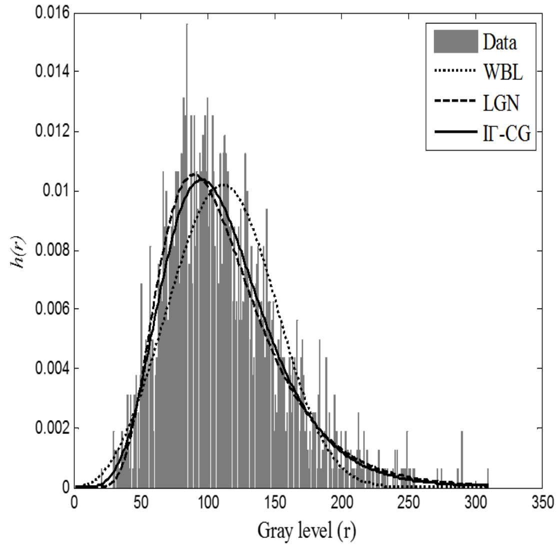
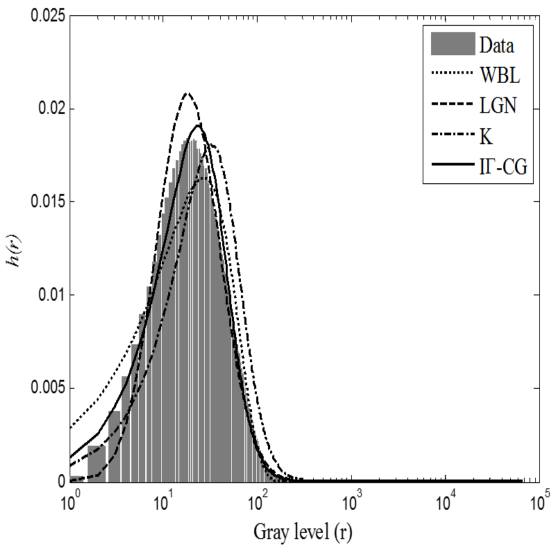
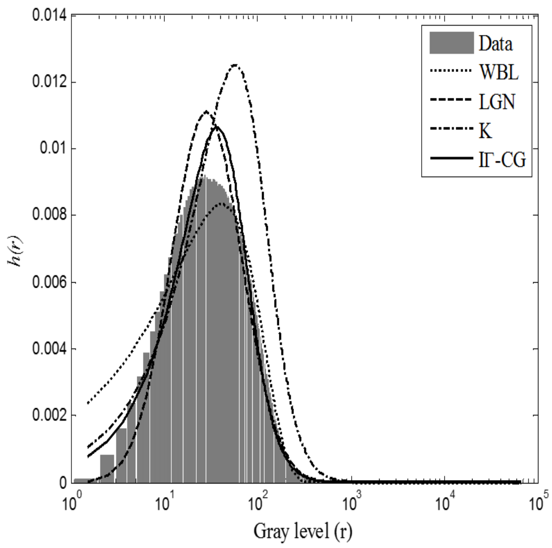


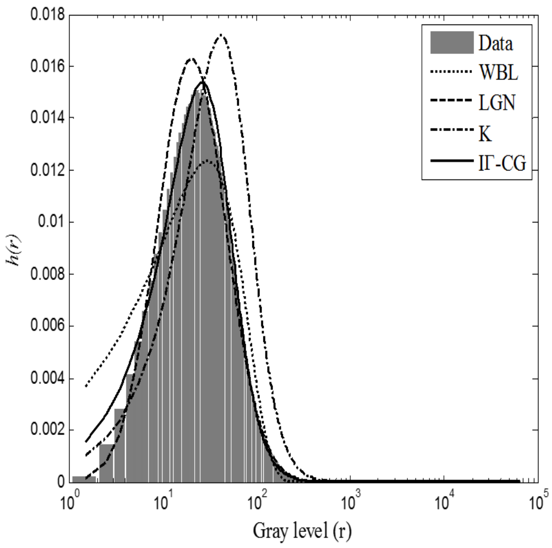
(a) Synthetic Image



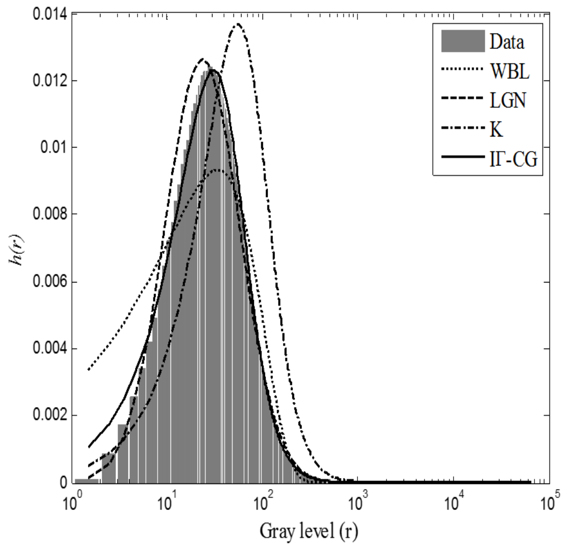
(b) “HarSitVeh” Image



(c) “TijArGolCou” Image



(d) “EubGatEnt” Image



(e) “NatGuaArm” Image

Fig. 2. Estimated pdfs and data histogram of experimental images