A new technique for the analysis of torque tube heat exchangers of superconducting generators

Pradeep Kumar Sahoo and Sunil Sarangi

Process Equipment and Design Laboratory, Cryogenic Engineering Centre, Indian Institute of Technology, Kharagpur 721 302, India

The torque tubes of a superconducting generator belong to a unique class of heat exchangers where a single fluid stream (boil off helium vapour) exchanges heat with a metallic wall conducting heat from the ambient to the cold (liquid helium) space. The rate of heat leak to the cold space is the chief indicator of the performance of the exchanger. This rate of heat inleak has been found to be a function of two dimensionless parameters: a generalized cooling parameter Λ^* and a temperature range parameter Ψ. A generalized design chart is presented which expresses the dimensionless heat flow rate Q_c in terms of the parameters Λ^* and Ψ. This chart will be useful for the design of torque tubes and vapour-cooled support members in cryogenic systems.

Keywords: superconducting generator; torque tubes; field winding; thermal modelling

Nomenclature

A'_c	Convective heat transfer area per unit length ($m^2\ m^{-1}$)
A_k	Cross sectional area of the torque tube (m^2)
C_p	Specific heat at constant pressure of the coolant vapour ($J\ kg^{-1}\ K^{-1}$)
h	Convective heat transfer coefficient ($W\ m^{-2}\ K^{-1}$)
k	Thermal conductivity of torque tube material ($W\ m^{-1}\ K^{-1}$)
l	Length of the torque tube heat exchange (m)
L	Latent heat of vaporization of coolant ($J\ kg^{-1}$)
$\dot{m}$	Mass flow rate of coolant ($kg\ s^{-1}$)
q	Axial heat flow rate through the torque tube (W)
Q	Heat leak ratio = $q/q_{c_{max}}$
T	Temperature (K)
x	Axial position (m)

Greek letters

β	Fluid flow rate factor = $\dot{m}/\dot{m}_{ss}$
Λ	Dimensionless cooling parameter
Λ^*	Generalized dimensionless cooling parameter
τ	Dimensionless temperature of the wall
θ	Dimensionless temperature of the fluid
ξ	Dimensionless length coordinate
Ψ	Dimensionless temperature range parameter $[C_p(T_h - T_C)/L]$

Subscripts

c	Cold end
f	Fluid
h	Warm end
ss	Self-sustained condition
w	Wall

The use of superconductors in the field winding of an electrical generator increases its efficiency and reduces the weight and the cost, but with the added complexity of keeping the field winding at a temperature below 5 K. To minimize the liquid helium requirement for cooling of the field winding, the heat leak to the cold space should be kept as low as possible. Convective heat transfer between the warm structure of the generator and the cold rotor is mostly eliminated by maintaining a high vacuum insulating space around the rotor, and radiation heat inleak is reduced substantially by the use of radiation shields. The rotor is isolated from the warm portion of the drive train by a pair of thin hollow cylinders called torque tubes which, as the name suggests, transmit the drive torque to the rotor. The field winding is energized by a pair of current leads. The torque tubes and the current leads play an important role in the operation of the generator. Most of the steady-state heat leakage into the cold space comes through these two subsystems. While the current lead contributes a significant portion of the heat load in a small generator, the heat leak scenario in large superconducting generators is dominated by the torque tubes.

The torque tubes should be strong enough in torsion to carry the operating and fault torques in the machine. In addition to the torque, the tubes carry the weight of the rotor and unbalanced centrifugal force. In addition, the tubes must be rigid enough to prevent the rotor from excessive vibration during operation. These requirements call for thick and short tubes, in direct conflict with the thermal isolation requirement for thin and long tubes. Torque tubes sustain a stiff temperature gradient—300 to 4 K over a short span. This results in significant heat leak into the cold space and consequent vaporization of liquid helium. In order to minimize the heat leak into the cold space through the torque tubes, the boiloff helium vapour is passed over the tube in a direction opposite to that of the conduction heat flux. The rate of helium gas flow is found from the relation:

$$\dot{m} = \frac{q_c}{L} \tag{1}$$

where q_c is the cold end heat inleak rate and L the latent heat of vaporization of liquid helium. This scenario can be called the 'self-sustained condition', because the boiloff stream is generated totally by the cold end heat inleak. In practice, the vapour flow rate can be higher or lower than this amount. Additional vapour can be generated by other sources of heat leak, and some of the vapour may be diverted through channels other than the torque tubes.

The cooling arrangement plays an important role in minimizing the heat leak to the cold zone. Materials of the torque tubes, as well as its dimensions—both the length and cross section, are determined from strength and vibration requirements. The geometry of the cooling channel is, however, determined from thermal and hydrodynamic considerations.

From a thermal analysis point of view, a torque tube constitutes a special class of counterflow heat exchangers where the heat exchange process takes place between the axially conducting metal tube and the helium stream flowing opposite to the direction of heat flow. The fluid and the wall temperature profiles are determined by heat transfer between the wall and the fluid, and by axial heat flow along the tubes. Reduction of cold end heat inleak by enhancing the heat transfer area or by using smaller channel cross section results in a higher pressure drop and consequent increase in the operating pressure of the liquid helium in the core of the rotor. This leads to a higher operating temperature and reduced current-carrying capacity of the superconducting winding. Therefore it is important to provide an optimized design for the torque tubes, which reduces both the cold end heat inleak as well as the pressure drop in the channels.

Based on Second law considerations, Bejan and Smith[1] have calculated the heat leak rate for minimum net refrigerating power consumption of a vapour-cooled cryogenic support. This can be achieved by providing a series of infinitely small Carnot refrigerators or by a stream of vapour with variable (increasing) mass flow rate. Both the schemes are difficult to attain; however, a single vapour stream, with a mass flow rate 10% above that dictated by self-sustained condition, has been found to provide close to ideal cooling.

The performance of the torque tubes has been analysed by several authors[2–4]. They have numerically solved the governing equations to provide temperature and heat-flux profiles. In this paper we have attempted to present a generalized formulation of the heat transfer mechanism in an exchanger where a single fluid stream exchanges heat with an axially conducting wall. The performance of the torque tube has been expressed in terms of two dimensionless parameters: a generalized cooling parameter Λ^* and a temperature range parameter Ψ. It has been possible, under certain valid assumptions, to present the results in the form of a single chart which can be used for both rating and sizing of torque tube heat exchangers.

Analysis

The analysis of the torque tube heat exchanger has been done in two parts. First an ideal cooling (infinite convective heat transfer coefficient) model is considered. This model predicts $q_{c_{min}}$, the theoretically minimum heat leak rate to the helium space. In the second part, the real cooling (finite convective heat transfer coefficient) situation is considered. In both cases constant cross section and constant (independent of temperature) thermophysical properties have been assumed. Under real operating conditions the cold end heat leak rate takes a value between $q_{c_{min}}$ and $q_{c_{max}}$, the latter being the rate of heat flow in the absence of vapour cooling of the tubes:

$$q_{c_{max}} = kA_k \frac{T_h - T_c}{l} \tag{2}$$

Ideal cooling model

An ideal cooling model, leading to minimum heat leak rate into the coolant medium under self-sustained condition, is provided by the assumption of an infinite heat transfer coefficient between the wall and the fluid. Such a system can be described by a single equation[5] derived from energy balance over a differential element of the toreque tube (*Figure 1*):

$$kA_k \frac{d^2T}{dx^2} - \dot{m}C_p \frac{dT}{dx} = 0 \tag{3}$$

where T represents both the fluid and the wall temperatures. Under the assumption of an infinite heat transfer coefficient there can be no difference between the two temperatures. A_k, the wall cross section and k, the thermal conductivity of the wall are assumed constant. Equation (3) is subject to the boundary conditions:

$$T = T_c \text{ at } x = 0$$
$$T = T_h \text{ at } x = l \tag{4}$$

The axial heat flow rate is described by the Fourier equation:

$$q = kA_k \frac{dT}{dx} \tag{5}$$

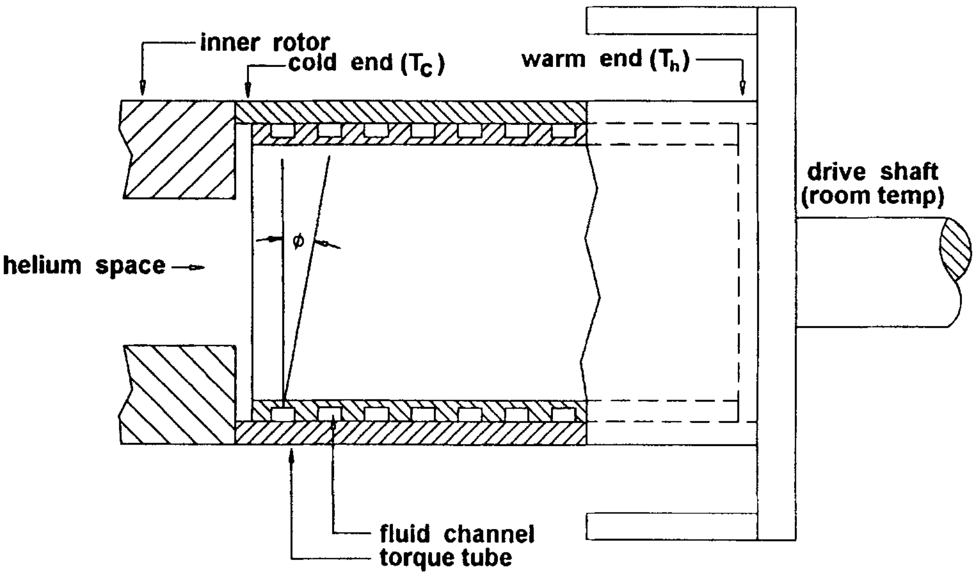

Figure 1 Schematic diagram of a torque tube

q being measured along the −ve x direction (hence the +ve sign). Equations (3) and (5) can be rewritten in dimensionless form as

$$\frac{d^2\tau}{d\xi^2} - Q_{c_{min}} \Psi \frac{d\tau}{d\xi} = 0 \tag{6}$$

and

$$Q = \frac{d\tau}{d\xi} \tag{7}$$

by using Equations (1) and (2), where

$$\xi = \frac{x}{l} \text{ (dimensionless length coordinate)}$$

$$\tau = \frac{T - T_c}{T_h - T_c} \text{ (dimensionless temperature)}$$

$$Q = \frac{q}{q_{c_{max}}} \text{ (dimensionless heat flux)}$$

$$\Psi = \frac{C_p(T_h - T_c)}{L} \text{ (dimensionless temperature range)} \tag{8}$$

As it has been assumed that the heat transfer is ideal, the heat inleak to the cold space will be the minimum possible. For this reason the heat leak ratio at the cold end is $Q_{c_{min}}$. The dimensionless temperature range Ψ is a key parameter in the operation of the torque tube exchanger. The boundary conditions (4) now reduce to:

$$\tau = 0 \text{ at } \xi = 0$$

$$\tau = 1 \text{ at } \xi = 1 \tag{9}$$

Using the method of integrating factor, the solution of Equation (6) is given in the form:

$$\tau = A_1 + A_2 e^{Q_{c_{min}} \Psi \xi} \tag{10}$$

A_1 and A_2 are constants of integration and can be obtained by using the boundary conditions. Differentiating Equation (10),

$$Q = \frac{d\tau}{d\xi} = A_2 Q_{c_{min}} \Psi e^{Q_{c_{min}} \Psi \xi} \tag{11}$$

Using the boundary conditions in Equations (10) and (11), we get

$$\tau = \frac{e^{Q_{c_{min}} \Psi \xi} - 1}{e^{Q_{c_{min}} \Psi} - 1} \tag{12}$$

and

$$Q = \frac{Q_{c_{min}} \Psi}{e^{Q_{c_{min}} \Psi} - 1} e^{Q_{c_{min}} \Psi \xi} \tag{13}$$

Equations (12) and (13) perfectly describe the temperature and heat-flux profiles in the exchanger as a function of the dimensionless length coordinate ξ. By substituting $\xi = 0$ in Equation (13), using the definition $Q_c = Q_{(\xi=0)}$, and solving the resulting equation for $Q_{c_{min}}$, we get:

$$Q_{c_{min}} = \frac{\ln(1 + \Psi)}{\Psi} \tag{14}$$

From Equation (14) it is clear that $Q_{c_{min}}$, the dimensionless cold end heat flow rate, is a function of the dimensionless parameter Ψ only. *Figure 2* shows the relationship between $Q_{c_{min}}$ and Ψ. Only a few discrete values of Ψ corresponding to common values of warm and cold end temperatures are important. They are summarized in *Table 1*.

Real cooling model

When the heat transfer coefficient between the torque tube wall and the fluid is finite, we define two temperatures T_w and T_f instead of the single temperature T used in the ideal model. Energy balance over differential wall and fluid elements gives the following equations:

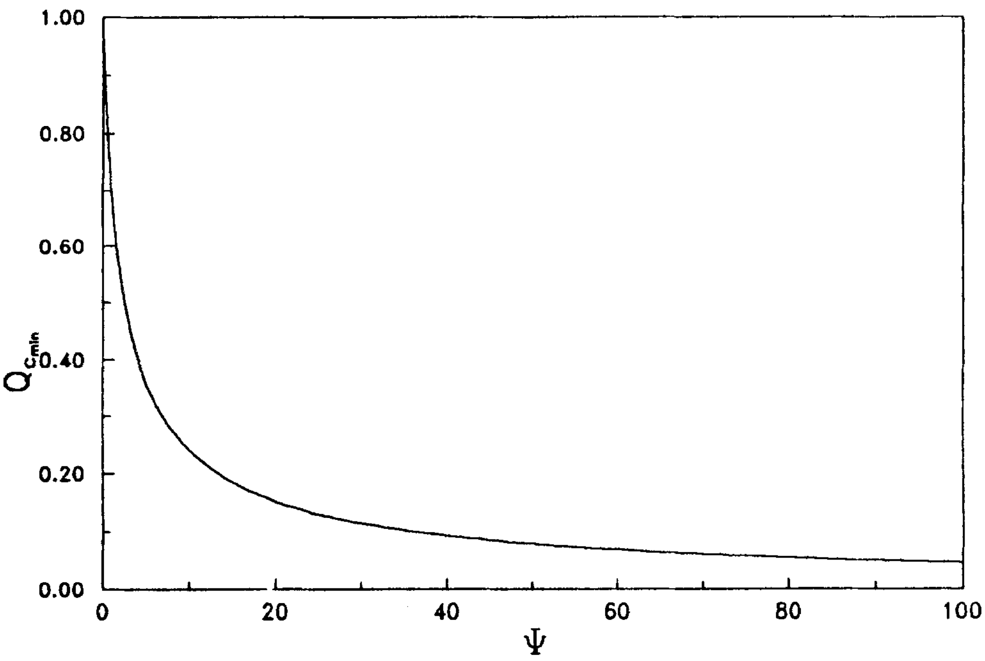

Figure 2 Variation of the dimensionless cold end heat inleak rate $Q_{c_{min}}$ with respect to the temperature range parameter Ψ

Table 1 Values of Ψ and $Q_{c_{min}}$ likely to be met in practice

T_h(K)	T_c(K)	Ψ	$Q_{c_{min}}$
80	4.2	18.90	0.158
300	4.2	73.65	0.059
350	4.2	86.00	0.052

$$k\,A_k\frac{d^2T_w}{dx^2} - hA'_c(T_w - T_f) = 0 \tag{15}$$

and

$$hA_c(T_w - T_f) = \dot{m}C_p\frac{dT_f}{dx} \tag{16}$$

with the boundary conditions:

$$\begin{aligned} T_w &= T_c \text{ at } x = 0 \\ T_f &= T_c \text{ at } x = 0 \\ T_w &= T_h \text{ at } x = l \end{aligned} \tag{17}$$

It should be noted that while $\dot{m}$ is constant over the length coordinate x, it is related to the cold heat inleak q_c by the relation given in Equation (1). This is the principal difference between the present derivation and conventional heat exchanger relations. Here q_c, and hence $\dot{m}$, is an unknown, to be determined from the overall solution of the problem. Using Equations (1), (2) and (8), we can express Equations (15), (16) and (17) in dimensionless form:

$$\frac{d^2\tau}{d\xi^2} - \Lambda(\tau - \theta) = 0 \tag{18}$$

and

$$\frac{d\theta}{d\xi} - \frac{\Lambda}{Q_c\Psi}(\tau - \theta) = 0 \tag{19}$$

with

$$\begin{aligned} \theta &= 0 \text{ at } \xi = 0 \\ \tau &= 0 \text{ at } \xi = 0 \\ \tau &= 1 \text{ at } \xi = 1 \end{aligned} \tag{20}$$

where

$$\Lambda = \frac{hA'_c l^2}{kA_k} \text{ (dimensionless cooling parameter)}$$

$$\theta = \frac{T_f - T_c}{T_h - T_c} \text{ (dimensionless fluid temperature)} \tag{21}$$

From Equation (19)

$$\tau = \theta + \frac{Q_c\Psi}{\Lambda}\frac{d\theta}{d\xi} \tag{22}$$

which is substituted in Equation (18) to give

$$\frac{d^3\theta}{d\xi^3} + \frac{\Lambda}{Q_c\Psi}\frac{d^2\theta}{d\xi^2} - \Lambda\frac{d\theta}{d\xi} = 0 \tag{23}$$

The solution of Equation (23) can be written as

$$\theta = A_1 + A_2e^{m_2\xi} + A_3e^{m_3\xi} \tag{24}$$

where

$$m_2, m_3 = \frac{-\frac{\Lambda}{Q_c\Psi} \pm \left[\left(\frac{\Lambda}{Q_c\Psi}\right)^2 + 4\Lambda\right]^{1/2}}{2} \tag{24a}$$

Substituting the expression for θ in Equation (22)

$$\tau = A_1 + A_2\left(1 + \frac{m_2Q_c\Psi}{\Lambda}\right)e^{m_2\xi} + A_3\left(1 + \frac{m_3Q_c\Psi}{\Lambda}\right)e^{m_3\xi} \tag{25}$$

The constants A_1, A_2, A_3 and the parameter Q_c can be obtained from the boundary conditions (20). Using the first two boundary condition from Equation (20) and the definition of Q_c from Equation (7),

$$A_1 = \frac{\Lambda}{\Psi}\frac{1}{m_2 m_3}$$

$$A_2 = \frac{\Lambda}{\Psi}\frac{1}{m_2(m_2 - m_3)}$$

$$A_3 = \frac{\Lambda}{\Psi}\frac{1}{m_3(m_3 - m_2)} \qquad (26)$$

where m_2 and m_3, defined in Equation (24a) contain the parameter Q_c. Now the variables θ, τ and Q can be cast in terms of the parameters Λ, Ψ and Q_c as

$$\theta = \frac{\Lambda}{\Psi}\left[\frac{1}{m_2 m_3} + \frac{e^{m_2\xi}}{m_2(m_2-m_3)} + \frac{e^{m_3\xi}}{m_3(m_3-m_2)}\right] \qquad (27)$$

$$\tau = \frac{\Lambda}{\Psi}\left[\frac{1}{m_2 m_3} + \frac{1+\frac{m_2 Q_c \Psi}{\Lambda}}{m_2(m_2-m_3)}e^{m_2\xi} + \frac{1+\frac{m_3 Q_c \Psi}{\Lambda}}{m_3(m_3-m_2)}e^{m_3\xi}\right] \qquad (28)$$

$$Q = \frac{\Lambda}{\Psi}\left[\frac{1+\frac{m_2 Q_c \Psi}{\Lambda}}{(m_2-m_3)}e^{m_2\xi} + \frac{1+\frac{m_3 Q_c \Psi}{\Lambda}}{(m_3-m_2)}e^{m_3\xi}\right] \qquad (29)$$

The exact value of Q_c can be calculated from Equation (28) by using the third boundary condition: $\tau = 1$ at $\xi = 1$. These functions are implicit in Q_c and can be solved by routine numerical techniques. It may be noted that Q_c lies between $Q_{c_{min}}$ (infinite convective heat transfer coefficient) and $Q_{c_{max}}$ (uncooled torque tube). Numerically, Q_c lies between $\ln(1+\Psi/)/\Psi$ and 1. So the Bisection method[6] can be conveniently used to solve Equations (28) and (24a) at $\xi = 1$.

Computed values of Q_c have been plotted in *Figure 3* as a function of Λ for Ψ = 18.9, 73.65 and 86.0 (see *Table 1*). In all three cases, as Λ increases Q_c decreases monotonically and asymptotically approaches $Q_{c_{min}}$. Changes in Q_c are imperceptible beyond Λ values of 300. This means it is futile to increase the heat transfer area density beyond a certain value.

Variable heat transfer coefficient

While deriving the above relations, a constant heat transfer coefficient has been assumed, but in reality h depends on the mass flow rate of the cooling gas. In order to simulate such a real process, a new variable Λ^* is introduced:

$$\Lambda = \Lambda^*\left(\frac{h}{h^*}\right) \qquad (30)$$

where h^* = heat transfer coefficient when the fluid flow rate $\dot{m} = q_{c_{max}}/L$ and

$$\Lambda^* = \frac{h^* A_c l}{k A_k} \qquad (31)$$

The relationship between h and $\dot{m}$ is generally given as

$$h \propto \dot{m}^n \qquad (32)$$

where n takes a value between 0 and 1 depending on the degree of turbulence. Hence using Equations (32) and (1) Λ can be written as

$$\Lambda = \Lambda^* Q_c^n \qquad (33)$$

Equation (28) can now be reframed in terms of Λ^* and n by using Equation (33). The use of Λ^* is advantageous over that of Λ, because unlike the latter, Λ^* can be readily calculated *a priori* for a given geometry. Λ, on the other hand, requires $\dot{m}$ or Q_c, which is unknown. *Figure 4* shows computed values of Q_c as a function of Λ^* and n for Ψ = 73.6. Three values of n have been used: 0 (constant heat

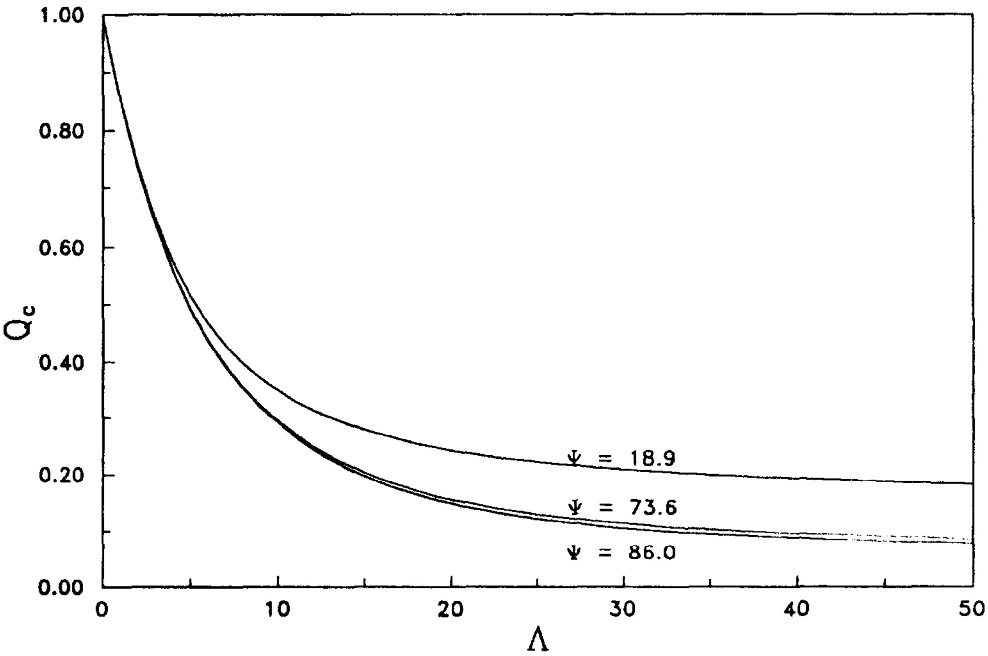

Figure 3 Variation of cold end heat inleak ratio Q_c with dimensionless cooling parameter Λ and temperature range parameter Ψ

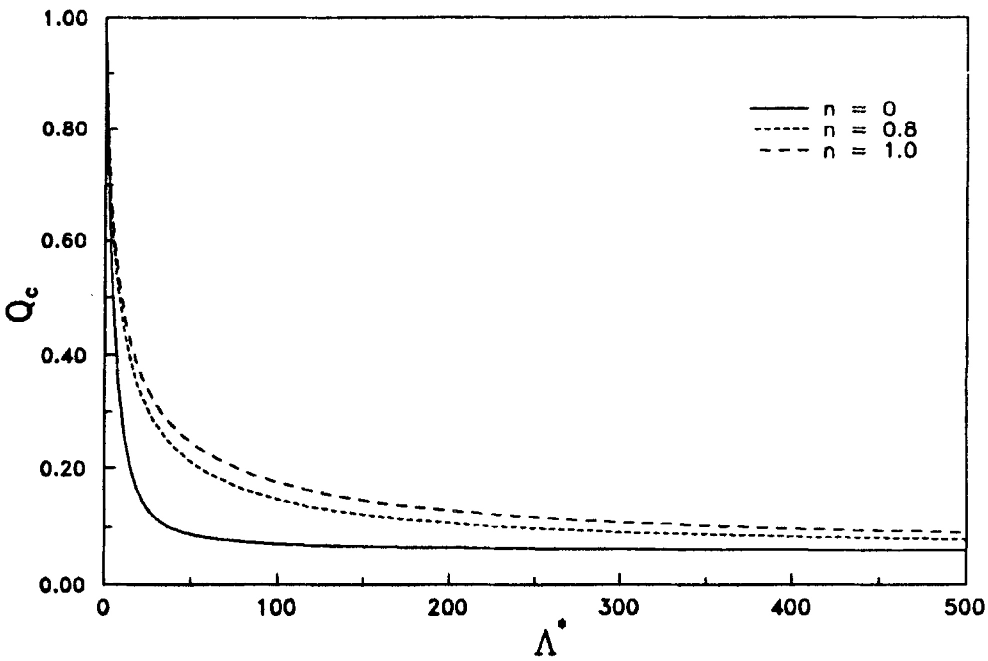

Figure 4 Variation of cold end heat inleak ratio Q_c with Λ^* and n for Ψ = 73.6

transfer coefficient), 0.8 and 1.0. Detailed results for other values of Ψ have been presented elsewhere[7].

Effect of vapour flow rate

The results presented so far describe the performance of the torque tube heat exchanger in self-sustained condition. The actual vapour flow rate in the exchanger, is, however, often quite different from this quantity because of additional generation and/or diversion of boiloff vapour. To account for this change in flow rate, a parameter β is introduced, defined by the relation

$$\dot{m} = \beta \dot{m}_{ss} \tag{34}$$

where $\dot{m}_{ss}$ = flow rate in self-sustained condition. Using equation (34)

$$\Lambda = \Lambda^* \beta^n Q_c^n \tag{35}$$

The performance of the torque tube exchanger under varying values of the mass flow rate parameter β has been presented in *Figure 5*.

The use of the design chart

The design chart presented in *Figures 3* to *5* can be used for rating and sizing of torque tube exchangers, as well as other vapour-cooled cryogenic support structures. As an example, let us consider a 300 MVA superconducting generator. From structural analysis[8,2], the torque tube cross section has the following geometry:

length	= 0.25 m
mean radius	= 0.29 m
wall thickness	= 0.055 m
cross-section area	= 0.04950 m^2

The heat transfer area to be provided depends on the cooling arrangement and the maximum pressure drop allowed in the torque tube. For the purpose of analysis it is assumed that the convective heat transfer area is 0.5 m^2. Thermal conductivity of the torque tube material generally depends on temperature. Following Chowdhury and Sarangi[9], let us use an average conductivity defined by the formula:

$$\frac{1}{k_{av}} = \frac{1}{T_h - T_c} \int_{T_c}^{T_h} \frac{dT}{k(T)}$$

The conductivity of stainless steel at different temperatures is taken from Johnson[10]. For T_h = 300 K and T_c = 4.2 K, k_{av} = 4.39 W m^{-1} K^{-1}.

From the above data, we get

$$q_{c_{max}} = 285.8 \text{ W}$$

$$h^* = 7012.75 \text{ W m}^{-2} \text{ K}^{-1}$$

$$\Lambda^* = 3630$$

From *Figure 5*, the cold end heat inleak ratio Q_c is 0.061. Hence the cold end heat inleak rate q_c ($= Q_c q_{c_{max}}$) is 17.43 W. A detailed numerical solution[7] of the governing equations with temperature-dependant properties predicts a cold heat leak rate of 15.8 W.

Conclusion

The analysis of the torque tube heat exchanger is simplified by the introduction of the parameters Λ^* and Ψ. The parameter Λ^* takes into account the torque tube geometry and represents the ratio of the convective heat transfer and the axial conduction rates, while Ψ stands for the operating temperature range suitably weighted by thermophysical properties of the flowing fluid. These two parameters, which can be computed early in a design or rating problem, completely describe the performance of the exchanger. The performance of the exchanger, i.e. the cold end heat inleak and resulting liquid boiloff can be read from the design

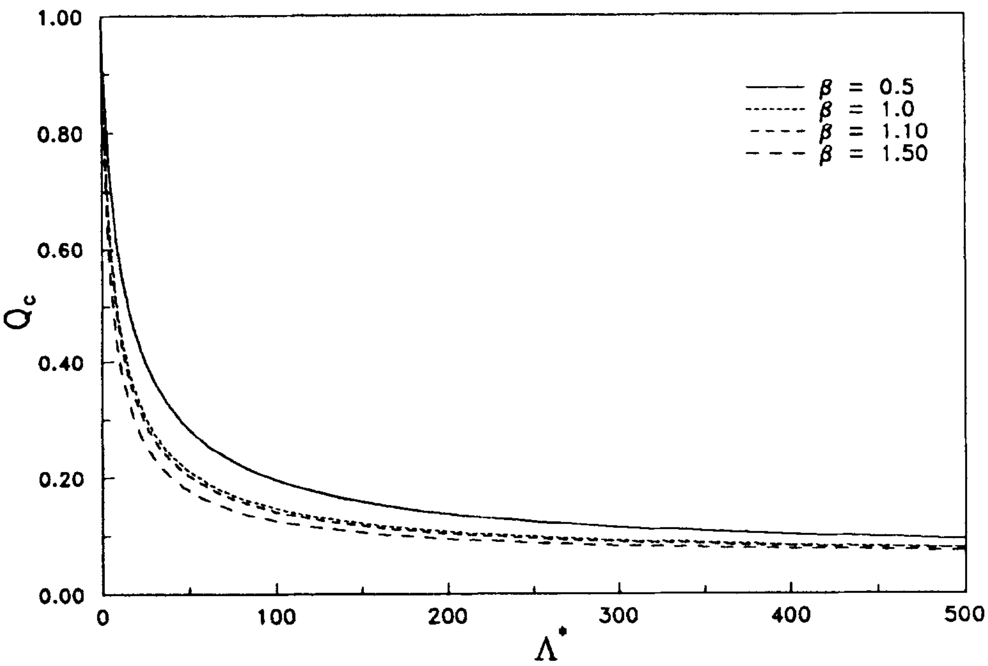

Figure 5 Variation of cold end heat inleak ratio Q_c with Λ^* and β for Ψ = 73.6 and n = 0.8

chart. This technique not only provides a new approach to analysis of a unique class of heat exchangers, but provides an easy and readily usable tool for analysis of torque tubes and other cryogenic vapour-cooled support structures.

References

1 **Bejan, A. and Smith, J.L. Jr.** Thermodynamic optimization of mechanical supports for cryogenic apparatus *Cryogenics* (1974) **14** 158–163

2 **Parker, J.H. Jr, Towne, R.A. and Rabinowitz, R.** Superconducting generator design, Technical report, Electric Power Research Institute (1977)

3 **Sato, K., Kumagai, M., Ito, K., Watanabe, Y. and Gocho, Y.** An approach to optimal design of superconducting generator rotor *IEEE Trans Energy Conversion* (1986) **1** 217–224

4 **Kirtley, J.L., Smith, J.L. Jr. and Umans, S.D.** Cryogenic isolating torque tubes for a superconducting generator detailed model and performance analysis *IEEE Trans Energy Conversion* (1991) **6** 267–272

5 **Sreedhar, R.** Thermal modelling of superconducting electrical generators, M. Tech thesis, Indian Institute of Technology, Karagpur (1992)

6 **Rajaraman, V.** *Computer Oriented Numerical Methods*, 2nd edn Prentice Hall of India, New Delhi (1989)

7 **Sahoo, P.K.** *Thermal and hydrodynamic modelling of superconducting AC generators*, PhD thesis, Indian Institute of Technology, Kharagpur, (1995) in preparation

8 **Bejan, A.** *Improved thermal design of the cryogenic cooling system for a superconducting synchronous generator*, PhD thesis, Massachusetts Institute of Technology (1974)

9 **Chowdhury, K. and Sarangi, S.** Effect of finite thermal conductivity of separating wall of the counter flow heat exchangers *Cryogenics* (1983) **23** 212

10 **Johnson, V.J.** A compendium of the properties of materials at low temperature (phase-I): Part-II. Properties of solids, Technical report, National Bureau of Standards, Cryogenic Engineering Laboratory (1960)