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PARAMETER IDENTIFICATION OF MULTISTOREY FRAME STRUCTURE FROM UNCERTAIN DYNAMIC DATA

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ABSTRACT

In general, identification of structural parameters may be categorized as inverse vibration problem. Usual method of identification uses the values of the parameters initially given to the structure by an engineer. It then modifies the original parameter values as per the observed values from test by an iteration process. The parameters involved in the said problems are traditionally considered as crisp. But, rather than crisp value we may have only the uncertain or incomplete information about the parameters being a result of errors in observations etc. These uncertainties may be modeled through probabilistic, interval or fuzzy theory. Unfortunately, probabilistic methods may not able to deliver reliable results with required precision without sufficient data. Hence interval and fuzzy theory are becoming powerful tools for handling the uncertainties in recent decades. This paper investigates the identification procedure of the uncertain column stiffness of multistorey frame structures by using prior known estimates of the uncertain parameters and uncertain dynamic data. Uncertainties are modeled through triangular convex normalized fuzzy sets. Bounds of the uncertain parameters are obtained by using a proposed fuzzy based iteration algorithm. Example problems are solved to demonstrate the reliability and accuracy of the identification process.

1. INTRODUCTION

Structural Dynamics problems may be categorized as direct or inverse problems. The direct problem consists of finding the response for a specified input or excitation. In the inverse problem the response is known then to develop a mathematical model of the system. The modelling problem may also be divided into two categories. In the first category the nature of the process is completely unknown. But in the second category, a considerable knowledge of the nature of the system may be available, whereas the particular values of the system parameters are unknown. In this paper the second category has been studied, where system equations are known or deducible from the physics of the system, with coefficients remaining to be estimated and modified as per the known initial dynamic characteristics.

In this context various workers [1-3] have reviewed the state of the art of system identification in structural dynamics. Developments and various methods for studying this important field are available in literature [4]. More recent methods and practical guidelines for linear systems may be found in the work of Schoukens and Pintelon [5].

Although few researchers have studied the above issues but, at present also lot of efforts are being made to refine and develop the analytical models for the accurate results. Some representative works on the subject are available [6-9]. Related work done is discussed subsequently.

Loh and Ton [10] have studied a system identification approach to detect changes in structural dynamic characteristics on the basis of measurements. They used the recursive instrumental variable method and extended Kalman filter algorithm for the identification algorithm. The potential of using neural network to identify the internal forces of typical systems has been investigated by Chassiakos and Masri [11]. A localized identification of many degrees of freedom structures is investigated by Zhao et al. [12] and a memory-matrix based identification methodology for structural and mechanical systems is studied by Udwadia and Proskurowski [13]. Notable studies in this field have also been done by other workers [14-18].

In addition to the above literatures, there exist other research works in the present area of study. However the fundamental concepts are similar to those mentioned above. As regards the objective of the structural dynamic analysis related to identification is to develop an analytical model of a structure which can be verified and adjusted by actual test results. However, this adjustment is not easy and can be done by computer and convergence algorithm in terms of some iterative cycles.

In the above studies, authors have considered the variables and parameters as crisp. Generally, the parameters are taken as constant or crisp for simplifying the problem. Actually there are incomplete information about the variables being a result of errors in measurements, observations, applying different operating conditions or it may be maintenance induced error, etc. Rather than the particular value of the properties, we may have only the imprecise bounds of the values. These are called uncertain in nature. So for various scientific and engineering problems, it is an important issue how to deal with variables and parameters of uncertain value. Recently some effort has been made by various researchers throughout the globe to handle these uncertainties by taking the parameters in terms of interval and fuzzy numbers.

Recently, few authors [19, 20] have studied the solution method for fuzzy system of linear equations. They have considered the system of linear equations with fuzzy number. Also vibration analysis of structures with imprecise material properties is also done by few authors. As such papers that are related to interval and fuzzy eigenvalue problems are discussed here. An excellent paper by Chen et al. [21] who has presented a new method for calculating the upper and lower eigenvalue bound of structures with interval parameter. Uncertain bunds of eigenvalue are also studied by Friswell et al. [22]. Dimarogonas [23] discussed the vibration problem using interval analysis. Cechlarova [24] investigated the eigenvectors of the interval matrix using max-plus algebra. Recently, modal analysis of structures by using interval analysis is also studied by Sim et al. [25].

Fuzzy material and geometric properties have been considered by various authors for finite element analysis. Both static and dynamic analyses of structures are excellently explained by Akpan et al. [26] using fuzzy finite element analysis. An important paper is that of Hanss et al. [27] who proposed the application of fuzzy arithmetic in the finite element analysis. There after Qui et al. [28] presented a paper which gives detailed analysis for exact bounds for the static response of structures with uncertain-but-bounded parameters.

In view of the above, the main aim of the present study is to develop a systematic mathematical model for the identification of uncertain structural parameters which can provide the vibration characteristics consistent with the uncertain experimental data. The method first uses the values of the uncertain structural parameters viz. as triangular fuzzy numbers initially given to the structure by an engineer. It then modifies the original parameter values as per the observed values from test by an iteration process, giving uncertain bound of the modified values of the parameters.

2. PRELIMINARIES

In the following paragraph some definitions related to the present work are given [18, 19, 20, 29, 30].

Definition 2.1 Fuzzy number

A fuzzy number U is convex normalised fuzzy set U of the real line R such that

$$\{\mu_U(x) : R \to [0,1], \forall\ x \in R\}$$

where, μ_U is called the membership function of the fuzzy set and it is piecewise continuous.

Definition 2.2 Triangular fuzzy number (TFN)

A fuzzy number U is said to be triangular if

i. There exists exactly one $x_0 \in R$ with $\mu_U(x_0) = 1$ (x_0 is called the mean value of U), where μ_U is called the membership function of the fuzzy set.

ii. $\mu_U(x)$ is piecewise continuous.

The membership function μ_U of an arbitrary triangular fuzzy number $U = (a,b,c)$ may be defined as follows

$$\mu_U(x) = \begin{cases} 0, & x \le a \\ \dfrac{x-a}{b-a}, & a \le x \le b \\ \dfrac{c-x}{c-b}, & b \le x \le c \\ 0, & x \ge c. \end{cases}$$

Any arbitrary triangular fuzzy number $U = (a,b,c)$ can be represented with an ordered pair of functions through $\alpha-$cut approach as $[\underline{u}(\alpha), \overline{u}(\alpha)] = [(b-a)\alpha + a, -(c-b)\alpha + c]$ where, $\alpha \in [0,1]$. This satisfies the following requirements

i. $\underline{u}(\alpha)$ is a bounded left continuous non-decreasing function over $[0,1]$.

ii. $\overline{u}(\alpha)$ is a bounded right continuous non-increasing function over $[0,1]$.

iii. $\underline{u}(\alpha) \le \overline{u}(\alpha)$, $0 \le \alpha \le 1$.

Definition 2.3 Fuzzy arithmetic

As discussed above, fuzzy numbers may be transformed into an interval through $\alpha-$cut approach. So, for any arbitrary fuzzy number $x = [\underline{x}(\alpha), \overline{x}(\alpha)]$, $y = [\underline{y}(\alpha), \overline{y}(\alpha)]$ and scalar k, we have the interval based fuzzy arithmetic as

i. $x = y$ if and only if $\underline{x}(\alpha) = \underline{y}(\alpha)$ and $\overline{x}(\alpha) = \overline{y}(\alpha)$

ii. $x + y = [\underline{x}(\alpha) + \underline{y}(\alpha), \overline{x}(\alpha) + \overline{y}(\alpha)]$

iii. $x - y = [\underline{x}(\alpha) - \overline{y}(\alpha), \overline{x}(\alpha) - \underline{y}(\alpha)]$

iv. $x \times y = \begin{bmatrix} \min\ (\underline{x}(\alpha) \times \underline{y}(\alpha), \underline{x}(\alpha) \times \overline{y}(\alpha), \overline{x}(\alpha) \times \underline{y}(\alpha), \overline{x}(\alpha) \times \overline{y}(\alpha), \\ \max\ (\underline{x}(\alpha) \times \underline{y}(\alpha), \underline{x}(\alpha) \times \overline{y}(\alpha), \overline{x}(\alpha) \times \underline{y}(\alpha), \overline{x}(\alpha) \times \overline{y}(\alpha) \end{bmatrix}$

v. $kx = \begin{cases} [k\overline{x}(\alpha), k\underline{x}(\alpha)], & k < 0, \\ [k\underline{x}(\alpha), k\overline{x}(\alpha)], & k \ge 0. \end{cases}$

3. MATHEMATICAL MODELLING AND METHOD OF IDENTIFICATION

To investigate the present method, a two-storeyed frame structure, as shown in Figure 1 is considered. However the general multistorey frame structure modeling may easily be extended from this example of two storey frame. This is investigated for the sake of demonstration of the procedure. The uncertain floor masses, $\tilde{m}$ are assumed to be the same and the uncertain column stiffnesses $\tilde{k}_1, \tilde{k}_2, \tilde{k}_3$ and $\tilde{k}_4$ (as labelled in Figure 1) are the structural parameters which are to be identified. Corresponding uncertain dynamic equation of motion in matrix form for these two degrees of freedom system may be written as:

$$[\tilde{M}]\{\ddot{\tilde{x}}\} + [\tilde{K}]\{\tilde{x}\} = \{0\} \tag{1}$$

where $[\tilde{M}] = 2\times 2$ fuzzy mass matrix of the structure $= \begin{bmatrix} \tilde{m} & 0 \\ 0 & \tilde{m} \end{bmatrix}$, $[\tilde{K}] = 2\times 2$ fuzzy stiffness matrix of the structure $= \begin{bmatrix} (\tilde{k}_1 + \tilde{k}_2 + \tilde{k}_3 + \tilde{k}_4) & -(\tilde{k}_3 + \tilde{k}_4) \\ -(\tilde{k}_3 + \tilde{k}_4) & (\tilde{k}_3 + \tilde{k}_4) \end{bmatrix}$ and $\{\tilde{X}\} = 2\times 1$ fuzzy vector of displacements.

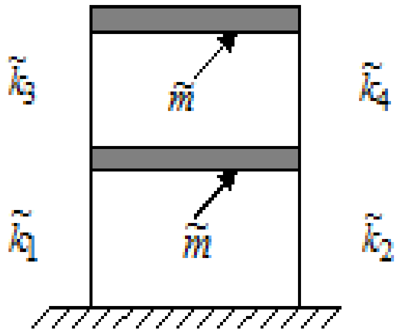

Figure 1. Two storey frame structure

Considering the simple harmonic motion, Eq.(1) can be written as a fuzzy eigenvalue problem as

$$[\tilde{K}]\{\tilde{X}\} = \tilde{\lambda}[\tilde{M}]\{\tilde{X}\}\,. \tag{2}$$

By using the parametric form of fuzzy numbers, Eq. (2) will be

$$[\underline{K}(\alpha), \overline{K}(\alpha)]\{\underline{X}(\alpha), \overline{X}(\alpha)\} = [\underline{\lambda}(\alpha), \overline{\lambda}(\alpha)][\underline{M}(\alpha), \overline{M}(\alpha)]\{\underline{X}(\alpha), \overline{X}(\alpha)\}\,.$$

Now our aim is to solve the above fuzzy eigenvalue problem to get the lower and upper bounds of the fuzzy eigenvalues.

With the above in mind let us proceed now with the identification procedure which can handle the uncertain data. Let us assume that the uncertain structural parameters to be identified are denoted by $\tilde{P}_i$, for $i = 1,2,3,4$. The uncertain value of the structural parameters of the prior original structure given initially are denoted by $\hat{\tilde{P}}_i$, for $i = 1,2,3,4$ and the corresponding fuzzy eigenvalues are symbolized as, $\hat{\tilde{\lambda}}_i(\hat{\tilde{P}})$.

Next the well-known Taylor's series expansion of the fuzzy modal parameters about the initial estimates of the parameters gives:

$$\left\{\tilde{\lambda}(\tilde{P})\right\}=\left\{\hat{\tilde{\lambda}}(\hat{\tilde{P}})\right\}+\left[\tilde{S}\right]\left(\left\{\tilde{P}\right\}-\left\{\hat{\tilde{P}}\right\}\right) \tag{3}$$

where $\left\{\tilde{P}\right\}=\left[\tilde{P}_1,\tilde{P}_2,\tilde{P}_3,\tilde{P}_4\right]^T$, $\left\{\hat{\tilde{P}}\right\}=\left[\hat{\tilde{P}}_1,\hat{\tilde{P}}_2,\hat{\tilde{P}}_3,\hat{\tilde{P}}_4\right]^T$ and $[\tilde{S}]$ is the fuzzy eigenvalue partial derivative matrix, $[\partial(\tilde{\lambda})/\partial(\tilde{P})]$.

Let us now denote experimentally measured uncertain eigenvalues by $\{\tilde{\lambda}_E\}$. It is interesting to note here that if the values of the initial and experimental parameters are equal, then no modification is done. But if the values are different then we denote this difference by

$$\left\{\delta\tilde{\lambda}\right\}=\left\{\tilde{\lambda}_E\right\}-\left\{\hat{\tilde{\lambda}}\right\}. \tag{4}$$

Now let us denote the modified parameters as

$$\{\bar{\tilde{P}}\}=[\bar{\tilde{P}}_1,\bar{\tilde{P}}_2,\bar{\tilde{P}}_3,\bar{\tilde{P}}_4]^T \tag{5}$$

and, in general, for $n-$degrees of freedom system the expression for the uncertain modified parameters from Eq. (3) can be written as :

$$\left\{\bar{\tilde{P}}\right\}=\left\{\hat{\tilde{P}}\right\}+\left[\tilde{Q}\right]\left\{\delta\tilde{\lambda}\right\} \tag{6}$$

where $\left[\tilde{Q}\right]=\left(\left[\tilde{S}\right]^T\left[\tilde{S}\right]\right)^{-1}\left[\tilde{S}\right]^T$.

In order to have the uncertain bounds of the identified parameters with acceptable accuracy, here an iterative procedure is proposed. After finding the modified parameters from Eq. (6), these are substituted in Eq. (2) to get revised uncertain vibration characteristics viz. $\{\bar{\tilde{\lambda}}\}$.

The new fuzzy eigenvalue partial derivative matrix $\{\bar{\tilde{S}}\}$ is then obtained using the current values of $\{\bar{\tilde{P}}\}$ and $\{\bar{\tilde{\lambda}}\}$. From Eq. (6), the modified parameters $\{\bar{\tilde{P}}_t\}$ are again found by utilizing the above values and then the new (revised) estimates of fuzzy eigenvalue are obtained as $\{\bar{\tilde{\lambda}}_t\}$.

If the vector norm of $\{\bar{\tilde{\lambda}}\}$ and $\{\bar{\tilde{\lambda}}_t\}$ is less than some specified accuracy then the procedure is stopped and the revised parameter viz. $\{\bar{\tilde{P}}_t\}$ is identified, otherwise the next iteration is to be followed.

4. NUMERICAL RESULTS

As mentioned earlier the procedure is demonstrated for a two storeyed frame structure. Implementing the above procedure with the proposed iterative cycle for the revised uncertain frequencies and parameters, computer programs have been written and tested for the above example problem.

In this example, floor masses, $m=(3550,3600,3650)$ Kg and the column stiffnesses $\tilde{k}_1=\tilde{k}_2=(5350,5400,5450)$ N/m, $\tilde{k}_3=\tilde{k}_4=(3550,3600,3650)$ N/m have been taken as triangular fuzzy number. Through $\alpha-$cut these may represented as $m=[50\alpha+3550,-50\alpha+3650]$ Kg, $\tilde{k}_1=\tilde{k}_2=[50\alpha+5350,-50\alpha+5450]$ N/m and $\tilde{k}_3=\tilde{k}_4=[50\alpha+3550,-50\alpha+3650]$N/m. From these prior mass and stiffness parameters, the uncertain vibration characteristics may be computed from Eq. (2) as $\tilde{\lambda}_1=(0.9314,1,1.0703)$ and $\tilde{\lambda}_2=(5.8906,6,6.1128)$.

Using the above sets of initial data of the fuzzy parameters with different uncertain experimental (hypothetical) test data for the frequencies, viz. $\tilde{\lambda}_{1E}=(0.65,0.7,0.75)$ and $\tilde{\lambda}_{2E}=(5.3,5.5,5.7)$ (i.e. first and second experimental eigenvalues of the system) the bounds of the stiffness parameters of the structure have been identified and these are reported in Table 1. Corresponding plot for identified stiffness parameters are depicted in Figs. 2 and 3.

Bounds of stiffness parameters (N/m)	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 1$
$\underline{k}_1 = \underline{k}_2$	5232	5274	5316
$\overline{k}_1 = \overline{k}_2$	5400.5	5358	5316
$\underline{k}_3 = \underline{k}_4$	3614.5	3629.8	3645
$\overline{k}_3 = \overline{k}_4$	3674.7	3659.9	3645

Table 1: Identified lower and upper bounds of stiffness parameters

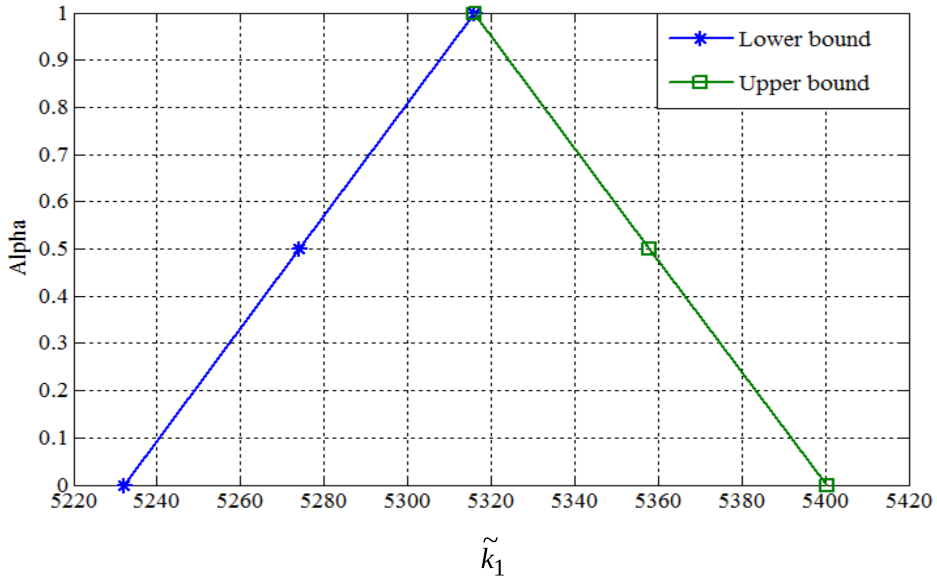

Figure 2. Identified lower and upper bounds of stiffness parameter $\tilde{k}_1$(N/m)

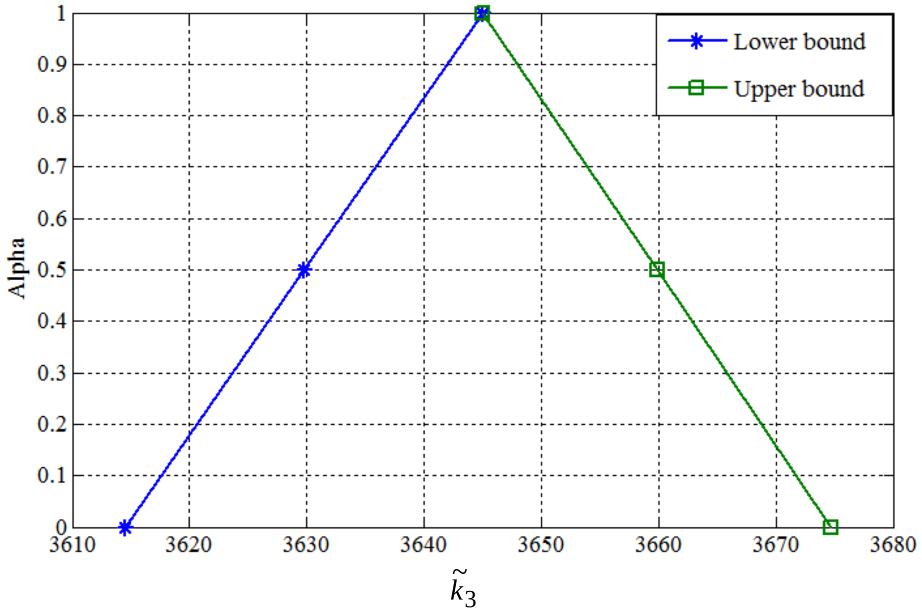

Figure 3. Identified lower and upper bounds of stiffness parameter $\tilde{k}_3$(N/m)

Similarly for another set of experimental (hypothetical) fuzzy data of the natural frequencies $\tilde{\lambda}_{1E} = (0.88, 0.9, 0.92)$ and $\tilde{\lambda}_{2E} = (5.3, 5.5, 5.7)$ the identified bounds of the stiffness parameters are tabulated in Table 2 and the revised frequencies are also shown in Table 3. Corresponding plot for identified stiffness parameters are given in Figs. 4 and 5.

Bounds of stiffness parameters (N/m)	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 1$
$\underline{k}_1 = \underline{k}_2$	5286	5324	5362
$\overline{k}_1 = \overline{k}_2$	5440	5401	5362
$\underline{k}_3 = \underline{k}_4$	3590.6	3607.7	3624.5
$\overline{k}_3 = \overline{k}_4$	3657.6	3641	3624.5

Table 2: Identified lower and upper bounds of stiffness parameters

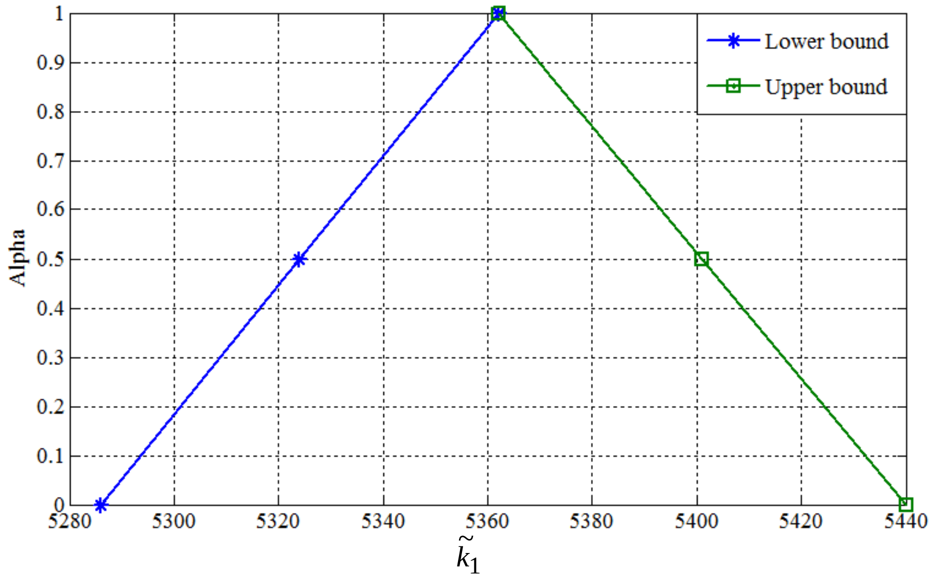

Figure 4. Identified lower and upper bounds of stiffness parameter $\tilde{k}_1$(N/m)

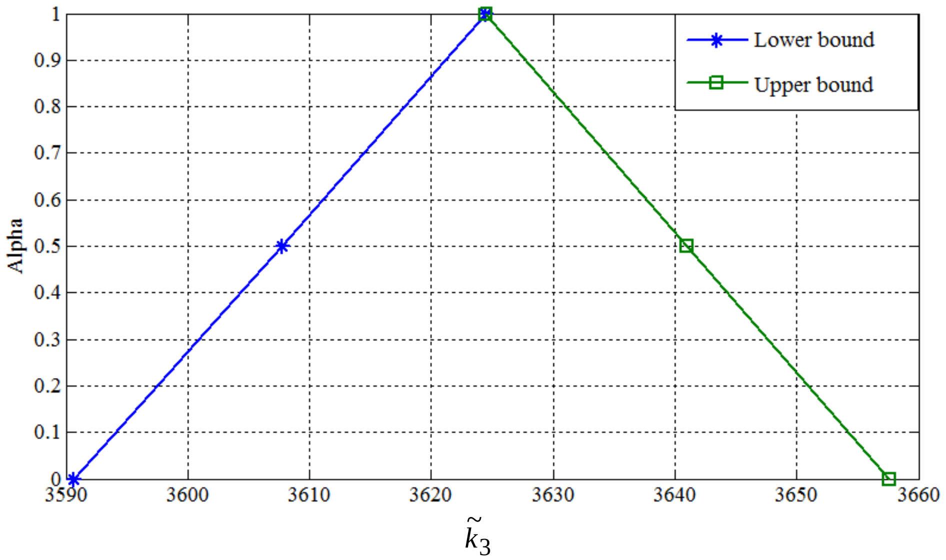

Figure 5. Identified lower and upper bounds of stiffness parameter $\tilde{k}_3$ (N/m)

α	$\underline{\lambda}_{1E}$	$\overline{\lambda}_{1E}$	$\underline{\lambda}_{2E}$	$\overline{\lambda}_{2E}$	$\underline{\lambda}_{1R}$	$\overline{\lambda}_{1R}$	$\underline{\lambda}_{2R}$	$\overline{\lambda}_{2R}$
0	0.88	0.92	5.3	5.7	0.9430	1.0552	5.8884	6.1306
0.5	0.89	0.91	5.4	5.6	0.9706	1.0267	5.9476	6.0688
1	0.9	0.9	5.5	5.5	0.9985	0.9985	6.0078	6.0078

Table 3: Experimental and revised lower and upper bounds of frequencies

5. DISCUSSIONS AND CONCLUSIONS

The presented procedure systematically modifies and identifies the uncertain structural parameters, viz. the column stiffness for a frame structure. It uses the prior known estimates of uncertain parameters and corresponding uncertain vibration characteristics and then the algorithm estimates the bounds of present parameters utilizing the known uncertain dynamic data from some experiments. Numerical procedure is tested by incorporating two sets of data. The uncertainties present in the parameters are considered as triangular convex normalized fuzzy sets. It is worth mentioning that if the input data set viz. the design frequency is near to the experimental frequency data then the modified stiffness data bound has less width. This is expected as the design and experimental frequency are close means that the structure has not deteriorated much. On the other hand when the experimental data is taken bit far from the deigned one then the estimated stiffness parameters give larger bound. These effects may be clearly seen from Tables 1 and 2. It may be noted that the accuracy of the results depends upon many factors viz. on the uncertain bound of the experimental data, initial design values of the parameters, the fuzzy computation, norm as defined etc. The present investigation may be a first of its kind to handle the identification procedure for uncertain data. As such there may be many improvements in the present procedure keeping in mind the accuracy and efficiency in fuzzy computation and others. Present investigators are working in these lines and will communicate the findings in more detail in future communications. Although the method has been demonstrated for a simple problem of two storey, but the method may very well be extended to higher storey frames and other structures in a similar fashion.

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