Non-linear Channel Equalization using Computationally Efficient Neuro-Fuzzy Channel Equalizer

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ABSTRACT

This paper investigates the problem of channel equalization in digital cellular radio (DCR). These channels are affected by inter symbol interference (ISI) with non-linearity in presence of additive white Gaussian noise (AWGN). Here we propose a computationally efficient neuro- fuzzy system based equalizer for use in communication channels with these anamolies. This equalizer performs close to the optimum maximum a-posteriori probability (MAP) equalizer with a substantial reduction in computational complexity and can be trained with supervised scalar clustering algorithm. These features can make the equalizer very suitable for mobile communication applications. Simulation studies indicate that this equalizer performs close to optimal equalizer.

1 Introduction:

The baseband model of the Digital communication systems (DCS) discussed in this paper is presented in Figure1. The channel constituite the physical channel through which signal propapgates, the transmitter and receiver filters and amplifiers used in them. The channel suffer from inter symbol interference (ISI) due to non-ideal channel characteristics. The problem becomes more severe in the presence of aditive white Gaussian noise (AWGN). A digital equaliser is called for in these circumstances to equalise the channel so as to recover the signal from the corrupted received signal with minimum error probability. The optimal equaliser decision function in these circumstances is provided by the maximum likelihood sequence estimator (MLSE) implemented with Viterbi algorithm. Implementation of Viterbi algorithm needs channel estimate and the equalizer performance improves with increase in decision delay. In mobile communication environment channel is changing with time and this limits the equalizer performance. MLSE is computationally complex and hence symbol-by-symbol spaced equalisers are often used instead. The most popular form of a symbol-by-symbol spaces equaliser is an adaptive filter trained with a suitable algorithm. Additionally when communication channel suffers from non-linearities channel estimation becomes very difficult and hence the performance of equalizer requiring channel estimation suffers.

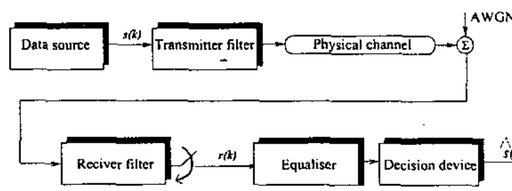

Figure 1: *A digital communication system*

The communication system discussed here is depicted in Figure.1 .The information symbol $s(k)$ is transmitted through a non-linear dispersive channel described by

$$r(k)=\sum_{i=0}^{n_h-1} a_i s(k-i)+K_1\left[\sum_{i=0}^{n_h-1} a_i s(k-i)\right]^3+e(k) \quad (1)$$

Here $s(k)$ forms the transmitted signal constellation taken from a set of alphabet $+1/-1$, r (k) is the received channel vector with its i th scalar components denoted by $r(k-i)$, n_h is the channel tap length, a_i are the channel impulse response coefficients, K_1 forms the non-linear term of third order and other non-linear terms have been neglected, $e(k)$ is the AWGN and $\hat{s}(k-d)$ forms the estimated sample corresponding to transmitted sample $s(k-d)$ and d is the equaliser decision delay. The non-linearity in a channel can be due to non-linearities associated with non-linear devices used in transmitter and receiver.

This channel can also represented by the channel transfer function

$$C(z)=H(z)+K_1 H^3(z)+e(k) \quad (2)$$

where

$$H(z)=\sum_{i=0}^{n_h-1} a_i z^{-i} \quad (3)$$

The optimum symbol-by-symbol spaced equaliser decision function is provided by the maximum a-posteriori probability criteria and is called Bayesian equaliser. The equaliser

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0-7803-7569-6/02/$17.00

decision function using a Bayesian equaliser is nonlinear and hence the performance of linear equalisers is far from optimal. For this reason several nonlinear equalisation techniques using artificial neural networks [1], radial basis functions (RBFs)[2] , recurrent neural networks [3], fuzzy filters [4] have been developed successfully. The work reported in this paper is an extension of the work reported in [5]. In [5] fuzzy implementation of Bayesian equaliser was proposed for linear channels. Here we present the performance of the same equalizer for non-linear channels.

The paper is organised in 6 sections. Following the introduction, the RBF equalizer and its fuzzy implementation is discussed, following this the channel state estimation has been discussed. Section 4 discusses the neuro fuzzy equalizer design while the advantages are discussed next. Simulation results have been presented in section 6 and the last section provides the concluding remarks.

2 RBF Equaliser and its Fuzzy Implementation [5]

The general symbol decision equalizer 1 is characterised by equalizer order m and delay d. The optimal decision function for this equalizer can be represented as [6]

$$\begin{aligned} f(\mathbf{r}(k)) &= \sum_{i=1}^{N_s^+} \exp\left(\frac{-\|\mathbf{r}(k)-\mathbf{c}_i^+\|^2}{2\sigma_e^2}\right) \\ &- \sum_{j=1}^{N_s^-} \exp\left(\frac{-\|\mathbf{r}(k)-\mathbf{c}_i^+\|^2}{2\sigma_e^2}\right) \end{aligned} \quad (4)$$

Here σ_e^2 represents the channel noise variance, c_i^+ and c_i^- are the positive and negative channel states respectively. The terms n_s^+ and n_s^- are the number of positive and negative channel states respectively and they are equal. Here it is assumed that the transmitted symbol $s(k)$ is binary taking the value from $+1/-1$. This equation can also be presented as :

$$f(\mathbf{r}(k)) = \sum_{i=1}^{N_s} w_i \exp\left(\frac{-\|\mathbf{r}(k)-\mathbf{c}_i\|^2}{2\sigma_e^2}\right) \quad (5)$$

Here N_s is the number of channel states equal to 2^{n_h+m-1}, w_i are the weights associated with each of the centers. w_i is +1 if $\mathbf{c}_i$ correspond to a positive channel state and -1 if it represents a negative channel state. It is also observed that each of the channel state vector has m components. We can represent any channel state $\mathbf{c}_i$ as $\mathbf{c}_i = [c_{i0}, c_{i1}, c_{i2}, ..., c_{i(m-1)}]$. Rewriting the squared norm in Eqn(5) as a summation and exploiting the properties of the exp function yields:

$$f(\mathbf{r}(k)) = \frac{\sum_{i=1}^{N_s} w_i \left\{\prod_{l=0}^{m-1} \phi_{il}\right\}}{\sum_{i=1}^{N_s} \left\{\prod_{l=0}^{m-1} \phi_{il}\right\}} \quad (6)$$

$$\psi_l^j = \exp\left\{-\frac{1}{2}\left(\frac{|r(k-l)-C_j|^2}{\sigma_\eta^2}\right)\right\} \quad (7)$$

$$\phi_{il} \in \psi_l^j \quad (8)$$

No.	*s(k)*	*s(k-1)*	*s(k-2)*	$\hat{r}(k)$	$\hat{r}(k-1)$	
1	1	1	1	1.5	1.5	Positive
2	1	1	-1	1.5	-0.5	channel
3	1	-1	1	-0.5	0.5	states
4	1	-1	-1	-0.5	-1.5	
5	-1	1	1	0.5	1.5	Negative
6	-1	1	-1	0.5	-0.5	channel
7	-1	-1	1	-1.5	0.5	states
8	-1	-1	-1	-1.5	-1.5	

Table 1: *The channel states calculation*

Here C_j is the $(j+1)$th component of channel state vector $\mathbf{c}_i$.

3 Channel States Estimation

From the previous section it has been observed that knowledge of the channel states is essential for evaluation of the optimum decision function for the equalizer. The channel state estimation needs the knowledge of the channel. But under most circumstances knowledge of the channel may not be available. Additionally estimation of channels for non-linear channels is very difficult. Under these circumstances the channel states can be estimated during the training period when the transmitted symbols are known to the receiver with the aod of supervised clustering algorithm [6].

The channel states can be computed from the scalar channel states. The scalar channel states refers to the possible noise free received samples. The scalar states can be calculated by a clustering algorithm. Calculation of the scalar channel states is simple and computational complexity for this is independent of the order of the equalizer. These scalar states can be suitably combined in the fuzzy if-then rule for generate the channel states. Once the channel state vectors have been estimated finding the decision function of the equalizer is straightforward.

We take an example to illustrate the relationship of scalar and vector channel states. Table 1 provides the channel state calculation for a equalizer of order $m = 2$, delay $d = 0$. The channel transfer function is $H(z) = 0.5 + z^{-1}$. For the analysis we assume that the channel is linear. Here n_h is 2. Following observations are made from the channel state calculation:

- There are 2^{n_h+m-1}=8 vector channel states which can be represented as $[\hat{r}(k), \hat{r}(k-1)]$.
- There are 2^{n_h}=4 possible scalar channel states which correspond to each of the elements of $\hat{r}(k)$ or $\hat{r}(k-1)$.
- The weights w_i of the decision functions eqns.5 assume the value $+1$ or -1 for positive and negative states respectively.
- A change in the decision delay only changes some of the positive states to negative states and equal number of negative states to positive state. The decision function can be obtained by suitable adjustment of the parameter p_i for the states that have changes from positive to negative states or vice-verse.

4 Neuro Fuzzy Equalizers with Scalar Centers

The structure of the fuzzy equaliser is presented in Figure 2. Here, the incoming signal sample is presented to the membership function generator. Each of the components of the membership function generator produces an output ψ_i^j, characterised by its centres C_i^j which are positioned at the scalar channel states. Here j represents the fuzzy centre at the scalar channel states. The membership functions from $r(k-i)$, $1 \le i \le m-1$ are generated by passing the membership function from $r(k)$ through a TDL.

The inference block of the equaliser has N_s fuzzy *if then* rules with product inference and the rule base is generated from the information of the combination of scalar channel states forming the channel states. Each of these rules uses only one of the ψ_i^j terms corresponding to each of the m inputs to the equaliser. The output of the inference units are suitably weighted and added to provide a and b which provide the function of the defuzzifier. The output of the equaliser is computed by the equaliser function presented in (2) which is $(a-b)/(a+b)$. The output of the decision function passed through a hard limiter to forms the detected sample. An example is considered to illustrate the working of this equaliser:

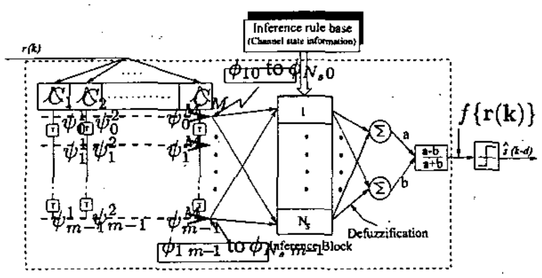

Figure 2: Structure of fuzzy implemented Bayesian equaliser

Example:

The channel considered here is $H(z) = 0.5 + 1.0z^{-1}$. The equaliser is characterised by $m = 2$, $d = 0$ and SNR= 8 dB. This provides $N_s = 8$ channel states and $M = 4$ scalar channel states. The channel states for this equaliser have been presented in Table 3. It is also seen that the m-dimensional N_s channel states take their components from the available M scalar channel states. The weights w_i of the equaliser decision function are $+1$ for $\mathbf{c}_1$, $\mathbf{c}_2$, $\mathbf{c}_3$, $\mathbf{c}_4$ and -1 for $\mathbf{c}_5$, $\mathbf{c}_6$, $\mathbf{c}_7$, $\mathbf{c}_8$.

For fuzzy implementation the centres for membership functions are positioned at scalar channel states $+1.5$, -0.5, $+0.5$ and -1.5. The membership functions ψ_1^1, ψ_1^2, ψ_1^3 and ψ_1^4 corresponding to $r(k-1)$, are delayed samples of ψ_0^1, ψ_0^2, ψ_0^3 and ψ_0^4 corresponding to $r(k)$. The inference block consist of $N_s = 8$ fuzzy IF ... THEN ... rules. Here $\phi_{10} = \phi_{20} = \psi_0^1$, $\phi_{30} = \phi_{40} = \psi_0^2$, $\phi_{50} = \phi_{60} = \psi_0^3$, $\phi_{70} = \phi_{80} = \psi_0^4$, $\phi_{11} = \phi_{51} = \psi_1^1$, $\phi_{21} = \phi_{61} = \psi_1^2$, $\phi_{31} = \phi_{71} = \psi_1^3$, and $\phi_{41} = \phi_{81} = \psi_1^4$. The products $\phi_{10}\phi_{11}$, $\phi_{20}\phi_{21}$, $\phi_{30}\phi_{31}$, $\phi_{40}\phi_{41}$ constitute the rules for $\mathbf{C}_d^+$, are added to provide a and $\phi_{50}\phi_{51}$, $\phi_{60}\phi_{61}$, $\phi_{70}\phi_{71}$, $\phi_{80}\phi_{81}$ constitute the rules for $\mathbf{C}_d^-$ and are added to provide b. The calculation of the decision function is straightforward.

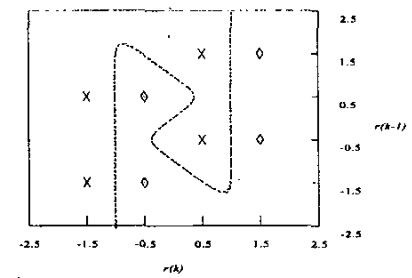

Figure 3: Fuzzy equaliser decision boundary for channel $H(z) = 0.5 + 1.0z^{-1}$ with $m = 2$, $d = 0$ and SNR= 8 dB.$\diamond$ positive channel states and $\times$ negative channel states with actual channel states

The decision boundary of this equaliser is presented in Figure 3. Figure 3(a) presents the decision boundary of the fuzzy equaliser and the Bayesian equaliser when the channel states and noise statistics are known, whereas in Figure 3(b) the fuzzy equaliser uses the estimated channel states and noise statistics and the Bayesian equaliser uses the true channel parameters. The positive and negative channel states are shown with $\diamond$ and $\times$ respectively. A study of the decision boundaries shows that, the fuzzy equaliser is able to provide a near optimal decision boundary even at a low SNR of 8 dB.

The fuzzy equaliser developed here, uses FBF with product inference and COG defuzzifier. Owing to the close relationship of this fuzzy equaliser with the Bayesian equaliser, the NBESS has been implemented using a RBF network with scalar centres [5]. However, the use of a fuzzy system to implement this equaliser provides the possibility of using other forms of inference rules and defuzzification processes. This can provide some of the alternate forms of fuzzy implementation of the Bayesian equaliser.

5 Advantages of Neuro Fuzzy Equalizer

We have seen in the previous sections that the neuro fuzzy equalizer provides the same decision function as the RBF implementation of Bayesian equaliser. Additionally they provide Bayesian equalizer implementation with computational complexity reduction. Some of the advantages associated with the neuro fuzzy equalizer for non-linaer channels are as under,

- the equalizer does not need channel identification;
- in mobile environment decision directed training can be employed to take care of the channel variation;
- provides scope for performance tradeoff with complexity;

6 Simulation Results

In order to demonstrate the performance of the proposed equalizer for non-linear channels following simulations were carried out. In all the tests $s(k)$ was an equiprobable random binay number taking the value from $+1/-1$. In the first test the decision boundary of the proposed equalizer was compared with MAP equalizer. For this $H(z) = 0.5 + z^{-1}$ was selected with non-linearity K_1 was taken as 0.9. The decision delay used was 0. The decision function provided by the fuzzy equalizer and the MAP equalizer are presented in Figure.4. It is seen that the fuzzy equalizer is able to provide the nearly similar decision function as the optimal equalizer.

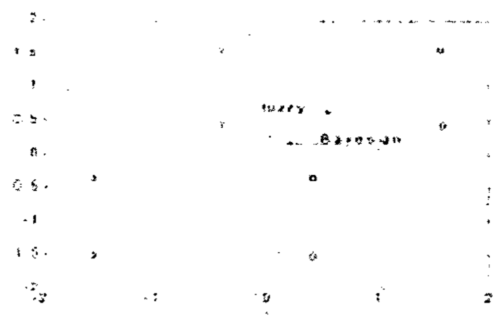

Figure 4: Fuzzy equaliser decision boundary

In the next part of simulation bit error rate (BER) was considered as the performance criterion. Equalizer order $m = 2$ with decision delay $d = 0$ was used. In the first case $H(z) = 0.5 + z^{-1}$ was used and subsequently $H(z) = 0.5 + 0.81z^{-1} + 0.31z^{-2}$ was used. In both cases channel non-linearity of 3rd order was used with $H_1 = 0.9$. The plot of the BER perdformance of the equalizers against the channel SNR for bothe the cases are presented in Figure. 5 and Figure. 6. From these it is observed that the neuro fuzzy filter performs very close to MAP equalizer.

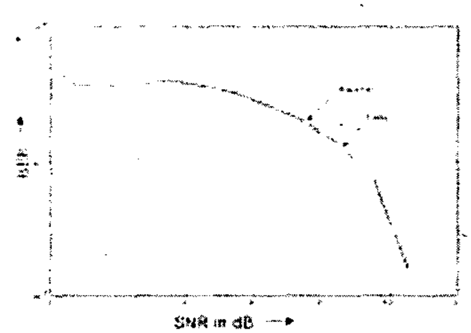

Figure 5: Fuzzy equalizer BER performance

7 Conclusion

The training of the proposed equalizer is very simple using supervised clustering algorithm. The performance of the equalizer has been demonstrated to be comparable to the optimal MAP equalizer. In line with the normalised radial basis function equalizer with scalar centres [7] this equalizer can also be usedful for co-channel interference suppression. We are investigating the performance of the equalizers for non-linear channels in presence of co-channel interference. We also plan to investigate the equalizer performance in fading

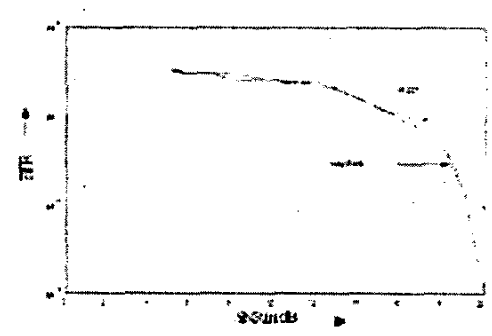

Figure 6: Fuzzy equaliser BER performance

envirinment. The work reported in this pape has been reported in [8].

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