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Decentralized Mixed Delay-Dependent/Delay-Independent Stabilization of Interconnected Time-Delay Systems

Sandip Ghosh * **Sarit K. Das** ** **Goshaidas Ray** **

* *Department of Electrical Engineering, National Institute of Technology, Rourkela, India, (e-mail: sandipg@nitrkl.ac.in).*
** *Department of Electrical Engineering, Indian Institute of Technology, Kharagpur, India, (e-mail: gray, skdas@ee.iitkgp.ernet.in)*

Abstract: The problem of decentralized stabilization of interconnected systems that are delay-dependent for some delays and delay-independent for the remaining ones is addressed in this paper. For the purpose, some of the subsystems are grouped to form new larger subsystems. Such a grouping allows all the interconnection delays present among the smaller subsystems to be divided into two groups in the new subsystem description — (i) intraconnection delays, within the larger subsystem, and (ii) interconnection delays, among the larger subsystems. This facilitates delay-dependent stabilization for the intraconnection delays and delay-dependent stabilization for the interconnection delays. An LMI based stabilization criterion is derived for such mixed delay-dependent/delay-independent stabilization of the overall system. A numerical example is presented demonstrating the effectiveness of the developed criterion.

Keywords: Interconnected systems, Decentralized control, Time-delay systems.

1. INTRODUCTION

Large-scale systems comprising of several interconnected systems inherits interconnection delays that are often time-varying in nature [Bakule, 2008]. Since analysis of even a simpler system with time-varying delays is complex and only sufficient conditions exist for them in the sense of Lyapunov, it is difficult to analyze large-scale systems with several delays. Mostly, for such systems, delay-independent stability analysis that does not require information on the size of the delay has been carried out based on Lyapunov-Krasovskii approach [Hmamed, 1986, Lee and Radovic, 1988, Hu, 1994, Trinh and Aldeen, 1995, Oucheriah, 2000, Nian and Li, 2001, Ghosh et al., 2009]. It may, however, be the case that some of these interconnection delays are small and then the above delay-independent methods may yield conservative results. The cases when all the interconnection delays are known have been analyzed by employing delay-dependent analysis in Tsay et al. [1996], de Souza and Li [1999], Fernando et al. [2012]. Note that, in such cases all the delays are required to be bounded.

The delay-dependent and delay-independent stability of large-scale systems can be interpreted corresponding to its cooperative and competitive stability [Šiljak, 1978]. The cooperative stability is encountered when subsystems co-operatively stabilize the overall system, e.g., in formation control of autonomous vehicles [Stipanovic et al., 2004]. On the other hand, the competitive (also see connective controllability [Sezer and Huseyin, 1981]) for which any combinations of interconnected subsystems are stable. Such connective stability can be inferred to the delay-independent stability for the interconnection delays.

For connective stability analysis, the decomposition and aggregation principle of Lyapuov functions are followed [Šiljak, 1978]. However, the same does not apply to co-operative stability problems since individual subsystems may be unstable for such cases. Analysis of systems using delay-dependent stability method developed by following the decomposition and aggregation principle in de Souza and Li [1999] and Fernando et al. [2012] does not work for some cooperative stability problems that involves unstable subsystems. To this end, it is shown in Ghosh et al. [2010] that grouping of subsystems beforehand on the basis of delay-dependent/independent features and then employing analysis is useful for stability analysis of such systems.

This paper considers the problem of decentralized state feedback stabilization of linear large-scale systems subjected to both finite and arbitrary interconnection delays. For the purpose, the subsystems with finite delays among themselves are grouped to form larger subsystems that contain multiple finite intraconnection delays within them but have multiple arbitrary interconnection delays among themselves. It is considered that the smaller subsystems within the larger ones share their state information for control. Based on stability criterion developed in Ghosh et al. [2010], a stabilization criterion is derived with appropriate linearization of the resulting nonlinear matrix inequality. A numerical example is presented to show the effectiveness of the proposed criterion in terms of exploiting the delay-dependent information for some delays and, at the same time, being delay-independent for the others.

2. STABILITY ANALYSIS

Stability analysis of large-scale systems in a generalized framework with respect to the interconnection delays has earlier been developed in Ghosh et al. [2010]. In this section, we briefly introduce this result retaining the notation used in Ghosh et al. [2010] which is further used to derive the stabilization result of this paper. It may be noted that such analysis requires grouping of the smaller subsystems to form larger subsystems that would be conducive to the desired analysis as presented in the following subsection.

2.1 Grouping of subsystems

Consider a system having N^* number of interconnected subsystems, the i^{th} one of which is described as:

$$S_i^* : \dot{x}_i^*(t) = G_i x_i^*(t) + \sum_{j=1}^{N^*} H_{ij} x_j^*(t - \tau_{ij}^*), \quad i = 1, 2, \ldots, N^*, \qquad (1)$$

where $x_i^*(t) \in \Re^{n_i^*}$ is the state vector of S_i^*; $\tau_{ij}^* \geq 0$, $j = 1, 2, ..., N^*$ represent the interconnection delays present in the system and G_i, H_{ij}, $j = 1, 2, ..., N^*$ are appropriate dimensional matrices. The overall system can alternatively be represented using larger subsystem (S_i), each of which may contain several S_i^* that shares finite interconnection delays among themselves. However, the delays among these S_i may be arbitrary. As a result, the subsystems S_is are having several finite intraconnection delays within themselves whereas the interconnection delays among them are arbitrary. Letting that the total number of such S_is is N, the i^{th} one consisting of q_i number of S_i^* of (1), the new subsystem description becomes

$$S_i : \dot{x}_i(t) = A_i x_i(t) + \sum_{k=1}^{q_i^2} C_{ik} x_i(t - \eta_{ik}) + \sum_{\substack{j=1 \\ j\neq i}}^{N} \sum_{k=1}^{q_i q_j} D_{ijk} x_j(t - \tau_{ijk}), i = 1, 2, \ldots, N, (2)$$

where $x_i(t) \in \Re^{n_i}$ with $n_i = \sum_{k=(r_{i-1}+1)}^{r_i} n_k^*$; A_i, C_{il} and D_{ijk} are matrices with proper dimensions. Note that, η_{ik}, $k = 1, 2, \ldots, q_i^2$, are the finite intraconnections delays and τ_{ijk}, $j = 1, 2, \ldots, N$, $k = 1, 2, \ldots, q_i q_j$, are arbitrary interconnection delays. Further, η_{ik} satisfies $0 \leq \eta_{ik} \leq \bar{\eta}_{ik}$ and τ_{ijk} satisfies $0 \leq \tau_{ijk} < \infty$. For stability, it is necessary that $\left(A_i + \sum_{k=1}^{q_i^2} C_{ik}\right)$, $i = 1, 2, \ldots, N$, are Hurwitz.

It may be noted that the above grouping of subsystems divides all the delays into two groups in the system representation (2). The intraconnection delays appear as the local delays in the larger subsystems whereas interconnection delays appear as it is for the smaller subsystems but with multiple delays in between two larger subsystems. The former one is important since treating the interconnection delays of the smaller subsystems as the local delays of the larger subsystems provides the benefit of exploiting the delay-dependent analysis with respect to the local delays in (2) since the decomposition and aggregation principle is now applied on the larger subsystems.

2.2 Criterion for Analysis

The following lemma presents a stability condition following the result in Ghosh et al. [2010] (equation (23) therein).

Lemma 2.1. System (2) is asymptotically stable if $Q_{ijk} = Q_{ijk}^T > 0$, $j = 1, 2, ..., N$, $k = 1, 2, ..., (q_i q_j)$, can arbitrarily be chosen such that there exist (i) positive definite symmetric matrices P_i, Q_{ik} and R_{ik}, (ii) matrices L_{1i}, L_{2i}, M_{il}, N_{1il}, N_{2il} and N_{3il}, $l = 1, 2, \ldots, q_i^2$, that satisfy the following LMI:

$$\begin{bmatrix} \Omega_i^{11} & \Omega_i^{12} & \Omega_i^{11} \\ * & \Omega_i^{22} & \Omega_i^{23} \\ * & * & \Omega_i^{33} \end{bmatrix} - \begin{bmatrix} \Omega_i^{14} \\ \Omega_i^{24} \\ \Omega_i^{34} \end{bmatrix} \left(\Omega_i^{44}\right)^{-1} \begin{bmatrix} \Omega_i^{14} \\ \Omega_i^{24} \\ \Omega_i^{34} \end{bmatrix}^T + \sum_{j=1}^{N} \sum_{k=1}^{q_i q_j} \mathcal{M}_i D_{ijk} Q_{ijk}^{-1} D_{ijk}^T \mathcal{M}_i^T < 0, \qquad (3)$$

where

$$\Omega_i^{11} = L_{1i} A_i + A_i^T L_{1i}^T + \sum_{k=1}^{q_i^2} (Q_{ik} + N_{1ik} + N_{1ik}^T) + \sum_{\substack{j=1 \\ j\neq i}}^{N} \sum_{k=1}^{q_i q_j} Q_{jik};$$

$$\Omega_i^{12} = P_i - L_{1i} + A_i^T L_{2i}^T + \sum_{k=1}^{q_i^2} N_{2ik}^T;$$

$$\Omega_i^{22} = -L_{2i} - L_{2i}^T + \sum_{k=1}^{q_i^2} \bar{\eta}_{ik} R_{ik};$$

$$\Omega_i^{13} = \left[\Omega_{i1}^{13} \; \Omega_{i2}^{13} \; \ldots \; \Omega_{iq_i^2}^{13} \right],$$

$$\Omega_{ik}^{13} = L_{1i} C_{ik} + A_i^T M_{ik}^T - N_{1ik} + N_{3ik}^T;$$

$$\Omega_i^{23} = \left[\Omega_{i1}^{23} \; \Omega_{i2}^{23} \; \ldots \; \Omega_{iq_i^2}^{23} \right], \Omega_{ik}^{23} = L_{2i} A_i - M_{ik}^T - N_{2ik};$$

$$\Omega_i^{33} = \begin{bmatrix} \Omega_{i11}^{33} & \Omega_{i12}^{33} & \ldots & \Omega_{i1q_i^2}^{33} \\ * & \Omega_{i22}^{33} & \ldots & \Omega_{i2q_i^2}^{33} \\ * & * & \ddots & \vdots \\ * & * & * & \Omega_{iq_i^2 q_i^2}^{33} \end{bmatrix}, \quad \begin{aligned} \Omega_{ikk}^{33} &= M_{ik} C_{ik} + C_{ik}^T M_{ik}^T - N_{3ik} - N_{3ik}^T - Q_{ik}, \\ \Omega_{ilk}^{33} &= M_{il} C_{ik} + C_{il}^T M_{ik}^T, \quad l \neq k; \end{aligned}$$

$$\Omega_i^{14} = \left[N_{1i1} \; N_{1i2} \; \ldots \; N_{1iq_i^2} \right]; \Omega_i^{24} = \left[N_{2i1} \; N_{2i2} \; \ldots \; N_{2iq_i^2} \right];$$

$$\Omega_i^{34} = \mathrm{diag}\left\{ N_{3i1}, N_{3i2}, \ldots, N_{3iq_i^2} \right\};$$

$$\Omega_i^{44} = \mathrm{diag}\left\{ -\bar{\eta}_{i1}^{-1} R_{i1}, -\bar{\eta}_{i2}^{-1} R_{i2}, \ldots, -\bar{\eta}_{i(q_i^2)}^{-1} R_{iq_i^2} \right\},$$

$$\mathcal{M}_i = \left[L_{1i}^T \; L_{2i}^T \; M_{i1}^T \; M_{i2}^T \; \ldots \; M_{iq_i^2}^T \right]^T.$$

3. THE STABILIZATION PROBLEM

In this section, the problem of decentralized state feedback stabilization of interconnected systems with both finite and arbitrary interconnection delays is considered. For this, consider system (2) with each subsystem having a local control input. Then the i^{th} subsystem dynamics, $i = 1, \ldots, N$, may be written as:

$$\hat{S}_i : \quad \dot{x}_i(t) = A_i x_i(t) + \sum_{k=1}^{q_i^2} C_{ik} x_i(t - \eta_{ik})$$

$$+\sum_{\substack{j=1\\j\neq i}}^{N}\sum_{k=1}^{q_iq_j} D_{ijk}x_j(t-\tau_{ijk})+B_iu_i(t), \quad (4)$$

where all the terms are as defined in (2) with, in addition, $u_i(t)\in\Re^{m_i}$ is the local control input to the i^{th} subsystem and $B_i\in\Re^{n_i\times m_i}$ is the control input matrix.
The objective of this section is to design a stabilizing decentralized state feedback controller in the form of

$$u_i(t)=K_ix_i(t), \quad (5)$$

where $K_i\in\Re^{m_i\times n_i}$, $i=1,\ldots,N$. Note that, designing the control gains of (5) is equivalent to the synthesis of K_i for the i^{th} subsystem (4). For the existence of a stabilizing controller (5), it is assumed that (i) all subsystem states $x_i(t)$, $i=1,2,\ldots,N$, are locally measurable, and (ii) the pair $\left\{\left(A_i+\sum_{k=1}^{q_i^2}C_{ik}\right),B_i\right\}$, $i=1,2,...,N$, are controllable. The latter assumption arises from the particular consideration of $\eta_{ik}=0$, $k=1,2,\ldots,q_i^2$ and $\tau_{ijk}=\infty$, $j=1,2,...,N$, $k=1,2,...,q_iq_j$ for the i^{th} subsystem.

The controllability criterion above plays an important role in selection of q_is for grouping the smaller subsystems. An approach for this selection would be to start with a smaller subsystem and checking its controllability. If uncontrollable then it may be grouped with other subsystems still the criterion is satisfied for the whole system with minimum possible smaller subsystems in each group.

4. STABILIZATION CRITERION

Theorem 1. System (4) is stabilizable with the feedback (5) if δ_{i1}, $\delta_{i(k+1)}$, $k=1,2,\ldots,q_i^2$ and $Q_{ijl}=Q_{ijl}^T>0$, $j=1,2,...,N$, $l=1,2,...,q_iq_j$, can first be arbitrarily chosen such that there exists (a) positive definite symmetric matrices $\hat{P}_i$, $\hat{Q}_{ik}$ and $\hat{R}_{ik}$, (b) invertible matrix $\hat{M}_i$, (c) matrices $\hat{N}_{1ik}$, $\hat{N}_{2ik}$, $\hat{N}_{3ik}$ and $\hat{Y}_i$ that satisfy the following LMI:

$$\begin{bmatrix}\Xi_i^{11} & \Xi_i^{12} & \Xi_i^{13} & \Xi_i^{14} & \Xi_i^{15}\\ * & \Xi_i^{22} & \Xi_i^{23} & \Xi_i^{24} & 0\\ * & * & \Xi_i^{33} & \Xi_i^{34} & 0\\ * & * & * & \Xi_i^{44} & 0\\ * & * & * & * & \Xi_i^{55}\end{bmatrix}<0, \quad (6)$$

where

$$\Xi_i^{11}=A_i\hat{M}_i^T+\hat{M}_iA_i^T+B_i\hat{Y}_i+\hat{Y}_i^TB_i^T+\sum_{k=1}^{q_i^2}\left(\hat{Q}_{ik}+\hat{N}_{1ik}+\hat{N}_{1ik}^T\right)+D_i;$$

$$\Xi_i^{12}=\hat{P}_i-\hat{M}_i^T+\delta_{i1}\hat{M}_iA_i^T+\delta_{i1}\hat{Y}_i^TB_i^T+\sum_{k=1}^{q_i^2}\hat{N}_{2ik}^T+\delta_{i1}D_i;$$

$$\Xi_i^{22}=-\delta_{i1}\hat{M}_i^T-\delta_{i1}\hat{M}_i+\sum_{l=1}^{q_i^2}\bar{\eta}_{ik}\hat{R}_{ik}+\delta_{i1}\delta_{i1}D_i;$$

$$\Xi_i^{13}=\left[\,\Xi_{i1}^{13}\ \Xi_{i2}^{13}\ \ldots\ \Xi_{iq_i^2}^{13}\,\right],$$

$$\Xi_{ik}^{13}=C_{ik}\hat{M}_i^T+\delta_{i(k+1)}\hat{M}_iA_i^T+\delta_{i(k+1)}\hat{Y}_i^TB_i^T-\hat{N}_{1ik}+\hat{N}_{3ik}^T+\delta_{i(k+1)}D_i;$$

$$\Xi_i^{23}=\left[\,\Xi_{i1}^{23}\ \Xi_{i2}^{23}\ \ldots\ \Xi_{iq_i^2}^{23}\,\right]^T,$$

$$\Xi_{ik}^{23}=\delta_{i1}A_i\hat{M}_i^T+\delta_{i1}B_i\hat{Y}_i-\delta_{i(k+1)}\hat{M}_i-\hat{N}_{2ik}+\delta_{i1}\delta_{i(k+1)}D_i;$$

$$\Xi_i^{33}=\begin{bmatrix}\Xi_{i11}^{33} & \Xi_{i12}^{33} & \ldots & \Xi_{i1q_i^2}^{33}\\ * & \Xi_{i22}^{33} & \ldots & \Xi_{i2q_i^2}^{33}\\ & & \ddots & \vdots\\ * & * & & \\ * & * & * & \Xi_{iq_i^2q_i^2}^{33}\end{bmatrix},$$

$$\Xi_{ikk}^{33}=\delta_{i(k+1)}C_{ik}\hat{M}_i^T+\delta_{i(k+1)}\hat{M}_iC_{ik}^T-\hat{N}_{3ik}-\hat{N}_{3ik}^T-\hat{Q}_{ik}+\delta_{i(k+1)}\delta_{i(k+1)}D_i,$$

$$\Xi_{ilk}^{33}=\delta_{i(l+1)}C_{ik}\hat{M}_i^T+\delta_{i(k+1)}\hat{M}_iC_{il}^T+\delta_{i(l+1)}\delta_{i(k+1)}D_i,\quad l\neq k;$$

$$\Xi_i^{14}=\left[\,\hat{N}_{1i1}\ \hat{N}_{1i2}\ \ldots\ \hat{N}_{1iq_i^2}\,\right];$$

$$\Xi_i^{24}=\left[\,\hat{N}_{3i1}\ \hat{N}_{3i2}\ \ldots\ \hat{N}_{3iq_i^2}\,\right];$$

$$\Xi_i^{34}=diag\left\{\hat{N}_{2i1},\hat{N}_{2i2},\ldots,\hat{N}_{2iq_i^2}\right\};$$

$$\Xi_i^{44}=diag\left\{-\bar{\eta}_{i1}^{-1}\hat{R}_{i1},-\bar{\eta}_{i2}^{-1}\hat{R}_{i2},\ldots,-\bar{\eta}_{iq_i^2}^{-1}\hat{R}_{iq_i^2}\right\};$$

$$\Xi_i^{15}=\left[\,\Xi_{i1}^{15}\ \Xi_{i2}^{15}\ \ldots\ \Xi_{iN}^{15}\,\right],$$

$$\Xi_{ij}^{15}=\left[\,\hat{M}_iQ_{ji1}\ \ \hat{M}_iQ_{ji2}\ \ \ldots\ \ \hat{M}_iQ_{ji(q_iq_j)}\,\right];$$

$$\Xi_i^{55}=diag\left\{\Xi_{i1}^{55},\Xi_{i2}^{55},\ldots,\Xi_{iN}^{55}\right\},$$

$$\Xi_{ij}^{55}=diag\left\{-Q_{ji1},-Q_{ji2},\ldots,-Q_{ji(q_iq_j)}\right\};$$

$$D_i=\sum_{j=1}^{N}\sum_{k=1}^{q_iq_j}D_{ijk}Q_{ijk}^{-1}D_{ijk}^T.$$

Further, the stabilizing control gains for the i^{th} subsystem can be written as:

$$K_i=\hat{Y}_i\left(\hat{M}_i^T\right)^{-1}. \quad (7)$$

Proof. The closed-loop system (4) along with the controller (5) can be written as

$$\dot{x}_i(t)=\bar{A}_ix_i(t)+\sum_{k=1}^{q_i^2}C_{ik}x_i(t-\eta_{ik})+\sum_{\substack{j=1\\j\neq i}}^{N}\sum_{k=1}^{q_iq_j}D_{ijk}x_j(t-\tau_{ijk}), \quad (8)$$

where $\bar{A}_i=A_i+B_iK_i$.
Then, following Lemma 1, one may write the stability criterion for system (8), in view of (3), as:

$$\begin{bmatrix}\Omega_i^{11} & \Omega_i^{12} & \Omega_i^{11}\\ * & \Omega_i^{22} & \Omega_i^{23}\\ * & * & \Omega_i^{33}\end{bmatrix}-\begin{bmatrix}\Omega_i^{14}\\ \Omega_i^{24}\\ \Omega_i^{34}\end{bmatrix}\left(\Omega_i^{44}\right)^{-1}\begin{bmatrix}\Omega_i^{14}\\ \Omega_i^{24}\\ \Omega_i^{34}\end{bmatrix}^T+\sum_{j=1}^{N}\sum_{k=1}^{q_iq_j}\mathcal{M}_iD_{ijk}Q_{ijk}^{-1}D_{ijk}^T\mathcal{M}_i^T<0, \quad (9)$$

where

$$\Omega_i^{11} = L_{1i}\bar{A}_i + \bar{A}_i^T L_{1i}^T + \sum_{k=1}^{q_i^2}\left(Q_{ik} + N_{1ik} + N_{1ik}^T\right) + \sum_{\substack{j=1\\ j\neq i}}^{N}\sum_{k=1}^{q_iq_j} Q_{jik},$$

$$\Omega_i^{12} = P_i - L_{1i} + \bar{A}_i^T L_{2i}^T + \sum_{k=1}^{q_i^2} N_{2ik}^T;$$

$$\Omega_i^{22} = -L_{2i} - L_{2i}^T + \sum_{k=1}^{q_i^2} \bar{\eta}_{ik} R_{ik},$$

$$\Omega_i^{13} = \left[\, \Omega_{i1}^{13}\ \Omega_{i2}^{13}\ \dots\ \Omega_{iq_i^2}^{13} \,\right],$$

$$\Omega_{ik}^{13} = L_{1i}C_{ik} + \bar{A}_i^T M_{ik}^T - N_{1ik} + N_{3ik}^T,$$

and all other terms are as defined in (3). It may be noted that, (9) involves nonlinear terms since it has multiplication of L_{1i}, L_{2i} and M_{ik}, $k = 1, 2, \dots, q_i^2$ with B_iK_i. Now, to take care of this we employ the following procedure. First, assume $\hat{M}_i = L_{1i}^{-1}$, $\hat{M}_i L_{2i} = \delta_{i1} I$, $\hat{M}_i M_{ik} = \delta_{i(k+1)} I$ and define $\hat{\mathcal{M}}_i = \text{diag}\{\hat{M}_i, \hat{M}_i, \tilde{\mathcal{M}}_i\} \in \Re^{(2+q_i^2)n_i \times (2+q_i^2)n_i}$, $\tilde{\mathcal{M}}_i = \text{diag}\{\hat{M}_i, \hat{M}_i, \dots, \hat{M}_i\} \in \Re^{q_i^2 n_i \times q_i^2 n_i}$. Next, pre- and post-multiply (9), respectively, with $\hat{\mathcal{M}}_i$ and its transpose and then defining $\hat{P}_i = \hat{M}_i P_i \hat{M}_i^T$, $Y_i = K_i \hat{M}_i^T$, $\hat{N}_{1ik} = \hat{M}_i N_{1ik} \hat{M}_i^T$, $\hat{N}_{2ik} = \hat{M}_i N_{2ik} \hat{M}_i^T$, $\hat{N}_{3ik} = \hat{M}_i N_{3ik} \hat{M}_i^T$, $\hat{Q}_{ik} = \hat{M}_i Q_{ik} \hat{M}_i^T$ and $\hat{R}_{ik} = \hat{M}_i R_{ik} \hat{M}_i^T$, $k = 1, 2, \dots, q_i^2$, one obtains

$$\begin{bmatrix} \Pi_i^{11} & \Pi_i^{12} & \Pi_i^{13} \\ * & \Pi_i^{22} & \Pi_i^{23} \\ * & * & \Pi_i^{33} \end{bmatrix} - \hat{\mathcal{M}}_i \begin{bmatrix} \Omega_i^{14} \\ \Omega_i^{24} \\ \Omega_i^{34} \end{bmatrix} \left(\Omega_i^{44}\right)^{-1} \begin{bmatrix} \Omega_i^{14} \\ \Omega_i^{24} \\ \Omega_i^{34} \end{bmatrix}^T \hat{\mathcal{M}}_i^T + \sum_{j=1}^{N}\sum_{k=1}^{q_iq_j} \hat{\mathcal{M}}_i \mathcal{M}_i D_{ijk} Q_{ijk}^{-1} D_{ijk}^T \mathcal{M}_i^T \hat{\mathcal{M}}_i^T < 0, \quad (10)$$

where

$$\Pi_i^{11} = A_i\hat{M}_i^T + \hat{M}_i A_i^T + B_i\hat{Y}_i + \hat{Y}_i^T B_i^T + \sum_{k=1}^{q_i^2}\left(\hat{Q}_{ik} + \hat{N}_{1ik} + \hat{N}_{1ik}^T\right) + \sum_{j=1}^{N}\sum_{k=1}^{q_iq_j} \hat{M}_i Q_{jik} \hat{M}_i^T;$$

$$\Pi_i^{12} = \hat{P}_i - \hat{M}_i^T + \delta_{i1}\hat{M}_i A_i^T + \delta_{i1}\hat{Y}_i^T B_i^T + \sum_{k=1}^{q_i^2} \hat{N}_{2ik}^T;$$

$$\Pi_i^{13} = \left[\, \Pi_{i1}^{13}\ \Pi_{i2}^{13}\ \dots\ \Pi_{iq_i^2}^{13} \,\right],$$

$$\Pi_{ik}^{13} = C_{ik}\hat{M}_i^T + \delta_{i(k+1)}\hat{M}_i A_i^T + \delta_{i(k+1)}\hat{Y}_i^T B_i^T - \hat{N}_{1ik} + \hat{N}_{3ik}^T;$$

$$\Pi_i^{22} = -\delta_{i1}\hat{M}_i^T - \delta_{i1}\hat{M}_i + \sum_{l=1}^{q_i^2} \bar{\eta}_{ik}\hat{R}_{ik};$$

$$\Pi_i^{23} = \left[\, \Pi_{i1}^{23}\ \Pi_{i2}^{23}\ \dots\ \Pi_{iq_i^2}^{23} \,\right]^T;$$

$$\Pi_{ik}^{23} = \delta_{i1}A_i\hat{M}_i^T + \delta_{i1}B_i\hat{Y}_i - \delta_{i(k+1)}\hat{M}_i - \hat{N}_{2ik};$$

$$\Pi_i^{33} = \begin{bmatrix} \Pi_{i11}^{33} & \Pi_{i12}^{33} & \dots & \Pi_{i1q_i^2}^{33} \\ * & \Pi_{i22}^{33} & \dots & \Pi_{i2q_i^2}^{33} \\ * & * & \ddots & \vdots \\ * & * & * & \Pi_{iq_i^2q_i^2}^{33} \end{bmatrix},$$

$$\Pi_{ikk}^{33} = \delta_{i(k+1)}C_{ik}\hat{M}_i^T + \delta_{i(k+1)}\hat{M}_i C_{ik}^T - \hat{N}_{3ik} - \hat{N}_{3ik}^T - \hat{Q}_{ik},$$

$$\Pi_{ilk}^{33} = \delta_{i(l+1)}C_{ik}\hat{M}_i^T + \delta_{i(k+1)}\hat{M}_i C_{il}^T, \quad l \neq k.$$

Next, consider the second term in the LHS of (10). It can be rewritten as:

$$-\hat{\mathcal{M}}_i \begin{bmatrix} \Omega_i^{14} \\ \Omega_i^{24} \\ \Omega_i^{34} \end{bmatrix} \left(\Omega_i^{44}\right)^{-1} \begin{bmatrix} \Omega_i^{14} \\ \Omega_i^{24} \\ \Omega_i^{34} \end{bmatrix}^T \hat{\mathcal{M}}_i^T = - \begin{bmatrix} \hat{M}_i & 0 & 0 \\ * & \hat{M}_i & 0 \\ * & * & \tilde{\mathcal{M}}_i \end{bmatrix} \begin{bmatrix} \Omega_i^{14} \\ \Omega_i^{24} \\ \Omega_i^{34} \end{bmatrix} \tilde{\mathcal{M}}_i^T \left(\tilde{\mathcal{M}}_i \Omega_i^{44} \tilde{\mathcal{M}}_i^T\right)^{-1} \times \tilde{\mathcal{M}}_i \begin{bmatrix} \Omega_i^{14} \\ \Omega_i^{24} \\ \Omega_i^{34} \end{bmatrix}^T \begin{bmatrix} \hat{M}_i^T & 0 & 0 \\ * & \hat{M}_i^T & 0 \\ * & * & \tilde{\mathcal{M}}_i^T \end{bmatrix}. \quad (11)$$

Now, since $\tilde{\mathcal{M}}_i = \text{diag}\{\hat{M}_i, \hat{M}_i, \dots, \hat{M}_i\} \in \Re^{q_i^2 n_i \times q_i^2 n_i}$ and $\hat{N}_{jik} = \hat{M}_i N_{jik} \hat{M}_i^T$, in view of structure of Ω_i^{14}, Ω_i^{24}, Ω_i^{34} as in (3), one can write

$$\begin{bmatrix} \hat{M}_i & 0 & 0 \\ * & \hat{M}_i & 0 \\ * & * & \tilde{\mathcal{M}}_i \end{bmatrix} \begin{bmatrix} \Omega_i^{14} \\ \Omega_i^{24} \\ \Omega_i^{34} \end{bmatrix} \tilde{\mathcal{M}}_i^T = \begin{bmatrix} \Xi_i^{14} \\ \Xi_i^{24} \\ \Xi_i^{34} \end{bmatrix}. \quad (12)$$

where Ξ_i^{14}, Ξ_i^{24} and Ξ_i^{34} are as in (6). Further, in view of Ω_i^{34} as in (3) and Ξ_i^{44} in (6), one may write

$$\left(\tilde{\mathcal{M}}_i \Omega_i^{44} \tilde{\mathcal{M}}_i^T\right) = \Xi_i^{44}. \quad (13)$$

Using (12) and (13), LHS of (11) may be written as

$$-\hat{\mathcal{M}}_i \begin{bmatrix} \Omega_i^{14} \\ \Omega_i^{24} \\ \Omega_i^{34} \end{bmatrix} \left(\Omega_i^{44}\right)^{-1} \begin{bmatrix} \Omega_i^{14} \\ \Omega_i^{24} \\ \Omega_i^{34} \end{bmatrix}^T \hat{\mathcal{M}}_i^T = - \begin{bmatrix} \Xi_i^{14} \\ \Xi_i^{24} \\ \Xi_i^{34} \end{bmatrix} \left(\Xi_i^{44}\right)^{-1} \begin{bmatrix} \Xi_i^{14} \\ \Xi_i^{24} \\ \Xi_i^{34} \end{bmatrix}^T. \quad (14)$$

Now, one has to take care of the last term in the LHS of (10). In view of the structure of $\mathcal{M}_i$ as defined in (3), one obtains

$$\hat{\mathcal{M}}_i \mathcal{M}_i = \left[\, L_{1i}^T \hat{M}_i^T\ L_{2i}^T \hat{M}_i^T\ M_{i1}^T \hat{M}_i^T\ \dots\ M_{iq_i^2}^T \hat{M}_i^T \,\right]^T = \left[\, I\ \delta_{i1}I\ \delta_{i2}I\ \dots\ \delta_{i(q_i^2+1)}I \,\right]^T. \quad (15)$$

Note that, even though the above linearization is restrictive but it still helps to formulate the stabilization criterion in a convenient LMI form.

The last term in the LHS of (10) can be written in expanded form as

$$\sum_{j=1}^{N}\sum_{k=1}^{q_iq_j} \hat{\mathcal{M}}_i \mathcal{M}_i D_{ijk} Q_{ijk}^{-1} D_{ijk}^T \mathcal{M}_i^T \hat{\mathcal{M}}_i^T$$

$$= \left[\begin{array}{c|c|cccc} D_i & \delta_{i1}D_i & \delta_{i2}D_i & \dots & \delta_{i(q_i^2+1)}D_i \\ \hline * & \delta_{i1}\delta_{i1}D_i & \delta_{i1}\delta_{i2}D_i & \dots & \delta_{i1}\delta_{i(q_i^2+1)}D_i \\ \hline * & * & \delta_{i2}\delta_{i2}D_i & \dots & \delta_{i2}\delta_{i(q_i^2+1)}D_i \\ * & * & * & \ddots & \vdots \\ * & * & * & * & \delta_{i(q_i^2+1)}\delta_{i(q_i^2+1)}D_i \end{array}\right] \quad (16)$$

with $D_i = \sum_{j=1}^{N}\sum_{k=1}^{q_iq_j} D_{ijk}Q_{ijk}^{-1}D_{ijk}^T$.

Next, note that, Π_i^{11} of (10) is nonlinear due to the multiplication of $\hat{M}_i$ with its transpose. The summation of this term can be expressed as:

$$\sum_{j=1}^{N}\sum_{k=1}^{q_iq_j}\hat{M}_iQ_{jik}\hat{M}_i^T = \sum_{j=1}^{N}\Theta_{ji}\Phi_{ji}\Theta_{ji}^T$$
$$= \begin{bmatrix} \Theta_{1i}^T \\ \Theta_{2i}^T \\ \vdots \\ \Theta_{Ni}^T \end{bmatrix}^T \begin{bmatrix} \Phi_{1i} & 0 & \dots & 0 \\ * & \Phi_{2i} & \dots & 0 \\ * & * & \ddots & \vdots \\ * & * & * & \Phi_{Ni} \end{bmatrix} \begin{bmatrix} \Theta_{1i}^T \\ \Theta_{2i}^T \\ \vdots \\ \Theta_{Ni}^T \end{bmatrix}$$
$$= -\left[\Xi_i^{15}\right]\left(\Xi_i^{55}\right)^{-1}\left[\Xi_i^{15}\right]^T, \quad (17)$$

where $\Theta_{ji} = \left[\, \hat{M}_iQ_{ji1} \;\; \hat{M}_iQ_{ji2} \;\; \dots \;\; \hat{M}_iQ_{ji(q_iq_j)} \,\right], \Phi_{ji} = \text{diag}\left\{Q_{ji1}^{-1}, Q_{ji2}^{-1}, \dots, Q_{ji(q_iq_j)}^{-1}\right\}$, $j = 1, 2, \dots, N$, and Ξ_i^{15}, Ξ_i^{55} are as in (6).

Now, using (14), (16) and (17), inequality (10) may be written in partitioned matrix form as:

$$\begin{bmatrix} \Xi_i^{11} & \Xi_i^{12} & \Xi_i^{13} \\ * & \Xi_i^{22} & \Xi_i^{23} \\ * & * & \Xi_i^{33} \end{bmatrix} - \begin{bmatrix} \Xi_i^{14} \\ \Xi_i^{24} \\ \Xi_i^{34} \end{bmatrix} \left[\Xi_i^{44}\right]^{-1} \begin{bmatrix} \Xi_i^{14} \\ \Xi_i^{24} \\ \Xi_i^{34} \end{bmatrix}^T - \begin{bmatrix} \Xi_i^{15} \\ 0 \\ 0 \end{bmatrix} \left[\Xi_i^{55}\right]^{-1} \begin{bmatrix} \Xi_i^{15} \\ 0 \\ 0 \end{bmatrix}^T < 0. \quad (18)$$

Combining the second and third term in (18), one may write

$$\begin{bmatrix} \Xi_i^{11} & \Xi_i^{12} & \Xi_i^{13} \\ * & \Xi_i^{22} & \Xi_i^{23} \\ * & * & \Xi_i^{33} \end{bmatrix} - \begin{bmatrix} \Xi_i^{14} & \Xi_i^{15} \\ \Xi_i^{24} & 0 \\ \Xi_i^{34} & 0 \end{bmatrix} \begin{bmatrix} \Xi_i^{44} & 0 \\ 0 & \Xi_i^{55} \end{bmatrix}^{-1} \begin{bmatrix} \Xi_i^{14} & \Xi_i^{15} \\ \Xi_i^{24} & 0 \\ \Xi_i^{34} & 0 \end{bmatrix}^T < 0. \quad (19)$$

Finally, employing Schur Complement formula on (19), one obtains the LMI (6) that can be solved to obtain stabilizing control gains from (7).

5. NUMERICAL EXAMPLE

Example 1. Consider an interconnected system given by

$$S_i^* : \dot{x}_i^*(t) = G_ix_1^*(t) + \sum_{\substack{j=1 \\ j\neq 1}}^{3} H_{ij}x_j^*(t-\tau_{ij}^*) + \bar{B}_i\bar{u}_i(t), \quad i = 1, 2, 3 \quad (20)$$

with

$$G_1 = \begin{bmatrix} -2 & 0.1 \\ 0 & 0.2 \end{bmatrix},\; G_2 = \begin{bmatrix} -1 & 1 \\ -0.2 & 0 \end{bmatrix},\; G_3 = \begin{bmatrix} -1 & 1 \\ 0 & -3 \end{bmatrix},$$
$$H_{12} = \begin{bmatrix} 0 & 0 \\ -0.5 & 0 \end{bmatrix},\; H_{21} = \begin{bmatrix} 0 & 1.7 \\ -1 & 0 \end{bmatrix}, H_{13} = \begin{bmatrix} 0 & 0.8 \\ 0 & 0.1 \end{bmatrix},$$
$$H_{31} = \begin{bmatrix} 0 & 0 \\ 0.5 & 0 \end{bmatrix},\; H_{23} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0 \end{bmatrix},\; H_{32} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0 \end{bmatrix},$$
$$\bar{B}_1 = [\,1\;\,0\,]^T, \bar{B}_2 = \bar{B}_3 = [\,0\;\,0\,]^T.$$

Note that, the overall autonomous system is not delay-independently stable since G_1 is not Hurwitz. Moreover, for $\tau_{12}^* = \tau_{21}^* = 0$, the augmented matrix $\begin{bmatrix} G_1 & H_{12} \\ H_{21} & G_2 \end{bmatrix}$ is also not Hurwitz. Further, the pair $\{G_1, \bar{B}_1\}$ is uncontrollable but for $\tau_{12}^* = \tau_{21}^* = 0$, the augmented pair $\left\{\begin{bmatrix} G_1 & H_{12} \\ H_{21} & G_2 \end{bmatrix}, \begin{bmatrix} \bar{B}_1 \\ 0 \end{bmatrix}\right\}$ is controllable. Therefore, one may use both the states of 1^{st} and 2^{nd} subsystems to stabilize the system through the control input of the first subsystem and then the system has to be delay-dependently stable on τ_{12}^* and τ_{21}^*. The corresponding control law is described as:

$$u_1(t) = K_{11}^*x_1^*(t) + K_{12}^*x_2^*(t)$$
$$= [\,K_{11}^* \;\; K_{12}^*\,]\,[\,x_1^{*T}(t)\;\, x_2^{*T}(t)\,]^T.$$

The objective now is to design the stabilizing control gains K_{11}^* and K_{12}^*. By augmenting the 1^{st} and 2^{nd} subsystems, one obtains

$$\dot{x}_i(t) = A_ix_i(t) + \sum_{k=1}^{q_i^2} C_{ik}x_i(t-\eta_{ik}) + \sum_{\substack{j=1 \\ j\neq i}}^{2}\sum_{k=1}^{q_iq_j} D_{ijk}x_j(t-\tau_{ijk}) + B_iK_ix_i(t), \quad i = 1, 2,$$

with

$$q_1 = 2, q_2 = 1, B_2 = C_{13} = C_{14} = C_{21} = 0, \eta_{11} = \tau_{12}^*,$$
$$\eta_{12} = \tau_{21}^*,\; \tau_{121} = \tau_{13}^*,\; \tau_{122} = \tau_{23}^*,\; \tau_{211} = \tau_{31}^*,\; \tau_{212} = \tau_{32}^*,$$
$$A_1 = \begin{bmatrix} G_1 & 0 \\ 0 & G_2 \end{bmatrix}, A_2 = G_3, C_{11} = \begin{bmatrix} 0 & H_{12} \\ 0 & 0 \end{bmatrix},$$
$$C_{12} = \begin{bmatrix} 0 & 0 \\ H_{21} & 0 \end{bmatrix}, D_{121} = \begin{bmatrix} H_{13} \\ 0 \end{bmatrix},\; D_{122} = \begin{bmatrix} 0 \\ H_{23} \end{bmatrix},$$
$$D_{211} = [\,H_{31}\;\,0\,],\; D_{212} = [\,0\;\,H_{32}\,],$$
$$B_1 = [\,1\;\;0\;\;0\;\;0\,]^T, \quad B_2 = 0 \quad \text{and} \quad K_1 = [\,K_{11}^* \;\; K_{12}^*\,].$$

For $\eta_{11} = 0.1$ and $\eta_{12} = 0.2$, one may use Theorem 4.2 to design the controller. For solving the LMI, we consider

$$Q_{121} = 15I, \quad Q_{122} = I, \quad Q_{211} = Q_{212} = I.$$

Then solving LMI (6) for $\delta_{11} = \delta_{12} = \delta_{13} = 1$, one obtains a stabilizing control gain as:

$$K_{11}^* = [\,-9.2005 \;\;\; 231.1257\,], K_{12}^* = [\,23.3063 \;\;\; 49.6720\,].$$

The LMI criterion corresponding to the second new subsystem is also found to yield feasible solution. This ensures the system (20) with the designed controller is delay-dependently stable for $\eta_{11} = 0.1$ and $\eta_{12} = 0.2$. The system is now simulated with $\tau_{12}^* = 0.1$, $\tau_{21}^* = 0.2$, $\tau_{13}^* = \tau_{31}^* = 0.6$,

$\tau_{23}^* = \tau_{32}^* = 0.4$; $x_1^*(t) = [\,-1 \quad 2\,]^T$ for $-0.6 \leq t \leq 0$; $x_2^*(t) = [\,-4 \quad 1\,]^T$ for $-0.4 \leq t \leq 0$; $x_3^*(t) = [\,0 \quad -2\,]^T$ for $-0.6 \leq t \leq 0$. The corresponding norm of the state responses of the three subsystems and the control input are shown in Figs. 1 − 4, which shows that the controller effectively stabilizes the composite system.

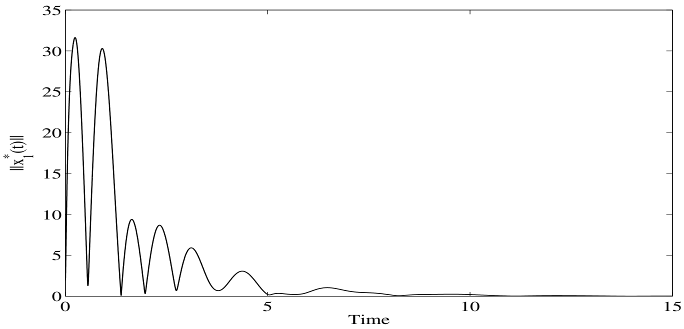

Fig. 1. Variation of norm of the state of S_1^*.

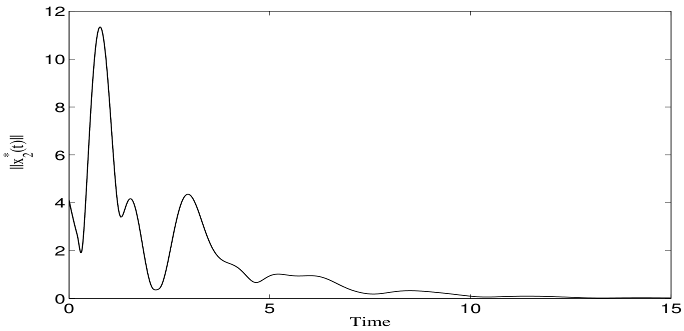

Fig. 2. Variation of norm of the state of S_2^*.

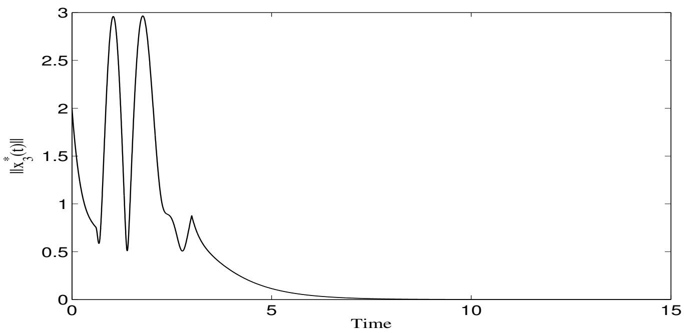

Fig. 3. Variation of norm of the state of S_3^*.

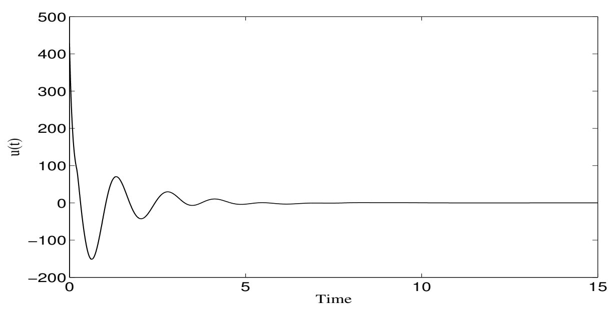

Fig. 4. Control input to the first subsystem S_1^*.

6. CONCLUSION

Mixed delay-dependent/delay-independent state feedback stabilization of interconnected time-delay system has been considered and a stabilization criterion has been derived in terms of LMIs for designing the decentralized controller. Formation of larger subsystems involving finite delays has been considered to obtain the desired delay-dependent stabilization. A numerical example has been presented to demonstrate the effectiveness of the proposed design.

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