Comparison of constants in wheeler model under single overload and spot heating conditions

ARVIND JONI\textsuperscript{1}, P K RAY\textsuperscript{1}, J R MOHANTY\textsuperscript{2}, B B VERMA\textsuperscript{2}
\textsuperscript{1}Dept of Mechanical Engineering
\textsuperscript{2}Dept of Metallurgical and Materials Engineering
National Institute of Technology, Rourkela 769008.

Abstract: In this paper fatigue crack growth retardation due to overloading and spot heating (to a sub critical temperature) ahead of crack tip have been addressed. The wheeler model for crack growth retardation has been used for the purpose of comparison. The shaping exponents ‘\(m\)’ of both the cases were numerically determined from experimental data and variations in the result have been analysed. It is found that shaping exponent ‘\(m\)’ due to spot heating at specific position is more than that due to overload.

Keywords: overload, spot heating, Wheeler model, fatigue crack retardation.

NOMENCLATURE

\(a_i\) Instantaneous crack length
\(a_0\) Initial crack length at which spot heating/overloading done
\(a_p\) Overload/spot heating affected crack length
\(C\) Constant in Paris equation
\(d\) Distance of heating spot from the crack tip
\(Cp_i\) Retardation parameter
\(da/dN\) Crack growth rate
\(D\) Button diameter
\(E\) Modulus of elasticity
\(m\) Shaping exponent in the Wheeler model
\(n\) Exponent used in Paris equation
\(N\) Number of cycles
\(N_D\) Number of delay cycles
\(r_p\) Plastic zone size
\(r_{pi}\) Current plastic zone size corresponding to the ‘\(i\)’th cycle
\(r_{po}\) Overload plastic zone size
\(R\) Stress ratio = \(\frac{\sigma_{\text{min}}}{\sigma_{\text{max}}}\)
\(T\) Spot temperature
\(w\) Width of specimen
\(\Delta K\) Stress intensity factor range = \(K_{\text{max}} - K_{\text{min}}\)
\(\sigma_{\text{max}}\) Maximum cyclic stress
\(\sigma_{\text{min}}\) Minimum cyclic stress
\(\sigma_y\) Yield stress
\(\Delta\sigma\) Stress range = \(\sigma_{\text{max}} - \sigma_{\text{min}}\)
\(\alpha\) Plastic zone size constant
\(\gamma\) Retardation correction factor
\(\lambda\) Plastic zone size correction factor

INTRODUCTION

In contrast to the uniform crack growth on structures due to constant amplitude loading, the crack growth retardation takes place due to an application of an overload. If the overload is large
enough, crack arrest can occur and the growth of the fatigue crack stops completely. The overload introduces a large plastic zone ahead of the crack tip and hence magnitude and size of compressive residual stress field in the vicinity of the crack tip increase. This enhanced compressive residual stress reduces crack tip driving force and thus retards the fatigue crack growth rate. The application of a single overload cycle was observed to cause a significant decrease in the crack growth rate subsequent to the overload\(^1\). Several models have been developed to estimate load interaction effects on crack growth. These models are classified into two categories: i) crack tip plasticity models, and ii) crack closure models. Crack tip plasticity models are based on the assumption that the crack growth rates under variable amplitude loading can be related to the interaction of the crack tip plastic zones.

Another technique of retarding crack growth is by heating a spot near the crack tip\(^4,5\). This technique involves heating a zone ahead of the crack tip and then cooling to ambient temperature. Localized heating followed by non-uniform cooling introduces uneven distribution of inelastic strains which develop residual stress field. Thus the plastic zone ahead of crack tip consists of a small residual tensile stress field surrounded by a large compressive stress field. This residual compressive stress field gives rise to retardation effect. However spot heating differs from overloading in that the compressive stress zone may extend into a region behind crack tip, bringing about an increase in crack closure level\(^7\). In this analysis, the retarded crack length \(a_d\) is used to determine the plastic zone size. Also comparison has been done for the constants of Wheeler model for overload and spot heating conditions.

**EXPERIMENTAL PROCEDURE**

The material used in the investigation was steel (yield strength 283.6 MPa) having composition shown in Table 1.

<table>
<thead>
<tr>
<th>Element</th>
<th>C</th>
<th>Mn</th>
<th>P</th>
<th>S</th>
<th>Si</th>
<th>Al</th>
<th>Nb</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wt. %</td>
<td>0.16</td>
<td>1.40</td>
<td>0.023</td>
<td>0.006</td>
<td>0.33</td>
<td>0.034</td>
<td>0.039</td>
<td>0.036</td>
</tr>
</tbody>
</table>

Single edge notched (SEN) specimens of dimension 50 mm x 170 mm x 7 mm were prepared and the fatigue tests were performed in the LT direction. Before actual test, the notched specimens were precracked up to a length of 17mm (including a notch length of 15 mm i.e \(a/w =0.2895\)). Precracking had done according to ASTM-E647-88a standard\(^6\). Then the specimen after single overload at 19.47 mm was cycled in an INSTRON-8502 in tension-tension constant stress amplitude mode at stress ratio \(R=0.1\) and stress range \(\Delta\sigma =42.55\) MPa until the unretarded crack growth rate was reached. The constant amplitude loading tests were done under a sinusoidal waveform loading. The effect of single overload on the fatigue crack growth behavior was obtained using spectrum of type as shown in fig. 1 The overload ratio of OLR=1.9 and the initial crack length of \(a_0=15\) mm was used in this investigation.

The specimens used for spot heating were of carbon steel having yield strength of 267 MPa. The details of the material and specimen are given elsewhere\(^6\).

![Fig 1. Schematic diagram of test spectrum](image)
ESTIMATION OF PLASTIC ZONE SIZE DUE TO OVERLOAD AND SPOT HEATING

In case of mode-I overload application, retardation effect increases with increase in the level of overload. The most famous reason is the plasticity induced crack closure. According to Wheeler, retardation effect ceases when the tip of the instantaneous cyclic plastic zone reaches the end of overload affected monotonic plastic zone. Due to similarity in the crack growth behavior due to overload spike and spot heating, it is assumed that retardation effect ceases when the tip of the instantaneous cyclic plastic zone reaches the end of the plastic zone due to spot heating / monotonic overload.
ESTIMATION OF SHAPING EXPONENT

The fatigue crack propagation rate under constant amplitude loading is given by Paris equation as follows

$$\frac{da}{dN} = C (\Delta K)^n$$

(1)

C and n are material constants. Wheeler proposed the following modified relation for the delay in crack growth due to tensile overload as follows

$$\left(\frac{da}{dN}\right)_{\text{retarded}} = (Cp) \left[ C (\Delta K)^n \right]$$

(2)

Where (Cp), is the retardation parameter which is given by

$$(Cp)_i = \left( r_p / (a_0 + r_p - a_i) \right)^m$$

(3)

From eqn (2)

$$(Cp)_i = \left( \frac{\left(\frac{da}{dN}\right)_{\text{retarded}}}{C (\Delta K)^n} \right) = \left( \frac{\left(\frac{da}{dN}\right)_{\text{nonretarded}}}{\left(\frac{da}{dN}\right)_{\text{law}} - a_0} \right)$$

(4)

The Wheeler model states that the crack retardation persists as long as the current plastic zone advances within the overload plastic zone, i.e. \(a_i + r_p < a_o + r_p\). Meanwhile when the boundary of the current plastic zone touches the boundary of the overload plastic zone retardation ceases with \((Cp)_i\) increases to unity i.e. \(a_i + r_p = a_o + r_p\). Thus from eqn(4) retardation parameter \((Cp)_i\) increases to unity when \((da/dN)_{\text{retarded}} = (da/dN)_{\text{law}}\). The minimum value of \((Cp)_i\) is given by \(\left(\frac{da}{dN}\right)_{\text{retarded}} = (da/dN)_{\text{law}} \). Plastic zone size can be generally expressed by the equation as follows

$$r_p = \alpha (\Delta K / 2\sigma_f)^{1/2}$$

(5)
The Plastic zone size constant, $\alpha$, can be established experimentally or analytically. According to Irwin’s scheme, the constant $\alpha$ is taken as $1/\pi$ for plane stress and $1/3\pi$ for plane strain. Due to the presence of a net compressive residual stress field around the crack tip in the plastic zone due to spot heating as well as single overloading, current plastic zone size in the compressive stress field is less than the usual size of the plastic zone for plane stress cyclic loading. A plastic zone correction factor $\lambda$ was introduced and the following equation was suggested for the instantaneous cyclic plane stress plastic zone size in a compressive stress field

$$r_{pi} = \lambda (1/\pi) \left( \frac{\Delta K}{2\sigma_y} \right)^2$$

Eqn (3) is modified as follows

$$(Cp)_i = \left[ \lambda \frac{r_{pi}}{r_{po} + (a_i - a_0)} \right]^{m} = \gamma \left[ \frac{r_{pi}}{r_{po} + (a_i - a_0)} \right]^{m}$$

where $\gamma$ is a correction factor which is given by $\gamma = \lambda^m$.

The values of $\lambda$, $m$ and $\gamma$ calculated from equations (4) & (7) are tabulated in Table 2. The values of these constants for spot heating case are shown in Table 3.

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$m$</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.03938</td>
<td>0.2513</td>
<td>0.4436</td>
</tr>
</tbody>
</table>

Table 3. Values of $\lambda$, $m$ and $\gamma$ due to spot heating [5]:

<table>
<thead>
<tr>
<th>Temperature</th>
<th>$\lambda$</th>
<th>$m$</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>0.06672</td>
<td>0.2814</td>
<td>0.2442</td>
</tr>
<tr>
<td>550</td>
<td>0.007075</td>
<td>0.2835</td>
<td>0.2457</td>
</tr>
<tr>
<td>620</td>
<td>0.010556</td>
<td>0.3336</td>
<td>0.2191</td>
</tr>
<tr>
<td>700</td>
<td>0.012982</td>
<td>0.4544</td>
<td>0.1389</td>
</tr>
</tbody>
</table>

**CONCLUSION**

The values of $\lambda$ and $m$ increase with increase in spot heating temperature shows an decrease in the value of retardation correction factor $\gamma$. The value of $m$ obtained due to overloading is less than that due to spot heating. A higher value of $m$ means a lower value of $Cp_i$ and higher rate at which it approaches unity. Thus spot heating causes a more effective retardation than overloading at 1.9 overload ratio.

**REFERENCES**