MIMO BEAMFORMING IN SPATIALLY AND TEMPORALLY CORRELATED CHANNEL

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***Abstract*—In modern wireless communication systems, multi-element antenna arrays are popularly used for improving the capacity and reliability of wireless communication links. An attractive candidate in this regard is multi input multi output (MIMO) system. The MIMO system rely upon two schemes such as spatial multiplexing which improves multiplexing gain and beamforming which increases diversity gain. In this paper we compare the performances of beamforming and spatial multiplexing keeping data rate equal for both the schemes in a spatially and temporally correlated flat fading MIMO channel. The bit error rate (BER) performance is evaluated using maximum likelihood (ML) decoding technique. For better performance power allocation strategy is implemented at the transmitting end. The experimental results show that for higher values of spatial and temporal correlation beamforming performs better than spatial multiplexing.**

***Keywords*—MIMO; beamforming; spatial multiplexing; spatial and temporal correlation; dynamic power allocation; ML detection**

I. INTRODUCTION

In recent years MIMO system is widely accepted to meet the increasing demand of transmitting high data rate in wireless communication. The channel capacity and also the reliability of the communication link is attractive in MIMO system [1][2]. The effective performance of MIMO system is found to be achieved through various techniques like spatial multiplexing, diversity and beamforming [3].

In comparison to single antenna system, the channel capacity of a multi antenna system with N_t transmit and N_r receive antenna can be increased by the factor of minimum number of N_t and N_r. Thus in a MIMO system the number of degrees of freedom available or the number of parallel spatial channels that exist, depend upon the scattering environment and the number of antenna incorporated. By transmitting independent data streams through those spatial channels, the capacity of the system can be improved in spatial multiplexing [1][2][3][4][5]. However, the major drawback in above spatial multiplexing technique is that, the performance of the system degrades severely in correlated fading channel [6][7]. On the other hand beamforming is able to provide high directional gain with reduced interference from other direction in highly correlated MIMO channel.

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According to Narula et al. [8] in beamforming, the transmissions from different antenna elements at the base station are to be made to add coherently at the intended receiver for yielding an average factor of enhancement of signal to noise ratio (SNR). For the same, authors formulate a mathematical expression and show that, in point-to-point scenario with side information, beamforming in a direction determined by eigen structure of posterior channel correlation matrix maximizes the desired SNR. However in the above, authors transmit a single data per channel-use to all the recipients. A generalized version of similar contribution is found from Sampath et al. in [9] where authors show that the optimum beamformer is likely to be the vector or matrix whose columns are the first desired number of dominant eigenvectors of MIMO channel wishart matrix. It is found in [10] that the above beamformer matrix is the optimal choice in the sense of maximizing channel capacity.

In most of the existing literatures, authors consider independent and identically distributed (i.i.d) MIMO channel which is not practical in strict sense. Due to space constraint, when the communicating antennas are placed close together or due to the presence of local scattering elements around transmitter or receiver, spatial correlation arises and the channel remains no longer i.i.d [11][12]. In addition to that the channel gets temporally correlated when the receiver is not fixed [13][14]. Experimental characterization, modelling and analysis on such MIMO wireless channels with spatial and temporal correlation are given in [15][16][17]. Based on knowledge in the existing literatures BER performance with respect to SNR are not available for a scenario where spatial and temporal correlation exists simultaneously. This motivates us to analyze the performance of MIMO beamforming in a spatially and temporally correlated channel and compare it with the same of spatial multiplexing.

This paper is organized as follows. Section II describes the MIMO system model. Section III describes the Overview of MIMO beamforming system. Structure of spatially and temporally correlated MIMO channel is given in section IV. Dynamic power allocation and bit allocation is discussed in section V. Simulation results are given in section VI. Finally conclusion is drawn in section VII.

II. SYSTEM MODEL

We consider a single user narrow-band downlink communication link with N_t transmitting and N_r receiving antenna elements. A schematic of MIMO system is shown in Fig.1. The associated wireless channel can be represented by $N_r \times N_t$ matrix H and the system model can be given as

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{N_r} \end{bmatrix} = \begin{bmatrix} h_{1,1} & h_{1,2} & \cdots & h_{1,N_t} \\ h_{2,1} & h_{2,2} & \cdots & h_{2,N_t} \\ \vdots & & & \\ h_{N_r,1} & h_{N_r,2} & \cdots & h_{N_r,N_t} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{N_t} \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_{N_r} \end{bmatrix} \tag{1}$$

where,
x_i = Signal transmitted from i^{th} transmitting antenna
y_j = Signal received at j^{th} receiving antenna
n_j = Noise at the j^{th} receiving antenna
$h_{(j,i)}$ = $(j,i)^{th}$ element of 'H' where $h_{j,i}$ is the transfer channel characteristic between j^{th} receiving and i^{th} transmitting antenna
In general the above equation can be represented as

$$Y = HX + N \tag{2}$$

where

$$\begin{array}{l} Y = [\ y_1 \quad y_2 \quad \ldots \quad y_{N_r}\ ]^T \\ X = [\ x_1 \quad x_2 \quad \ldots \quad x_{N_t}\ ]^T \\ N = [\ n_1 \quad n_2 \quad \ldots \quad n_{N_r}\ ]^T \end{array}$$

For the above MIMO communication system we assume the following constraints

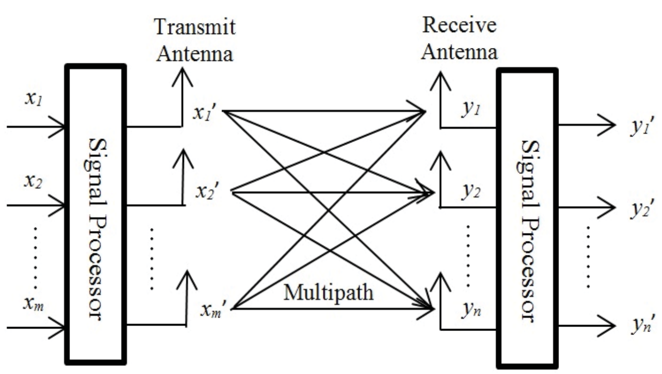

Fig. 1: $N_t \times N_r$ MIMO System

i. The channel matrix 'H' is Rayleigh distributed
ii. The matrix 'H' is of full rank.
iii. There is no line of sight between transmitting and receiving antenna array.
iv. Rich scattering environment is available.
v. The noise has Gaussian probability density function,'$\mathcal{N}(0,1)$'
vi. Channel is known both at the transmitter and receiver end.

III. Beamforming

Beamforming technique requires the channel state information CSI to be available at the transmitter. At the outset, beamformer matrix or precoding matrix is calculated at the transmitting end using the channel transfer matrix 'H'. Then this weight matrix is applied at each transmit antenna before transmitting the signal. Similarly a decoding matrix will be used at the receiving end during equalization. For the MIMO channel described earlier, the modified system equation will be

$$Y' = (ZHW)X + ZN \tag{3}$$

where the dimension of

$$\begin{array}{l} H \in C^{N_r \times N_t}, \quad W \in C^{N_t \times r} \\ (X, Y') \in C^{r \times 1}, \ N \in C^{N_r \times 1} \\ Z \in C^{r \times N_r} \end{array} \tag{4}$$

The term 'W' and 'Z' in (3) are called precoding and decoding matrix respectively. Here it is assumed that

$$\begin{array}{l} r = rank(H) \leq \min(N_r, N_t) \\ E(XX^\dagger) = I \\ E(XN^\dagger) = 0 \end{array} \tag{5}$$

where the symbol † denotes the hermitian operation of a given matrix.

If the channel matrix, H is decomposed as $H = UDV^\dagger$, where U and V are unitary matrices, then

$$H^\dagger H = VD^\dagger DV^\dagger = Q \wedge Q^\dagger \tag{6}$$

where $Q = V$ such that $Q^\dagger Q = I_{N_r}$ and $\wedge \in C^{N_r \times N_r}$ is a diagonal matrix with its diagonal elements given as

$$\lambda_i = \begin{cases} \sigma_i^2, \ if\, i = 1, 2, \cdots, r \\ 0, \ \ if\, i = r+1, \cdots, N_r \end{cases} \tag{7}$$

According to [5] beamforming is optimum when the signal is transmitted along the direction of first eigen vector of $H^\dagger H$ and hence minimum BER can be achieved.

IV. Spatial and temporal correlation

As discussed earlier, MIMO channel is not to be considered i.i.d, but correlated both in spatially and temporally [7]. In this section we discuss about the structure of correlation matrix with spatial correlation first and then the same with the spatio-temporal correlation.

A. Spatial Correlation

According to [6] the spatial correlation of MIMO channel depends upon angular spread, angle of arrival, antenna arrangement and antenna spacing. According to [17] if transmit and receive antenna correlation function are separable then, the correlation between two links, transmitter 'p' with receiver 'm' and transmitter 'q' with receiver 'n' can be defined as

$$R_{mp,nq} = E\left[H_t(m,p)H_t(n,q)^\dagger\right] \tag{8}$$

where $H_t(i,j)$ is the transfer channel matrix from j^{th} transmit antenna to i^{th} receive antenna at one particular instant of time 't'. If $vec(H) = (h_1^T, h_2^T, \cdots, h_{N_t}^T)^T$, where $h_1, h_2, \cdots, h_n$

are columns of matrix 'H', then covariance matrix 'H' is defined as $cov(vec(H))$ which is equal to

$$\mathrm{cov}(vec(H)) = E[vec(H)vec(H)^{\dagger}] = R_R \otimes R_T \quad (9)$$

where 'R_T' is transmit antenna correlation matrix and 'R_R' is receive antenna correlation matrix. Then the correlation matrix can be represented as

$$H = R_R^{1/2} H_W R_T^{1/2} \quad (10)$$

where H_W is the transfer channel matrix which contains i.i.d entries. In this paper we consider exponential model for the R_T matrix. For a channel matrix '$H_{(Nr \times Nt)}$' the dimension of 'R_T' matrix will be $(N_t \times N_t)$ which can be represented by

$$R_T = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \quad (11)$$

where $N_t = 2$ and 'ρ' is the correlation coefficients of neighboring branches.

In contrast to transmit antennas which undergoes correlated fading, we consider the receiver antenna elements to be independent fading. Therefore the structure of receiver covariance matrix will be an identity matrix given by

$$R_R = I_{M_r \times M_r} \quad (12)$$

B. Spatio-Temporal Correlation

According to [16][17] in addition to spatial correlation there also exists temporal correlation between two channel uses or between different instances of channel use. Similar to (8) the modified correlation matrix between two links in presence of temporal correlation can be expressed as

$$[R_{mp,nq} = E[H_t(m,p)H_{t+t_1}(n,q)^{\dagger}] \quad (13)$$

where 't' and '$t + t1$' are two different instances of time. We define $H_{eff} = [H_1, H_2, \cdots, H_K]$ the channels for 'K' channel uses where the dimension of H_{eff} will be $N_r \times KN_t$. Here it is assumed that, the temporal correlation of the channel is modelled by a $K \times K$ matrix R_t where $R_t(c,d)$ is the correlation of the channel between two channel uses c and d. If the covariance matrix 'R' is defined as

$$\mathrm{cov}(vec(H)) = E[vec(H)vec(H)^{\dagger}] = R_R \otimes R_T \otimes R_t \quad (14)$$

then the Kronecker correlation model can be given by

$$H = R_R^{1/2} H_{eff} (R_T \otimes R_t)^{1/2} \quad (15)$$

Here also we consider the exponential model for temporal correlation R_t. Although exponential model is not an accurate model but, it is simple and single parameter model that physically reasonable in the sense that correlation decreases with increase in distance [18] .

V. Power Allocation

Here we assume that complete CSI is available at the transmitter side also. Therefore before each instance of transmission, power can be allocated dynamically along all sub-channels keeping total transmitted power to be constant across all transmitting antenna. In this work rank one beamforming system is implemented where the symbols are transmitted along dominant eigen direction only. Therefore, total power is allocated only in a single direction. Therefore power allocation strategy will be applied to spatial multiplexing system only.

To achieve better BER, power can be allocated in each sub-channel based on the eigenmode realization [9][10]. More power is allocated to sub-channels having weaker eigenmodes whereas less power is allocated to sub-channels having stronger eigenmodes. If V is defined as eigen vector of $H^{\dagger}H$ such that

$$(H^{\dagger}H)\ V = VB \quad (16)$$

where B is a diagonal matrix with its main diagonal contain eigen values of $[H^{\dagger}H$, then for spatial multiplexing the modified optimum precoder and decoder matrix in (3) can be given by

$$\begin{aligned} W &= V\alpha \\ Z &= \beta V^{\dagger} H^{\dagger} \end{aligned} \quad (17)$$

where,

$$\begin{aligned} \alpha &= \sqrt{\gamma/B} \\ \beta &= \sqrt{\gamma/B}\ (1+\gamma)^{-1} \end{aligned} \quad (18)$$

If at any instant total power is given as 'P' then can be defined by

$$\gamma = \frac{P}{Tr(B^{-1})} \quad (19)$$

then the factor ZHW turns out to be $(\gamma\,/\gamma\,+1)\,I$, an identity matrix with constant scaling.

In addition to the above power allocation method we have also considered the issue of symbol transmission power. In rank one beamforming system number of symbols being transmitted at a time is one whereas in spatial multiplexing N_t numbers of symbols are transmitted at the same time. Therefore for better comparison a symbol of beamforming system are transmitted with ($N_t\times$ normalized symbol energy) amount of power.

In this paper to make the comparison fair fixed data rate is assumed to be transmitted according to TABLE I. Therefore dynamic bit allocation is not considered here.

VI. Simulation results and discussion

In this paper we consider a MIMO communication system where the number of receiving antenna is greater than number of transmitting antenna. Here we consider the case of N_t = 2 and N_r = 4. ML detection technique is used at the receiver to get the optimal output in the cost of higher complexity. For spatial multiplexing, power is allocated dynamically at the transmitting end according to (17).

Experimental results given in Fig.2 show the BER performance of a beamforming system. The results are obtained using (1) where channel is considered i.i.d.

The results obtained for beamforming are compared with same of spatial multiplexing for different bit rates. For better comparison we use different modulation in order to keep the data rate fixed for both spatial multiplexing and beamforming system. The modulation techniques which are used here are given in the TABLE I.

Two different cases are presented here. In case I. 4 bits of data are transmitted whereas in case II. 8 bits of data are

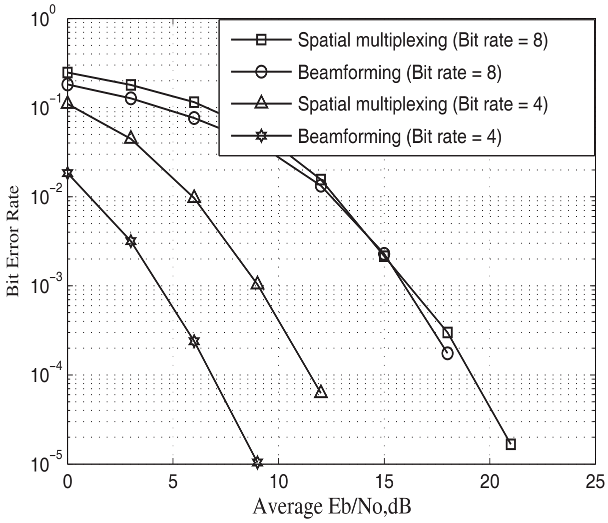

Fig. 2: BER of MIMO system for i.i.d channel

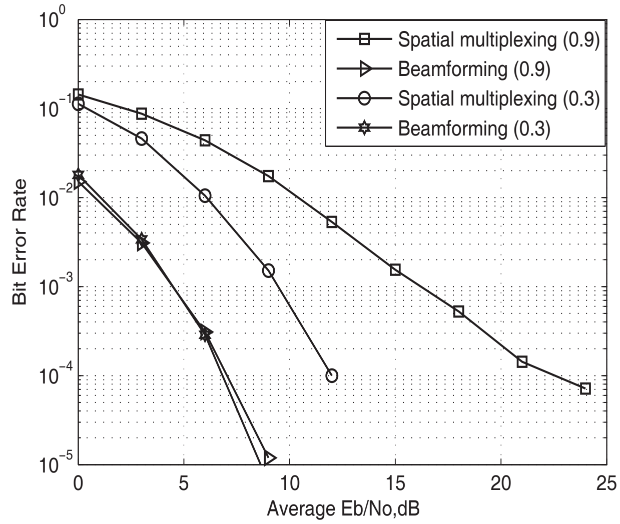

Fig. 3: BER of MIMO system for data rate of 4 bits per channel-use

TABLE I: Modulation order for beamforming and spatial multiplexing system

4×2 **MIMO System**	**4 Bit**	**8 Bit**
Spatial Multiplexing	4 QAM	16 QAM
Beamforming	16 QAM	256 QAM

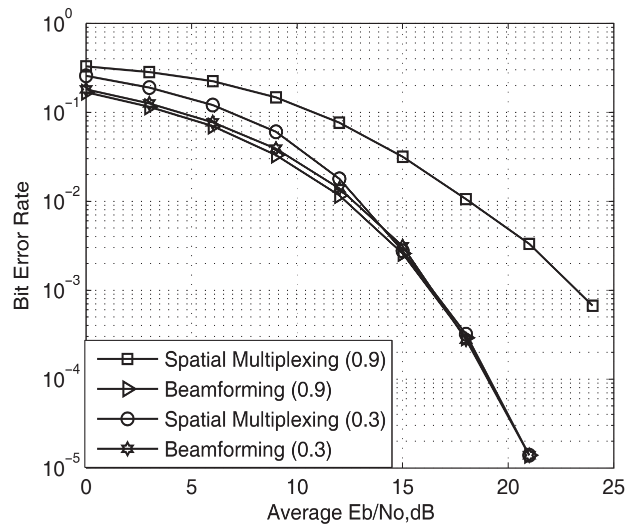

Fig. 4: BER of MIMO system for data rate of 8 bits per channel-use

transmitted at a particular instant of time. Results show that, beamforming performs better than spatial multiplexing system when the data rate is less.

The results presented in Fig.3 and Fig.4 are obtained for channel matrix given in (10) for different data rates where the effect of spatial correlation is included. The structure of transmit and receive correlation matrices are used as per (11) and (12) respectively. The correlation of receive antenna tends to be negligible in comparison to transmit antenna, and hence it is considered as identity matrix. Experiment is done for various values of transmit antenna correlation and results are presented for $\rho = 0.3$ and $\rho = 0.9$.

The experimental results show that for lower data rate, BER in beamforming system is lower as usual. However, for higher data rate, BER of spatial multiplexing and beamforming system is comparable for higher order spatial correlation. It can also be noted that beamforming is hardly affected by the presence of antenna correlation.

In order to assess the performance in a channel where both spatial and temporal correlation coexist, the transfer matrix 'H' given in (15) is considered for experimentation and results are presented in Fig.5 and Fig.6 for a fixed data rate of 8 bit per channel use. Here 'K' is considered 4 which means, effect of temporal correlation persists up to four consecutive channel uses.

The results presented in Fig.5 show the effect of temporal correlation coefficient values of 0.3 and 0.99 on BER performance keeping spatial correlation values fixed at 0.8. Similarly the results given in Fig.6 show the effect of spatial correlation coefficient values of 0.3 and 0.99 on BER performance keeping the temporal correlation coefficient value fixed at 0.8. The results show that impact of spatial correlation on BER performance is more than temporal correlation. In fact from Fig.5 it is clear that the effect of temporal correlation upon BER is negligible. Furthermore, performance of spatial multiplexing

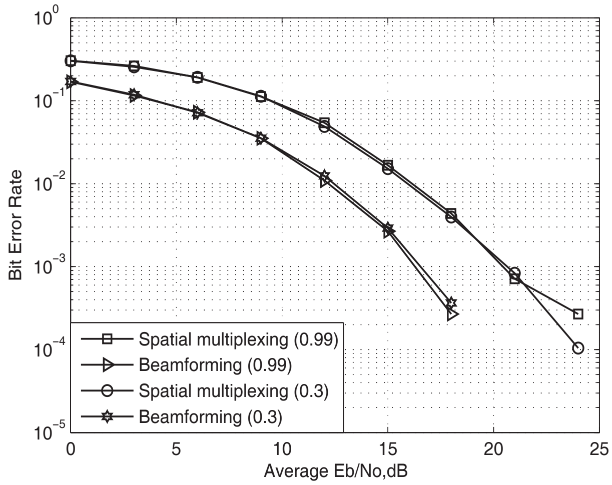

Fig. 5: Impact of temporal correlation with fixed spatial correlation

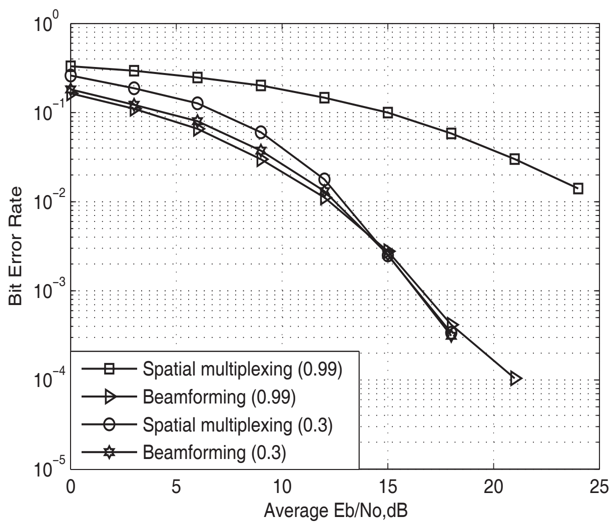

Fig. 6: Impact of temporal correlation with fixed spatial correlation

system degrades severely in highly correlated (spatially and temporally) channel, whereas beamforming system is relatively immune to such correlations.

VII. Conclusion

This paper presents a MIMO communication system in spatially and temporally correlated channel. The BER performance of beamforming system is compared with the results obtained for spatial multiplexing system. To make the comparison more accurate, data are transmitted keeping the transmit power and bit rate equal for both spatial multiplexing and beamforming system. Impact of simultaneous presence of spatial as well as temporal correlation is thoroughly examined from experimental results and a strong effect of those on MIMO communication system is noticed. The performance of such MIMO system in spatially and temporally correlated channel may possible to improve by beamforming suitably in the direction of dominant eigen structure of channel correlation matrix.

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