Combining Tone Injection and Companding Techniques for PAPR Reduction of FBMC-OQAM System

Rahul Gopal
Dept.of.Electronics & Communication
National Institute of Technology
Rourkela, Odisha 769008
Email:rahulgopal3@gmail.com

Sarat Kumar Patra
Dept.of.Electronics & Communication
National Institute of Technology
Rourkela, Odisha 769008
Email:skpatra@nitrkl.ac.in

Abstract—**Filter Bank Multicarrier(FBMC) is a strong waveform contender for 5G communication because of it's very low out of band radiation and high spectral efficiency. Like OFDM, FBMC also suffers from high peak to average power ratio(PAPR) problem which reduces power amplifier efficiency. Here we used FBMC-OQAM(offset quadrature amplitude modulation) which is the most efficient method among all FBMC schemes. There are several PAPR reduction techniques. Tone Injection(TI) is one of the PAPR reduction technique in which we add equivalent constellation points to the original constellation point in such a way that PAPR will reduce without compromising spectral efficiency. Companding is another PAPR reduction technique in which compression of low amplitude signal and expansion of large amplitude signal in the transmitter side and reverse operation in the receiver side is performed to reduce the PAPR of signal.In this paper, we proposed a PAPR reduction scheme based on the combination of tone injection and companding techniques. The simulation results show that the combined method gives better result compared to TI and Companding methods taken separately.**

Keywords- Companding,FBMC-OQAM,PAPR,Tone Injection,

I. INTRODUCTION

Today we are using OFDM for MCM in 4G communication. As we move towards 5G communication, OFDM will not be a suitable candidate for 5G because it uses cyclic prefix(CP) which reduces spectral efficiency and also high out of band radiation is a serious problem with OFDM. In [1] studied roll of FBMC for 5G mobile communication and also advantages of FMBC over OFDM. FBMC-OQAM is most efficient for transmission of complex data which we used here as FBMC technique. Different pros and cons of FBMC and OFDM are compared in [2]. FBMC system is characterized by it's prototype filter. In [3] a survey on different prototype filters is carried out and design technique of prototype filter is illustrated in [4]. PAPR is a serious problem for any MCM system. FBMC system also suffers from PAPR problem. In [5] different PAPR reduction schemes are studied. Companding is one of the low complexity methods for PAPR reduction [6] and it do not decrease BER performance like clipping method does. Tone injection is another method for PAPR reduction [7] [8]. It is an iterative method in which a cyclically extended QAM constellation is used to reduce PAPR of signal. Effect of PAPR on power amplifier efficiency is given in [9]. In this paper, we proposed a PAPR reduction scheme by combining tone injection and $\mu-$law companding technique.

Paper is organized as follows: In section II, FBMC-OQAM system is described with it's prototype filter design and transmitter design. In section III, PAPR problem and it's reduction techniques tone injection(TI) and $\mu-$law companding are described.In section IV, the proposed combined PAPR reduction scheme is described. In section V, simulation results are shown and finally conclusions are drawn in section VI

II. FBMC-OQAM SYSTEM

Filter Bank Multicarrier system consist of a bank of filters in transmitter and receiver side. These filters are frequency and phase shifted version of a prototype filter. Prototype filter is the basis of FBMC system which separate two symbols in such a way that minimum out of band radiation occurs. Length of filter is given by L=KN. Where N is the number of subcarriers and K is overlapping factor. To maintain same theoretical throughput as that of OFDM, overlapping of symbols required. FBMC-OQAM system uses staggering technique to achieve maximum spectral efficiency. In OQAM real and imaginary parts of a complex symbol are time staggered by T/2, where T is a symbol period. These symbols are given to the synthesis filter where spacing between two subcarriers is 1/T. This is shown in Fig 1.

FBMC-OQAM signal is given as

$$d[n] = \sum_{m=-\infty}^{\infty} \sum_{k=0}^{n-1} \left(\theta_k \left\{x_m[k]\right\} p[n-mn] + \left(\theta_{k+1} im \left\{x_m[k]\right\} p[n-mn-\tfrac{n}{2}]\right)\right) e^{jk(n-mn)\frac{2\pi}{n}} \tag{1}$$

Where

$$\theta_k = \begin{cases} 1, & \text{if k is even} \\ j, & \text{if k is odd} \end{cases} \tag{2}$$

$X_m[k]$ = complex input symbol at k^{th} subcarrier at time m.
P[n] = prototype filter

A. Prototype Filter Design

Prototype filter for FBMC-OQAM system is designed by using a windowing technique with PHYDYAS filter.

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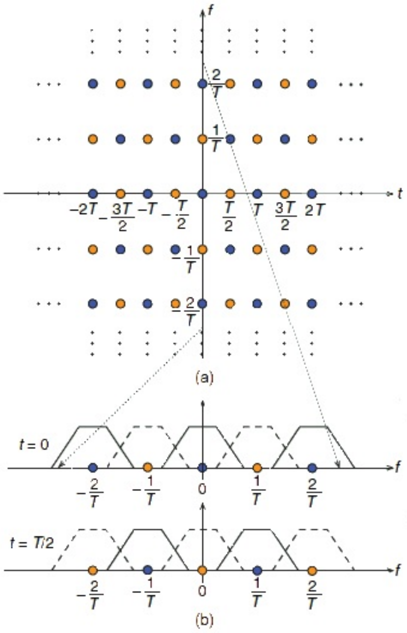

Fig. 1: (a)Frequency-time lattice representation of FBMC-OQAM symbols (b)Demonstration of the subcarriers spectra for t=0 and t=T/2

Blackman-Harris window is taken as a window function because it gives around 90 dB sidelobe reduction. Blackman-Harris window is given by

$$w(n) = a_o - a_1 cos\left(\frac{2\pi n}{N-1}\right) + a_2 cos\left(\frac{4\pi n}{N-1}\right) - a_3 cos\left(\frac{6\pi n}{N-1}\right) \tag{3}$$

where

$$a_o = 0.35875, a_1 = 0.48829, a_2 = 0.14128, a_3 = 0.01168$$

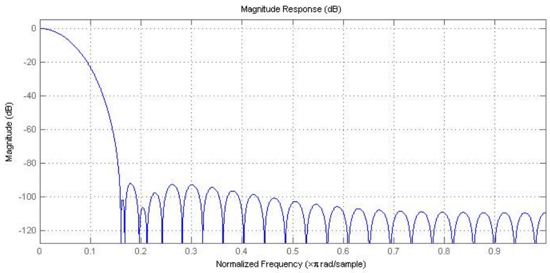

Fig. 2: Frequency response of the prototype filter using Blackman-Harris window.

PHYDYAS filter was originally designed by Bellanger [4] and later it was chosen to be the reference prototype filter of the European project PHYDYAS The impulse response of the PHYDYAS filter is given as [4]

$$h(t) = \begin{cases} \frac{1}{\sqrt{A}}[1 + 2\sum_{k=1}^{K-1}(-1)^k H_k cos(\frac{2\pi kt}{KT})] & t \in [0, KT] \\ 0 & \text{elsewhere} \end{cases} \tag{4}$$

where A is normalization constant

$$A = KT\left[1 + 2\sum_{k=1}^{K-1} H_k^2\right] \tag{5}$$

In our filter, filter coefficients for PHYDYAS filter are calculated by using Blackman-harris window technique to reduce out of band radiation.

B. *Transmitter and Receiver design*

A basic FBMC Transmitter given in [4] is shown in Fig.3. In the Transmitter side input complex data is first fed to OQAM pre-processing block, where it first converted from complex to real then multiplied with $\theta_{k,n}$ to get alternate real and imaginary valued symbols, So that orthogonality between neighbour symbols will maintain. Each sample modulates 2K-1 prototype filter coefficient. This is called weighted frequency spreading(WFS). Then these WFS signal is fed to IFFT of size KN to get FBMC-OQAM signal.

At the receiver side inverse operation is performed and i/p symbols are recovered by first weighted frequency despreading(WFDS) and then by OQAM post processing. WFDS depends on following property of Nyquist filter coefficients

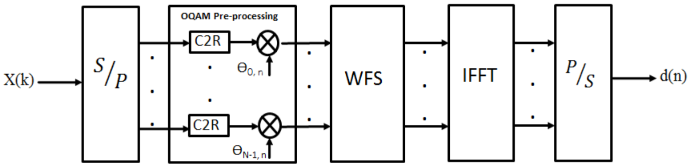

Fig. 3: Transmitter Block Diagram for FBMC-OQAM System.

$$\frac{1}{K}\sum_{k=-(K-1)}^{(K-1)} |H_k|^2 = 1 \tag{6}$$

III. PEAK TO AVERAGE POWER RATIO (PAPR)

High PAPR is an important issue in FBMC system which reduces the efficiency of power amplifier used in circuit. PAPR problem in any MCM system arises because of the fact that output symbol of MCM system is the summation of symbols modulated on different subcarriers and there is a probability that all symbols have same phase which leads to a very high peak compared to the average value of the symbol. PAPR of an FBMC system is defined as the ratio of peak power to the average power.

In general, the PAPR of a complex envelope d[n] with length N can be written as

$$PAPR = \left(\frac{max\{|d[n]|^2\}}{E\{|d[n]|^2\}}\right) \tag{7}$$

Where d[n] is amplitude of d[n] and E. denote the expectation of the signal. PAPR in dB can be written as:

$$PAPR(dB) = 10log10(PAPR) \tag{8}$$

The complementary cumulative density function(CCDF) is the probability that PAPR exceeds some threshold value. CCDF plot is used to measure the PAPR performance of PAPR reduction technique.

CCDF = Probability(PAPR>PAPR$_o$) (9)

In FBMC, as we increase the number of subcarrier PAPR also increases like in OFDM. Some of the PAPR reduction techniques that can be used in FBMC-OQAM system are discussed below.

A. COMPANDING TECHNIQUE

In companding technique, we enlarge the small signals while compressing the large signals to increase the immunity of small signals from noise. This compression is carried out at the transmitter end, after the output is taken from IFFT block There are two types of companders: $\mu-$law and A-law companders. In this paper, $\mu-$law compander is used because it gives better BER result compared to A-law compander. For a given input x, $\mu-$law encoding is:

$$F(x) = sgn(x)ln(1 + \mu\,|x|)/ln(1 + \mu), -1 \leq x \leq 1 \tag{10}$$

Where μ is the companding parameter (μ=255 is used, as it is the accepted U.S. and Japan standard for voice compander). The inverse operation is carried out at the receiver means compressing the small signals and enlarging the large signals to get back the original signal before giving the signals to the analysis filter bank. $\mu-$law expansion is then given by:

$$F^{-1}(y) = sgn(y)(\frac{1}{\mu})((1 + \mu)^{|y|} - 1), -1 \leq y \leq 1 \tag{11}$$

where y is the received signal.

B. TONE INJECTION

The basic idea is to increase the constellation size so that each of the points in the original basic constellation can be mapped into several equivalent points in the expanded constellation. Since each symbol in a data block can be mapped into one of several equivalent points, these extra degrees of freedom can be exploited for PAPR reduction. In this method, one tone is injected with proper phase and frequency in the symbol that corresponds to adding one of these equivalent points with original constellation point. Hence, these additional constellation points can be used to generate FBMC symbols with low PAPR. Fig.4 illustrate the extended constellation for 4-QAM, where one point in the original constellation can be replaced by any one of its equivalent point, these equivalent points are spaced in real and/or imaginary axes by the extension size D. So, any one of the nine equivalent points can be used to obtain signals with lower PAPR. If, for an M-ary QAM, d is the minimum distance between signal points, constant D must be equal to or larger than $d\sqrt{M}$ so that adding equivalent constellation point does not increase BER.

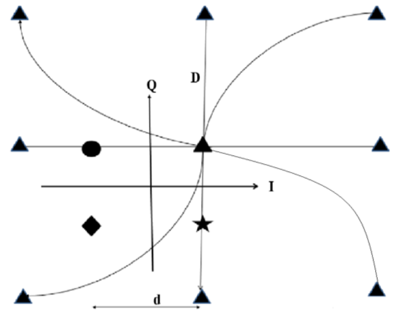

Fig. 4: Cyclically extended 4-QAM constellation diagram.

Adding these equivalent constellation points to the original constellation point has effect of increasing the transmitted power. However, probability that very large peaks occur is very less so overall average power will not increase significantly due to these tone modifications. After tone injection, the modified transmit signal is given by

$$\begin{aligned} s^*[n] &= s[n] + c[n] \\ &= \tfrac{1}{\sqrt{N}} \textstyle\sum_{k=0}^{N-1} (S_k + C_k) e^{j2\pi kn/N} \end{aligned} \tag{12}$$

where the extension vector C_k is defined as

$$C_k = D(p_k + jq_k) \tag{13}$$

wherep_k and q_k take values from the set1,0,1 to get any one of the extension constellation point However,our problem is to find the optimal values of p_k and q_k, which will give lowest PAPR for s[n]. Here we used TI with aggressive clipping projection (TI-ACP) to achieve a very good and fast, suboptimal solution for finding preferred subcarriers and extension vectors for tone injection.
A clipped signal $s_{clip}[n]$ is defined as

$$s_{clip}[n] = s[n] + c_{clip}[n] \tag{14}$$

where $c_{clip}[n]$ represents the clipped-off portion of the signal. In the , $C_{clip}[k]$ is the frequency domain representation of $c_{clip}[n]$. To obtain the peak reduction signal c[n], $C_{clip}[k]$ is projected onto the extension vectors to determine C_k in (12) . The iterative signal can be given as

$$s^{i+1}[n] = s^i[n] + c[n] \tag{15}$$

exact extension direction can be determined by $C_{clip}[k]$, and the extension vector is Chosen to be the projection with the largest magnitude.
Extension vector C_k contains the largest magnitude vector among all the projected vectors, projecting onto the allowable extension vectors. values of p_k and q_k in (13) can be given by

$$\begin{aligned} p_k &= sgn(Re(C_{clip}[k])) \\ q_k &= sgn(Im(C_{clip}[k])) \end{aligned} \tag{16}$$

where sgn represents the signum function.
For single-tone modification, maximum possible peak reduction can be given as

$$|s[n]| - |s^*[n]| \leq |c[n]| \tag{17}$$

$$= \left| \frac{D}{\sqrt{N}} (p_k + jq_k) e^{j2\pi kn/N} \right| \tag{18}$$

$$= \frac{D}{\sqrt{N}} \sqrt{p_k^2 + q_k^2} \tag{19}$$

For the complex baseband signal maximum peak reduction per tone injection can be achieved by choosing $|p_k|$ = 1 and $|q_k|$ = 1, which is given by

$$\delta_{max} = \frac{D}{\sqrt{N}} \sqrt{2} \tag{20}$$

Secondary peaks may grow For some values of p_k and q_k, above the current peak level, which can increase PAPR value of the signal. For a single-tone injection δ_{max} is also the maximum possible growth for a time-sample magnitude. So at each iteration, samples which exceed $2\delta_{max}$ below the largest peak level in magnitude is thus sufficient to consider because only these samples can cause an increase in PAPR.

The TI-ACP Algorithm

1. First we start with data block s[n] and set i=0 for first iteration s^0=s.
2. TO get $c_{clip}[n]$ all $\left|s^i[n]\right| \geq A$ in magnitude is clipped

$$c_{clip}[n] = \begin{cases} 0 & \left|s^i[n]\right| \leq A \\ (A - \left|s^i[n]\right|) e^{j\theta[n]} & \left|s^i[n]\right| \geq A \end{cases} \tag{21}$$

Where A is clipping level and

$$s[n] = |s[n]| \, e^{j\theta[n]} \tag{22}$$

3. Obtain C_{clip} via an FFT applied to c_{clip}.
4. Select the extension vector $C_k$for each subcarrier by projecting C_{clip} onto the allowable extension directions

5.Find E^i, the largest-magnitude sample

$$E^i = \max_n \left|s^i[n]\right| \tag{23}$$

6. Find the subcarrier k_1 which gives maximum peak reduction:

$$k_1 = \min_k |s_1[n_1]|^2 \tag{24}$$

Where

$$s_1[n] = \begin{cases} 0 & \left|s^i[n]\right| < E^i - 2\delta_{max} \\ s^i[n] + \frac{1}{\sqrt{N}} C_k e^{j2\pi kn/N}, & \left|s^i[n]\right| \geq E^i - 2\delta_{max} \end{cases} \tag{25}$$

and n_1 is the peak point.

7. Compute

$$s^{i+1}[n] = s^i[n] + c[n] \tag{26}$$

Where

$$c[n] = \frac{1}{\sqrt{N}} C_{k_1} e^{j2\pi k_1 n/N} \tag{27}$$

8. Increase iteration index i by 1 and go to Step 2 unless we reach a maximum iteration count or we get an acceptable PAPR level.Otherwise, stop PAPR reduction.

IV. COMBINING COMPANDING AND TONE INJECTION

Here we propose a new PAPR reduction scheme by combining companding and tone injection techniques which can reduce the PAPR value significantly.The block diagram of proposed scheme is shown below. Here first input complex signal is given to OQAM pre-processing block where input signal is staggered in time to get OQAM signal, and then this signal is weighted frequency spreaded(WFS) and fed to IFFT block. After this signal is converted from parallel to serial to get FBMC-OQAM signal. This FBMC-OQAM signal is then pass through compander block and after that tone injection block for PAPR reduction. This signal is then converted from D/A and then pass through AWGN channel.

At the receiver side,the incoming signal is first converted from A/D. After that signal is decompanded and then given to modulo-D block to remove injected constellation point. After that signal is fed to FFT block and then signal is weighted frequency despreaded (WFDS) and then given to OQAM post-processing block to get back the original signal.

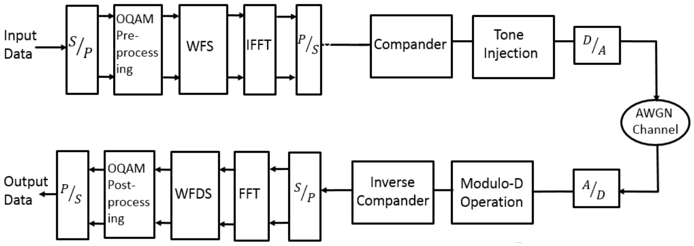

Fig. 5: Block Diagram of Proposed Scheme.

V. SIMULATION

Simulation results for different PAPR reduction scheme is shown in Fig.6 and Fig.7. In Fig.6 CCDF plot shows PAPR performance of original FBMC-OQAM signal, $\mu-$law companding and TI for 3 iterations. In Fig.7 CCDF plot shows PAPR performance of original FBMC-OQAM signal, TI with $\mu-$law companding for 3 iterations.For the simulation randomly generated 10^4 complex baseband symbols were modulated with 4-QAM and number of subchannels is fixed to 64. Overlapping factor K=4 is choosen, which gives IFFT length(KN)=512.Clipping level A is chosen to be 3 dB above average power. PAPR reduction of 5.9 dB is achieved with 3 tone injection in original FBMC-OQAM signal for 10^{-3} clip probability. With the combination of companding technique, this PAPR reduction reaches 12.7 dB so combined scheme gives 6.8 dB more PAPR reduction compared to TI technique.

A. Power Amplifier(PA) Efficiency Calculation

Operating point of PA should be adjusted to the average input power of the multicarrier signal in order to avoid clipping. An input back-off (IBO) is required in PA prior to amplification so that the input signal in the PA do not get distorted. The required amount of the IBO is closely related to the PAPR. For a linear class A amplifier, the PA efficiency η and the PAPR

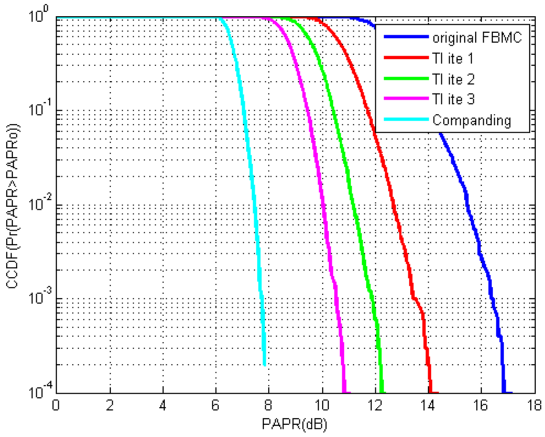

Fig. 6: CCDF plot for original FBMC signal,TI and Companding techniques.

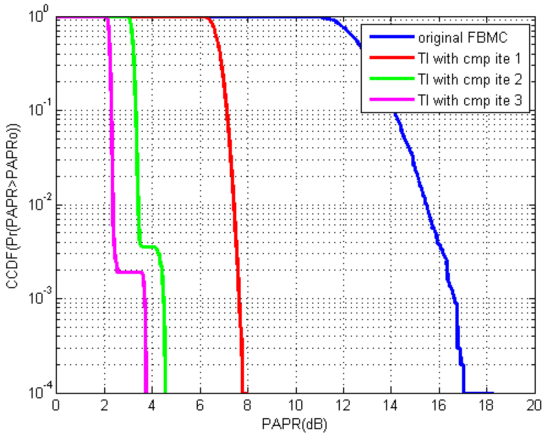

Fig. 7: CCDF plot for original FBMC signal and combined scheme of TI and Companding techniques.

are related by $\eta = \frac{\eta_{max}}{PAPR}$, where $\eta_{max} = 0.5$ for an ideal class A amplifier. Table I contains PAPR values for different PAPR reduction technique and original FBMC-OQAM signal for 10^{-3} clip probability and efficiency(in percent) of PA for different PAPR reduction techniques.

TABLE I:

PAPR of FBMC-OQAM signal=16.4 dB		η = 1.15
PAPR Reduction Technique	PAPR(dB)	η
Tone Injection iteration 1	13.4	2.29
Tone Injection iteration 2	11.9	3.23
Tone Injection iteration 3	10.5	4.46
$\mu-$law companding	7.7	8.49
Tone Injection with companding iteration 1	7.8	8.28
Tone Injection with companding iteration 2	4.5	17.74
Tone Injection with companding iteration 3	3.7	21.33

VI. CONCLUSION

In this paper, a FBMC-OQAM system is simulated using new prototype filter.This prototype filter is designed using Blackman-Harris window and PHYDYAS filter,to get low out of band radiation required for FBMC-OQAM system. One of the main problem in any MCM system is that it exibits a very high PAPR.So the main objective of this paper is to reduce PAPR value by using a combined scheme of Tone injection and Companding techniques.Individually Tone injection and Companding are efficient and low complexity techniques for PAPR reduction, but by combining them it is shown that PAPR value can be reduced at a much faster rate with same number of iterations.Simulation results show that PAPR value is decreasing from 16.4 dB to 3.7 dB whereas efficiency of power amplifier is increasing from 1.15 percent to 21.33 percent.Therefore, this technique can be efficiently used in any FBMC system for PAPR reduction.

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