Comparing the Performance analysis of three tank level control system using Feedback and Feedforward-feedback configuration

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Abstract- A multi tank level control system includes both the example of interacting and non-interacting system. In this system, we have considered three tanks each having equal area and each tank can be assumed as a first order system which are connected in interacting and non-interacting mode. The control system is intended to maintain the level of the third tank at some predefined value irrespective of changes of inflow of first tank. Conventional PID controller is a powerful controller used in process industries to regulate and control process variables. In this paper, we also consider the effect of the disturbance on the response of the system. According to these disturbances, we need to implement feed forward controller with better tuning algorithm. Thus, we implement transfer function of above system and behaviour is observed with step input.

Keywords—feedback, feed-forward, disturbance

I. INTRODUCTION

The liquid level control in a plant is crucial in order to provide desired specifications. The measurement and control of industrial process level parameter is of great importance in industrial field. The level of liquid may affect both the pressure and the rate of flow in and out of the tank. Hence, the quality of product may affect. So it is very important to control the level of liquid in process industries. PID controller are the most popular controllers used in process control because of their remarkable effectiveness and simplicity of implementation, more than 90% of existing control loop involve PID controller. In this paper, we used PID controller as feed-back controller and each tank is considered as a first order system. Transfer function between level of third tank and inflow to first tank is calculated[1], so whole system act as a third order system and the coefficient of P, I and D is chosen by Ziegler-Nichols tuning method[2-3]. Disturbance is applied to second tank in each case (interacting and non-interacting mode). Transfer function between level of third tank and disturbance is calculated and feed-forward controller is designed accordingly [4-5].

This paper is organized as follow: Section II deals with mathematical model of three tank process which is connected in interacting and non-interacting mode. Section III deals with calculation of transfer function of three tank system with respect to disturbance. Section IV consists of block diagram and tuning parameters of controller. Section V consists of simulation result and discussion. Section VI finally concludes the paper.

II. MATHEMATICAL MODELLING OF THREE TANK PROCESS

In process control industries, many physical parameters like flow, pressure, temperature, level etc. are the parameters that are controlled; control of liquid level in tank is a basic problem in process control industries. The process industries need to control the liquid level in interacting and non-interacting process, such as petroleum and chemical industries the following figure shows non-interacting mode of three tank process and various mode of interacting three tank process Fig. 1 shows three tank system connected in non-interacting mode in which input flow rate liquid is drained to tank 1 and outlet flow rate of liquid is non-interacted with tank 2 and in similar way tank 2 is connected to tank 3. Fig. 2 shows three tank system in interacting mode in which tank 1 is interaction with tank 2 and tank 3 with non-interaction with tank 2. Figure 3 shows tank 2 and tank 3 are connected in interaction mode and tank 1 is non-interaction mode with tank 2.

The objective is to control liquid level in tank 3 at some predefined value for all above defined combinations. Tank 3 levels is measured, A3, and compared with set point value and difference is calculated between actual and desired level and control input is obtained by PID algorithm. The control input will be feed-back to the control valve to change the input flow rate of tank 1 and hence, level of tank 3 can be controlled.

In this paper, we have applied extra input on tank 2 as a disturbance in each case. In order to eliminate the effect of these disturbances, feed-forward controller is designed.
**A. Three Tank Non-Interacting System**

Three tank non-interacting system is shown in figure below:

![Diagram of three tank non-interacting system](image)

Mass Balance around tank 1

\[ A_1 \frac{dh_1}{dt} = Q_1(t) - Q_i(t) \]  

Valve relation

\[ Q_i(t) = \frac{H_i}{R_i} \]

Where,

- \( Q_1(t) \) - Tank 1 inlet flow rate \( (m^3/s) \)
- \( Q_i(t) \) - Tank 1 outlet flow rate \( (m^3/s) \)
- \( R_i \) - Outlet flow rate resistance of tank 1 \( (m^3/s) \)
- \( A_1 \) - Area of tank 1 \( (m^2) \)
- \( H_i \) - Actual liquid level in tank 1 \( (m) \)

Mass Balance around tank 2

\[ A_2 \frac{dh_2}{dt} = Q_2(t) - Q_i(t) \]

Valve relation

\[ Q_i(t) = \frac{H_i}{R_i} \]

Where,

- \( Q_2(t) \) - Tank 2 inlet flow rate \( (m^3/s) \)
- \( Q_i(t) \) - Tank 2 outlet flow rate \( (m^3/s) \)
- \( R_i \) - Outlet flow rate resistance of tank 2 \( (m^3/s) \)
- \( A_2 \) - Area of tank 2 \( (m^2) \)
- \( H_i \) - Actual liquid level in tank 2 \( (m) \)

Mass Balance around tank 3

\[ A_3 \frac{dh_3}{dt} = Q_3(t) - Q_i(t) \]

Valve relation

\[ Q_i(t) = \frac{H_i}{R_i} \]

The overall transfer function of three tank non-interacting system is calculated by using equation (1) to equation (6),

\[ \frac{Q_3(s)}{Q_i(s)} = \frac{R_3}{(\tau_3s + 1)(\tau_2s + 1)(\tau_1s + 1)} \]

Where

\[ \tau_1 = A_1R_1, \]
\[ \tau_2 = A_2R_2 \]
and \( \tau_3 = A_3R_3 \).

Assume,

\[ R_1, R_2, R_3, A_1, A_2 \text{ and } A_3 = 1. \]

Equation (7) can be written as

\[ \frac{H_i(s)}{Q_i(s)} = \frac{1}{(s+1)^3} \]

**B. Three Tank Interacting System (Case 1)**

Three tank interacting system is shown in figure below:

![Diagram of three tank interacting system](image)

Putting the value of equation (9), (10) and (11) in equation (1), (3), (5) and rearranging them, we get overall transfer function

\[ Q_3(t) = \frac{H_3(t)}{R_3} \]
\[ H_i(s) = \frac{R_i R R_i}{Q(s)} \times \frac{(SA_i R_i + 1)(SA_i R R_i + R_i + R_i) - R_i)(SA_i R_i + 1) }{((SA_i R_i + 1)(SA_i R R_i + R_i + R_i) - R_i)(SA_i R_i + 1) } \]  

(12)

C. Three Tank Interacting System (Case 2)

\[ Q_i = \frac{H_i(t)}{R_i} \]  

(13)

Three tank interacting system is shown in figure below:

![Three tank process Interacting (case2)](image1)

Putting the value of equation (13), (14) and (15) in equation (1), (3), (5) and rearranging them, we get overall transfer function,

\[ C(s) = \frac{R_i R R_i}{R(s)} \times \frac{R_i}{((SA_i R_i + 1)(SA_i R R_i + R_i + R_i) - R_i)(SA_i R_i + 1) } \]  

(16)

III. Calculation of Transfer Function of Three Tank System with Respect to Disturbance

Case 1 - For non-interacting three tank system,

Disturbance is given as input flow to the second tank

\[ \frac{H_i(s)}{C_i(s)} = \frac{R_i}{(\tau_i s + 1)(\tau_2 s + 1)} \]  

(17)

As \( R_i, \tau_1, \tau_2 \) and \( \tau_3 = 1 \) have

\[ \frac{H_i(s)}{C_i(s)} = \frac{1}{(s+1)^2} \]  

(18)

Case 2 - For interacting three tank system (case 1),

\[ \frac{H_i(s)}{C_i(s)} = \frac{R_3}{(\tau_3 s + 1)(\tau_2 s + 1)} \]  

(19)

As \( R_3, \tau_2 \) and \( \tau_3 = 1 \) have

\[ \frac{H_i(s)}{C_i(s)} = \frac{1}{(s+1)^2} \]  

(20)

Case 3 - For interacting three tank system (case 2),

\[ \frac{H_i(s)}{C_i(s)} = \frac{R_3}{(\tau_1 \tau_2 s^2 + (\tau_1 + \tau_2 + A_1 R_2) s + 1)} \]  

(21)

As \( R_2, \tau_1, \tau_2 \) and \( A_1 = 1 \) we have

\[ \frac{H_i(s)}{C_i(s)} = \frac{1}{s^2 + 3s + 1} \]  

(22)

IV. Block Diagram of Feedforward Feedback Control System

![Fig.4 Block diagram of feedback-feed forward controller](image2)

A. Tuning parameter for feed-back controller is calculated using Ziegler-Nicholas tuning method.

| TABLE 1. TUNING PARAMETER FOR FEEDBACK PID CONTROLLER |
|-------------|-----------|----------|
| PID TYPE    | \( K_P \) | \( \tau_i \) | \( \tau_D \) |
| P           | 0.5 \( K_C \) | infinite | 0 |
| PI          | 0.45 \( K_C \) | \( T_C/1.2 \) | 0 |
| PID         | 0.6 \( K_C \) | \( T_C/2 \) | \( T_C/8 \) |

| TABLE 2. COEFFICIENT OF PI CONTROLLER FOR THREE TANK PROCESS |
|-------------|-----------|----------|
| Process type | \( K_P \) | \( \tau_i \) |
| Three tank non-interacting system | 3.6 | 3.02 |
| Three tank interacting system (case 1) | 6.75 | 2.61 |
| Three tank interacting system (case 2) | 6.75 | 2.61 |
B. Tuning parameter for feed-forward controller

For feed-forward-feedback control system it is assumed that $G_f(s)$ will be a lead-lag transfer function of the form:

$$G_f(s) = \frac{K_f(T_1s+1)}{(T_2s+1)}$$

Where $K_f$ - steady state gain of feed-forward controller

$T_1, T_2$ - Time constant of dynamic part of feed-forward controller.

**TABLE III: TUNING PARAMETER FOR FEED-FORWARD CONTROL**

<table>
<thead>
<tr>
<th>S.No</th>
<th>Predominant mode</th>
<th>$T_1$</th>
<th>$T_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Lead</td>
<td>1.5 $t_p$</td>
<td>0.7 $t_p$</td>
</tr>
<tr>
<td>2</td>
<td>Lag</td>
<td>0.7 $t_p$</td>
<td>1.5 $t_p$</td>
</tr>
</tbody>
</table>

**TABLE IV: COEFFICIENT OF FEED FORWARD CONTROLLER**

<table>
<thead>
<tr>
<th>S. No</th>
<th>Process type</th>
<th>Predominant mode</th>
<th>$K_f$</th>
<th>$T_1$</th>
<th>$T_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Three tank non-interacting system</td>
<td>Lead</td>
<td>-1</td>
<td>3</td>
<td>1.4</td>
</tr>
<tr>
<td>2</td>
<td>Three tank interacting system (case 1)</td>
<td>Lead</td>
<td>-1</td>
<td>5.3</td>
<td>2.5</td>
</tr>
<tr>
<td>3</td>
<td>Three tank interacting system (case 2)</td>
<td>Lead</td>
<td>-1</td>
<td>4.4</td>
<td>2</td>
</tr>
</tbody>
</table>

V. SIMULATION RESULT AND DISCUSSION

The closed loop simulation result has been carried out using feedback and with feedback-feed-forward controller for feedback controller the controller parameter are tuned using Ziegler Nichol method and controller parameters are shown in TABLE III and for feed forward controller parameter are tuned as mentioned in TABLE IV.

The response of conventional feedback control for three tank non-interacting system and interacting system is shown in Fig.5, Fig.6 and Fig.7. If we added disturbance to the process, the peak overshoot of system is increased. The response of feedback feed-forward combination is shown in Fig.5, Fig.6 and Fig.7.

VI. CONCLUSION

The simulation result for conventional feedback controller and feedback feed-forward controller for three tank system (interacting and non-interacting mode) were obtained and compared. The peak overshoot of system increases when we introduce disturbance in process and by using feedback feed-forward combination overshoot is minimized. Peak overshoot of interacting system is less than that of non-interacting system. The response of three tank system was analyzed...
through computer simulation using Matlab/Simulink software package.

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REFERENCES