Journal of Reinforced Plastics and Composites, 2001, Vol 20, Iss 18

Parametric Resonance Characteristics of Laminated Composite Doubly Curved Shells Subjected to Non-Uniform Loading

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ABSTRACT: The parametric resonance characteristics of laminated composite doubly curved panels subjected to various in-plane static and periodic compressive edge loadings, including partial and concentrated edge loading are studied using finite element analysis. The first order shear deformation theory is used to model the doubly curved panels, considering the effects of transverse shear deformation and rotary inertia. The theory used is the extension of dynamic, shear deformable theory according to the Sander's first approximation for doubly curved laminated shells, which can be reduced to Love's and Donnell's theories by means of tracers. The effects of number of layers, static load factor, side to thickness ratio, shallowness ratio, boundary conditions, degree of orthotropy, ply orientations and various load parameters on the principal instability regions of doubly curved panels are studied in detail using Bolotin's method. Quantitative results are presented to show the effects of shell geometry, lamination details and load parameters on the stability boundaries. Results of plates and cylindrical shells are also presented as special cases and are compared with those available in the literature.

KEY WORDS: composite, doubly curved shell, non-uniform loading, instability, finite element.

INTRODUCTION

STRUCTURAL ELEMENTS SUBJECTED to in-plane periodic forces may lead to parametric resonance, due to certain combinations of the values of load parameters. The instability may occur below the critical load of the structure under compressive loads over wide ranges of excitation frequencies. Several means of

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combating resonance such as damping and vibration isolation may be inadequate and sometimes dangerous with reverse results [1]. Thus the parametric resonance characteristics are of great importance for understanding the dynamic systems under periodic loads. The parametric instability characteristics of laminated composite flat panel subjected to uniform loads were studied by several authors [2–4]. Parametric resonance in shell structures under periodic loads has been of considerable interest since the subject was more elaborately introduced by Bolotin [5] and Yao [6]. The parametric instability of thick orthotropic cylindrical shells was studied analytically by Bert and Birman [7]. The dynamic instability of composite simply-supported circular cylindrical shell was analysed by the Method of Multiple Scale (MMS) by Cederbaum [8]. A perturbation technique was employed by Argento and Scott [9–11] to study the instability regions subjected to axial loading and torsional loading. The dynamic instability of laminated composite circular cylindrical shells was studied by Ganapathi and Balamurugan [12] using a C^0 shear flexible two noded axisymmetric shell element. The dynamic stability of cross-ply laminated composite cylindrical shells under combined static and periodic axial force was investigated by Ng, Lam and Reddy [13] using Love's classical theory of thin shells. Most of the above mentioned investigators studied the dynamic stability of uniformly loaded closed cylindrical shells with a simply supported boundary condition. Recently the parametric instability of laminated composite conical shells under periodic edge loading was studied by Ganapathi et al. [14]. The practical importance of stability analysis of doubly curved panels/open shells has been increased in structural, aerospace (skin panels in wings, fuselage etc.), submarine hulls and mechanical applications. The static [15] and free vibration [16–19] characteristics of doubly curved shallow shells/curved panels were studied by a number of researchers and well reviewed [20,21]. The vibration and buckling stresses were studied for cylindrical panels [22–24] and isotropic thick simply-supported double curved open shells/panels [25]. The buckling characteristics of flat panel [26,27] and closed cylindrical shell [28,29] due to concentrated loadings were also investigated. The study of the parametric instability behaviour of curved panels is new. Recently, the effects of curvature and aspect ratio on dynamic instability for a uniformly loaded laminated composite thick cylindrical panel were studied by Ganapathi et al. [30]. However, no results have been reported in the literature on parametric instability of doubly curved composite shells. Besides this, the applied load and boundaries are seldom uniform in practice. The application of non-uniform loading and boundary conditions on the structural component will alter the global quantities such as free vibration frequency, buckling load and dynamic instability region (DIR).

In the present study, the parametric instability of doubly curved composite panels subjected to various in-plane non-uniform loads, including partial and concentrated edge loadings are investigated. The influences of various parameters like effects of number of layers, static and dynamic load factors, side to thickness ratio, shallowness ratio, various boundary conditions, ply orientations, load bandwidth and position of concentrated loads on the instability behaviour of curved panels have been examined. The present formulation of the problem is made general to accommodate a doubly curved panel with finite curvatures in both the directions having arbitrary load and boundary conditions.

THEORY AND FORMULATIONS

The basic configuration of the problem considered here is a doubly curved panel subjected to various non-uniform in-plane edge loadings, boundary conditions and is shown in Figure 1.

Governing Equations

The equation of equilibrium for free vibration of a structure subjected to in-plane loads can be written as:

$$[M]\{\ddot{q}\}+[[K_e]-P[K_g]]\{q\}=0 \tag{1}$$

The in-plane load $P(t)$ is periodic and can be expressed in the form

$$P(t)=P_s+P_t\cos\Omega t \tag{2}$$

where P_s is the static portion of P. P_t is the amplitude of the dynamic portion of P and Ω is the frequency of excitation. The static buckling load of elastic shell P_{cr} is the measure of the magnitude of P_s and P_t,

$$P_s=\alpha P_{cr}, \qquad P_t=\beta P_{cr} \tag{3}$$

where α and β are termed as static and dynamic load factors respectively. Using Equation (2) the equation of motion is obtained as:

$$[M]\{\ddot{q}\}+[[K_e]-\alpha P_{cr}[K_g]-\beta P_{cr}[K_g]\cos\Omega t]\{q\}=0 \tag{4}$$

Equation (4) represents a system of second order differential equations with periodic coefficients of the Mathieu-Hill type. The development of regions of instability arises from Floquet's theory which establishes the existence of periodic

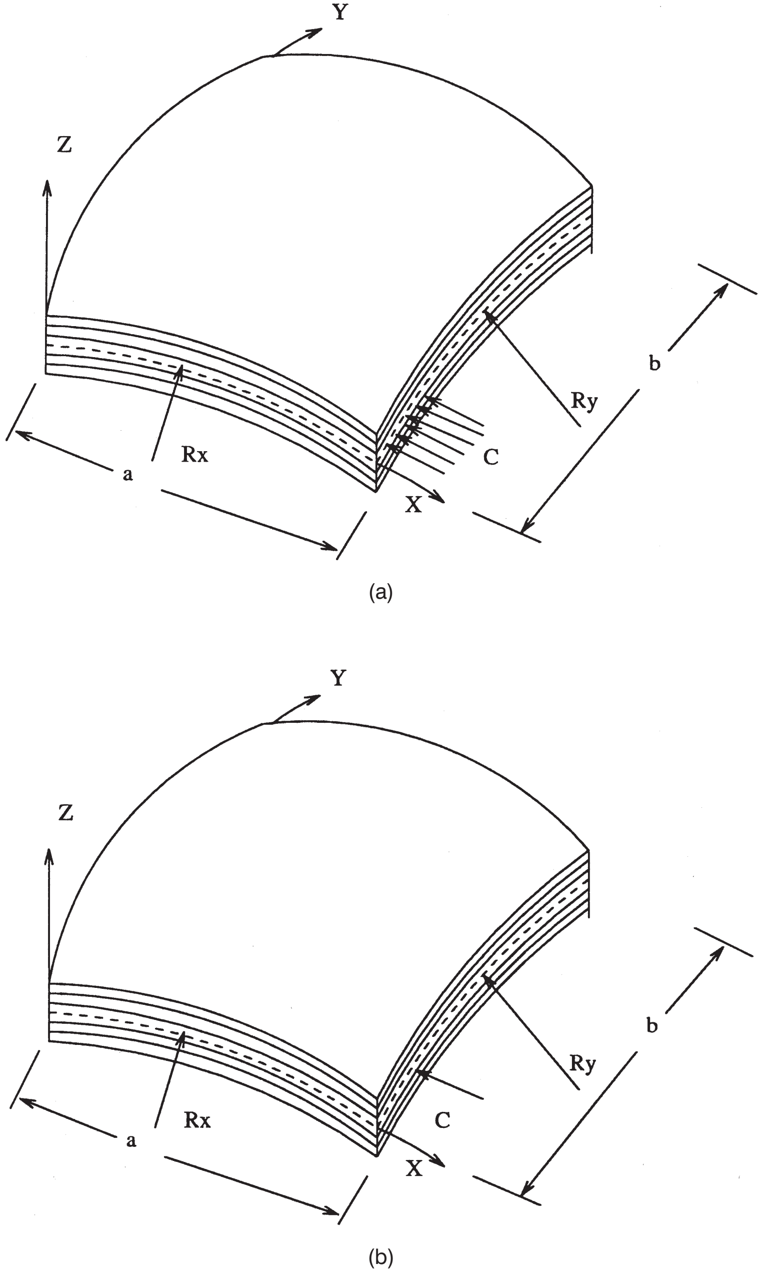

Figure 1. *Geometry and co-ordinate systems of a doubly curved panel: (a) partial loading; (b) concentrated loading.*

solutions. The boundaries of the dynamic instability regions are formed by the periodic solutions of period T and $2T$, where $T = 2\pi/\Omega$. The boundaries of the primary instability regions with period $2T$ are of practical importance [3] and the solution can be achieved in the form of the trigonometric series

$$q(t) = \sum_{k=1,3,5}^{\infty} \left[\{a_k\} \sin \frac{k\Omega t}{2} + \{b_k\} \cos \frac{k\Omega t}{2} \right] \tag{5}$$

Putting this in Equation (4) and if only first term of the series is considered, equating coefficients of sin $\Omega t/2$ and cos $\Omega t/2$, the Equation (4) reduces to

$$\left[[K_e] - \alpha P_{cr} [K_g] \pm \frac{1}{2} \beta P_{cr} [K_g] - \frac{\Omega^2}{4} [M] \right] \{q\} = 0 \tag{6}$$

Equation (6) represents an eigenvalue problem for known values of α, β and P_{cr}. The two conditions under a plus and minus sign correspond to two boundaries of the dynamic instability region. The eigenvalues are Ω, which give the boundary frequencies of the instability regions for given values of α and β. In this analysis, the computed static buckling load of the panel is considered as the reference load [31,32].

An eight-noded curved isoparametric element is employed in the present analysis with five degrees of freedom u, v, w, θ_x and θ_y per node. First order shear deformation theory is employed and the shear correction coefficient accounts for the nonlinear distribution of the thickness shear strains through the total thickness. The displacement field assumes that mid-plane normal remains straight before and after deformation, but not necessarily normal after deformation, so that

$$\begin{aligned} \bar{u}(x,y,z) &= u(x,y) + z\theta_x(x,y) \\ \bar{v}(x,y,z) &= v(x,y) + z\theta_y(x,y) \\ \bar{w}(x,y,z) &= w(x,y) \end{aligned} \tag{7}$$

where θ_x, θ_y are the rotations of the mid surface.

Also $\bar{u}$, $\bar{v}$, $\bar{w}$ and u, v and w are displacement components in the x, y, z directions at any point and at the mid surface respectively. The constitutive relationships for the shell becomes

$$\begin{Bmatrix} N_i \\ M_i \\ Q_i \end{Bmatrix} = \begin{bmatrix} A_{ij} & B_{ij} & 0 \\ B_{ij} & D_{ij} & 0 \\ 0 & 0 & S_{ij} \end{bmatrix} \begin{Bmatrix} \varepsilon_j \\ k_j \\ \gamma_m \end{Bmatrix} \tag{8}$$

where A_{ij}, B_{ij}, D_{ij} and S_{ij} are the extension-bending coupling, bending and transverse shear stiffness respectively [33]. A shear correction factor of 5/6 is included in S_{ij} for all the numerical computations. Extension of shear deformable Sander's kinematic relations for doubly curved shells [15–17] are used in the analysis. The linear strain displacement relations are

$$\varepsilon_{xl} = \frac{\partial u}{\partial x} + \frac{w}{R_x} + z\kappa_x$$

$$\varepsilon_{yl} = \frac{\partial v}{\partial y} + \frac{w}{R_y} + z\kappa_y$$

$$\gamma_{xyl} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + z\kappa_{xy} \tag{9}$$

$$\gamma_{yz} = \frac{\partial w}{\partial y} + \theta_y - C_1 \frac{v}{R_y}$$

$$\gamma_{xz} = \frac{\partial w}{\partial x} + \theta_x - C_1 \frac{u}{R_x}$$

where

$$\begin{gathered} \kappa_x = \frac{\partial \theta_x}{\partial x}, \quad \kappa_y = \frac{\partial \theta_y}{\partial y} \\ \kappa_{xy} = \frac{\partial \theta_x}{\partial y} + \frac{\partial \theta_y}{\partial x} + \frac{1}{2} C_2 \left(\frac{1}{R_y} - \frac{1}{R_x} \right) \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \end{gathered} \tag{10}$$

and C_1 and C_2 are tracers by which the analysis can be reduced to that of shear deformable Love's first approximation and Donnell's theories. The element geometric stiffness matrix for the doubly curved panel is derived using the nonlinear strain components as:

$$\varepsilon_{xnl} = \frac{1}{2}\left(\frac{\partial u}{\partial x} + \frac{w}{R_x}\right)^2 + \frac{1}{2}\left(\frac{\partial v}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial w}{\partial x} - \frac{u}{R_x}\right)^2 + \frac{1}{2}z^2\left[\left(\frac{\partial \theta x}{\partial x}\right)^2 + \left(\frac{\partial \theta_y}{\partial x}\right)^2\right]$$

$$\varepsilon_{ynl} = \frac{1}{2}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{1}{2}\left(\frac{\partial v}{\partial y} + \frac{w}{R_y}\right)^2 + \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{v}{R_y}\right)^2 + \frac{1}{2}z^2\left[\left(\frac{\partial \theta_x}{\partial y}\right)^2 + \left(\frac{\partial \theta_y}{\partial y}\right)^2\right] \quad (11)$$

$$\gamma_{xynl} = \left[\left(\frac{\partial u}{\partial x} + \frac{w}{R_x}\right)\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\left(\frac{\partial u}{\partial y} + \frac{w}{R_y}\right) + \left(\frac{\partial w}{\partial x} - \frac{u}{R_x}\right)\left(\frac{\partial w}{\partial y} - \frac{v}{R_y}\right)\right]$$

$$+ z^2\left[\left(\frac{\partial \theta_x}{\partial x}\right)\left(\frac{\partial \theta_x}{\partial y}\right) + \left(\frac{\partial \theta_y}{\partial x}\right)\left(\frac{\partial \theta_y}{\partial y}\right)\right]$$

The overall matrices $[K_e]$, $[K_g]$ and $[M]$ are obtained by assembling the corresponding element matrices.

Computer Program

A computer program has been developed to perform all the necessary computations. The element stiffness and mass matrices are derived using a standard procedure. The geometric stiffness matrix is essentially a function of the in-plane stress distribution in the element due to applied edge loading. Since the stress field is non-uniform, plane stress analysis is carried out using the finite element techniques to determine the stresses and these stresses are used to formulate the geometric stiffness matrix. Reduced integration technique is adopted for the element matrices in order to avoid possible shear locking. The overall matrices $[K_e]$, $[K_g]$ and $[M]$ are obtained by assembling the corresponding element matrices, using skyline technique. Subspace iteration method is adopted throughout to solve the eigenvalue problems.

RESULTS AND DISCUSSIONS

The convergence studies are made for non-dimensional fundamental frequencies of vibration of doubly curved cross-ply and angle-ply shells. Shells of various

Table 1. Convergence of non-dimensional frequencies without in-plane load of doubly curved 0°/90°/0° cross ply shells/panels.

$a/b = 1$, $b/h = 100$, $b/R_y = 0.5$
$E_{11} = 138$ GPa, $E_{22} = 8.96$ GPa, $G_{12} = 7.1$ GPa, $\nu_{12} = 0.3$

Non-dimensional frequency, $\omega = \bar{\omega} b^2 \sqrt{\left(\frac{\rho}{E_{11} h^2}\right)}$

	Non-Dimensional Frequencies of Shells			
Mesh Division	**Plate**	**Cylindrical**	**Spherical**	**Hyperbolic Paraboloid**
4 × 4	0.9988	1.8607	1.5428	1.4776
8 × 8	0.9986	1.8606	1.5313	1.4586
10 × 10	0.9986	1.8606	1.5305	1.4577
Reference [18]	(0.9998)	(1.8652)	(1.5362)	(1.4645)

geometry such as cylindrical ($R_y/R_x = 0$), spherical ($R_y/R_x = 1$), and hyperbolic paraboloidal shells ($R_y/R_x = -1$) are studied. The results are shown in Table 1 and Table 2. This shows good convergency of the numerical solutions. A 10×10 mesh has been employed to idealise the panel in the subsequent analysis. This idealisation is chosen in order to apply compression to a small fraction of the edge length and also for convergence criterion. The converged results compare well with the results by Qatu and Leissa [18]. The present formulation is validated for free vibration analysis of a doubly curved cross ply and angle-ply shells. The results are

Table 2. Convergence of non-dimensional frequencies without in-plane load of doubly curved 45°/–45°/45° angle ply shells/panels.

$a/b = 1$, $b/h = 100$, $b/R_y = 0.5$
$E_{11} = 138$ GPa, $E_{22} = 8.96$ GPa, $G_{12} = 7.1$ GPa, $\nu_{12} = 0.3$

Non-dimensional frequency, $\omega = \bar{\omega} b^2 \sqrt{\left(\frac{\rho}{E_{11} h^2}\right)}$

	Non-Dimensional Frequencies of Shells			
Mesh Division	**Plate**	**Cylindrical**	**Spherical**	**Hyperbolic Paraboloid**
4 × 4	0.4600	1.6990	1.3507	0.9703
8 × 8	0.4581	1.6587	1.2977	0.9223
10 × 10	0.4577	1.6545	1.2941	0.9183
Reference [18]	(0.4607)	(1.6574)	(1.3063)	(0.9234)

Table 3. Non-dimensional fundamental frequencies for the simply supported four layered cross-ply (0°/90°/90°/0°) spherical shell.

$E_{11}/E_{22} = 25$, $G_{23} = 0.2E_{22}$, $G_{12} = G_{13} = 0.5E_{22}$, $\nu_{12} = 0.25$

Non-dimensional frequency, $\omega = \bar{\omega}a^2\sqrt{\left(\frac{\rho}{E_{22}h^2}\right)}$

	$a/h = 100$			$a/h = 10$		
R/b	Ref. [16]	Ref. [17]	Present FEM	Ref. [16]	Ref. [17]	Present FEM
1	126.33	126.7	126.32	16.172	16.195	16.146
2	68.294	68.284	68.293	13.459	13.459	13.440
3	47.415	47.553	47.415	12.795	12.805	12.793
4	37.082	37.184	37.082	12.552	12.560	12.551
5	31.079	31.159	31.079	12.437	12.444	12.436
10	20.380	20.417	20.380	12.280	12.286	12.280
10^{30} (plate)	15.184	15.195	15.184	12.226	12.233	12.226

presented in Tables 3 and 4, showing good comparison with the literature. To validate the formulation further, the buckling results of singly and doubly curved shells/panels are compared with those available in the literature. The buckling results for singly and doubly curved shells are shown in Table 5 and Table 6 respectively. The above studies indicate good agreement exits between the present study and those from the literature. Once the free vibration and buckling results are validated, the dynamic instability studies are made.

Table 4. Non-dimensional fundamental frequencies for the various simply supported doubly curved angle-ply shells/panels.

$a/b = 1$, $h/R_x = 1/500$, $a/h = 100$
$E_{11}/E_{22} = 25$, $G_{23} = 0.2E_{22}$, $G_{12} = G_{13} = 0.5E_{22}$, $\nu_{12} = 0.25$

Non-dimensional frequency, $\omega = \bar{\omega}a^2\sqrt{\left(\frac{\rho}{E_{22}h^2}\right)}$

	Spherical		Elliptical Paraboloid		Hyperbolic Paraboloid	
Laminations	Ref. [33]	Present FEM	Ref. [33]	Present FEM	Ref. [33]	Present FEM
0/45/0	41.748	41.6659	33.468	33.291	15.335	15.2847
$(0/45)_s$	45.535	45.3005	36.634	36.3207	15.756	15.6999
45/–45	49.289	49.1690	37.985	37.8152	15.174	15.1429
$(45/–45)_2$	58.94	58.8263	50.421	49.3048	19.393	19.3909

Table 5. Non-dimensional buckling loads for the simply supported singly-curved cylindrical panel.

$a = 0.25\ m$, $b = 0.25\ m$, $h = 2.5\ mm$, $a/R_x = 0$, $E_{11} = 2.07 \times 10^{11}\ N/m^2$, $E_{22} = 5.2 \times 10^9 N/m^2$, $G_{12} = 2.7 \times 10^9 N/m^2$, $\nu_{12} = 0.25$

Non-dimensional buckling load, $\lambda = \dfrac{N_x b^2}{E_{22} h^3}$

b/R_y	Orientation	Non-Dimensional Buckling Loads	
		Present FEM	Ref. [22]
0	90/0	12.63	12.63
	0/90	12.63	12.63
0.1	90/0	17.629	17.51
	0/90	17.612	17.49
0.2	90/0	32.565	32.06
	0/90	32.5027	32.17
0.3	90/0	57.28	56.28
	0/90	57.117	56.62

Table 6. Non-dimensional buckling stresses for the simply supported doubly-curved shell/panel.

$a/b = 1$, $E_{11} = E_{22} = 70\ GPa$, $G_{12} = G_{23} = 26.923\ GPa$, $\nu_{12} = \nu_{21} = 0.3$

Non-dimensional buckling stress, $\lambda = \dfrac{N_x b^2}{\Pi^2 D_s}$

$$D_s = \frac{E_{11} h^3}{12(1.0 - \nu_{12}\nu_{21})}$$

a/h	a/R_x	b/R_y	Non-Dimensional Buckling Stresses	
			Present FEM	Ref. [25]
10	0	0	3.7315	3.7412
	0.2	0.2	4.1591	4.1394
	0	0.2	3.8307	3.8391
	–0.2	0.2	3.7006	3.7100
20	0	0	3.9288	3.9307
	0.2	0.2	5.6957	5.6904
	0	0.2	4.3627	4.3636
	–0.2	0.2	3.897	3.9012

Parametric Instability Studies

The parametric instability regions are plotted for a uniaxially loaded doubly curved panel with/without static component to consider the effect of various parameters. A simply supported doubly curved panel of $a = b = 500$ mm, $h = 2$ mm, $\rho = 1580$ kg/m^3, $E_{11} = 141.0$ GPa, $E_{22} = 9.23$ GPa, $G_{12} = G_{13} = 5.95$ GPa, $G_{23} = 2.96$ GPa, $\nu_{12} = 0.313$, $R_x = R_y = 2000$ mm is described as standard case and the computed buckling load of this eight layer antisymmetric angle ply panel is taken as the reference load for all further computations. The non-dimensional excitation frequency $\Omega = \bar{\Omega} a^2 \sqrt{\rho / E_{22} h^2}$ is used throughout the dynamic instability studies, (unless otherwise mentioned) where $\bar{\Omega}$ is the excitation frequency in radian/second. The effect of number of layers is shown in Figure 2. The dynamic instability regions have been plotted for 2, 4, 8 and 10 layer antisymmetric angle-ply doubly curved shell panel. It is seen that the effect of number of layers does not vary much beyond 8 layers. Hence, the further parametric studies are carried out for 8 layer angle ply shells only. The effect of static component of load for $\alpha = 0.0$, 0.2 and 0.4 on the instability region is shown in Figure 3. Due to increase of static component, the instability regions tend to shift to lower frequencies and become wider. The effect of degree of orthotropy is studied for E_{11}/E_{22} = 40, 25, 15, keeping other material parameters constant and is shown in Figure 4. The study shows an decrease of excitation frequencies with decrease of degree or orthotropy. Figure 5

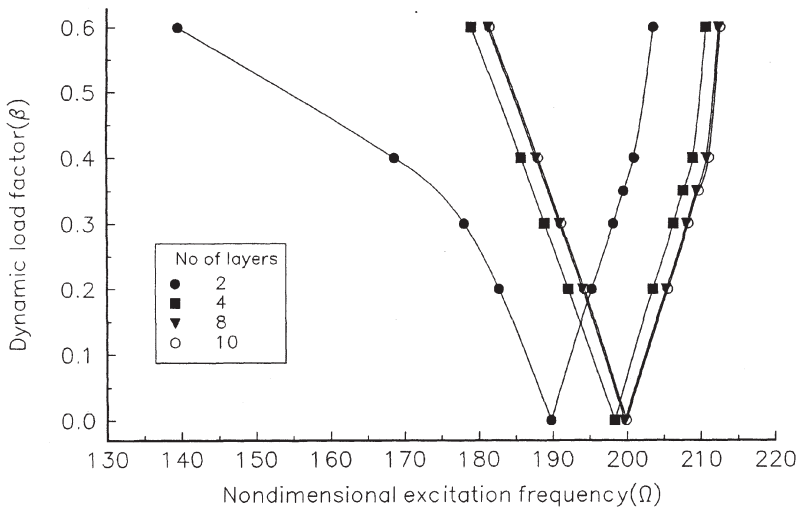

Figure 2. *Effect of number of layers on instability region of an antisymmetric angle-ply shell:* a/b = *1,* b/h = *250,* $a/R_x = b/R_y$ = *0.25,* α = *0.2,* N = *2, 4, 8, 10.*

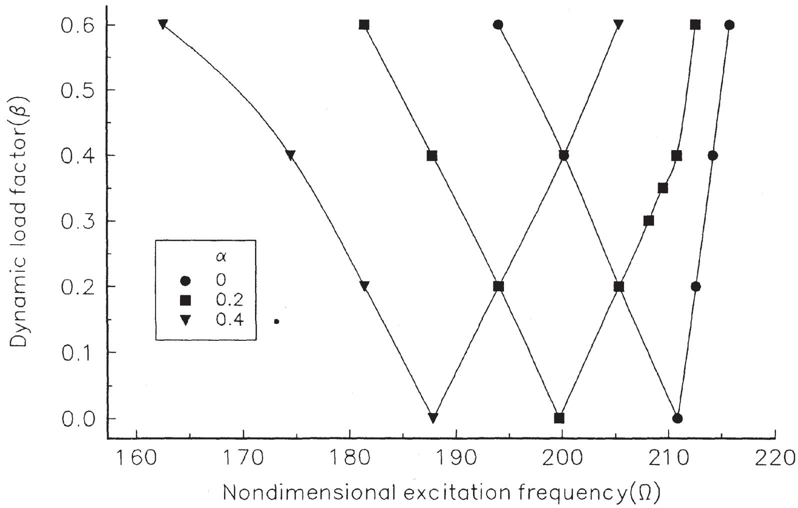

Figure 3. *Effect of static load on instability region of a fully-loaded antisymmetric angle-ply shell:* a/b = *1,* b/h = *250,* $a/R_x = b/R_y$ = *0.25,* α = *0, 0.2, 0.4.*

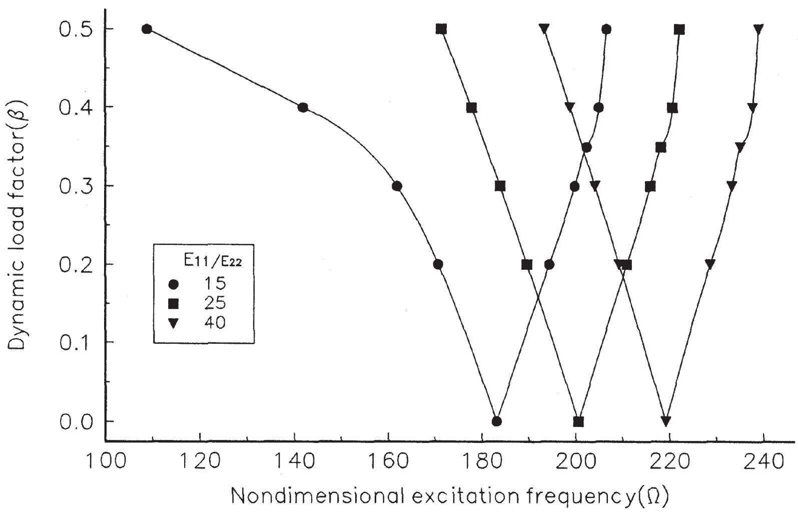

Figure 4. *Effect of degree of orthotropy on instability region of a* $(\pm 45°)_2$ *antisymmetric angle-ply shell:* a/b = *1,* b/h = *250,* $a/R_x = b/R_y$ = *0.25,* α = *0.2,* $G_{12}/E_{22} = G_{13}/E_{22}$ = *0.6,* G_{23}/E_{22} = *0.5,* E_{11}/E_{22} = *40, 25, 15.*

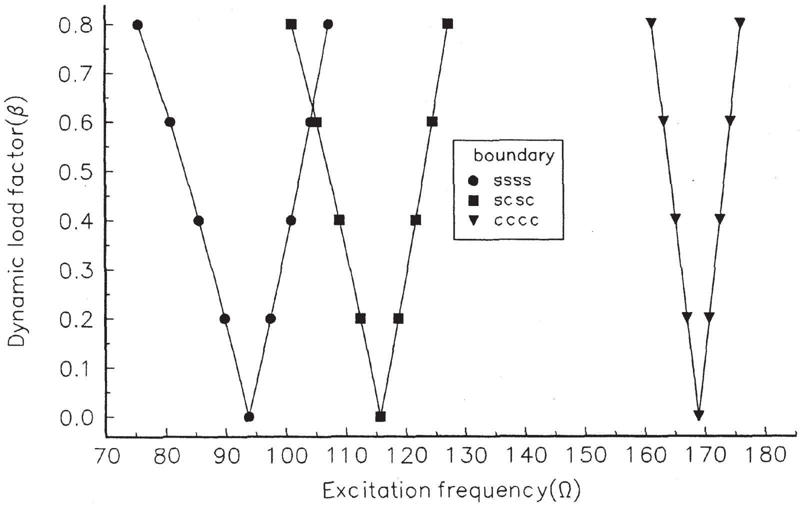

Figure 5. *Effect of boundary conditions (SSSS, SCSC, CCCC) on instability region of the curved panel for* a/b = *1,* b/h = *250,* a/R_x *=0.0,* b/R_y = *0.25 and* α = *0.2.*

shows the influence of different boundaries on the instability regions. The boundaries considered are: simply supported on all sides (SSSS), simply supported on curved surfaces and clamped boundary conditions on straight edges (SCSC) and all sides clamped (CCCC). As expected the excitation frequencies increase from simply supported to clamped edges due to the restraint at the edges. The effect of the shallowness of curved panel on the instability regions is shown in Figure 6. It is observed that, the instability excitation frequency is higher by decreasing R_x and R_y. The effect of lamination angle has been studied for uniaxial loading with static component. The lamination angles considered are: 0°, 15°, 30°, 45°, 60°, 75°, 90°. As observed in Figure 7, the instability region is smaller and starts at higher frequencies with greater the lamination angle for this range of thickness ratio *b/h* and material properties for uniaxially loaded shell. The ply orientation of 45° seems to be a preferential orientation for this type of panel. For this orientation, the instability region shifts to higher frequencies and becomes narrower in comparison with other orientations. Similar trends are also observed by Narita and Zhao [34] based on an optimisation study on free vibration characteristics of this range of shells. Studies have also been made (Figure 8) for comparison of instability regions for different shell geometries. It is observed that the excitation frequency increases with introduction of curvatures from cylindrical panel to doubly curved panel. However, the hyperbolic paraboloid shows intermediate stiffness between a singly cylindrical panel to a spherical shell panel. The

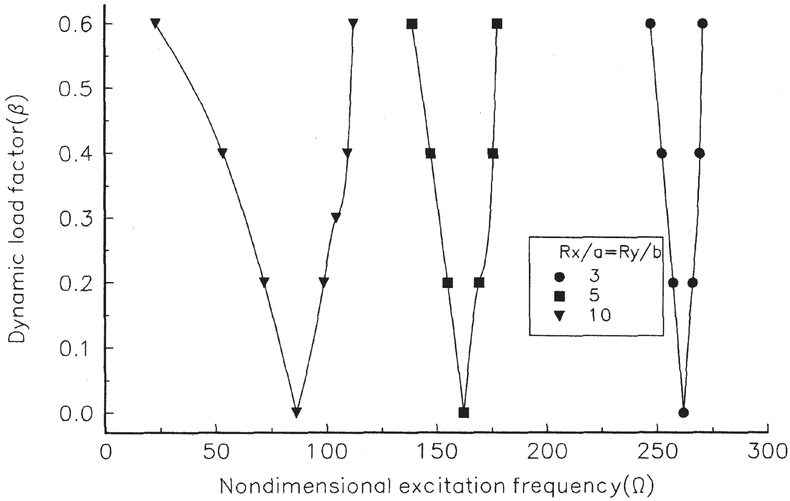

Figure 6. *Effect of shallowness on instability region of the curved panel for* a/b = *1,* b/h = *250,* $a/R_x = b/R_y$ = *0.3, 0.2, 0.1,* α = *0.2.*

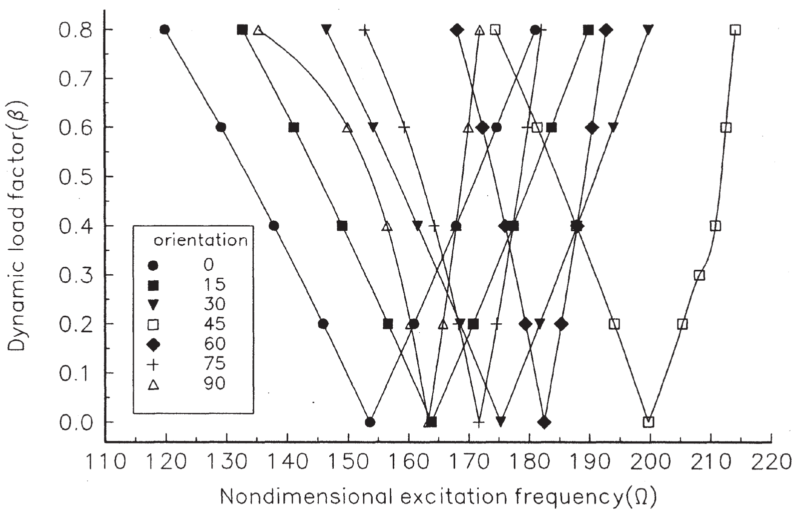

Figure 7. *Effect of different ply orientations on instability region of a* $(\pm 45)_2$ *antisymmetric angle-ply simply supported cylindrical panel:* a/b = *1,* b/h = *250,* $b/R_y = a/R_x = 0.25$, $\alpha = 0.2$.

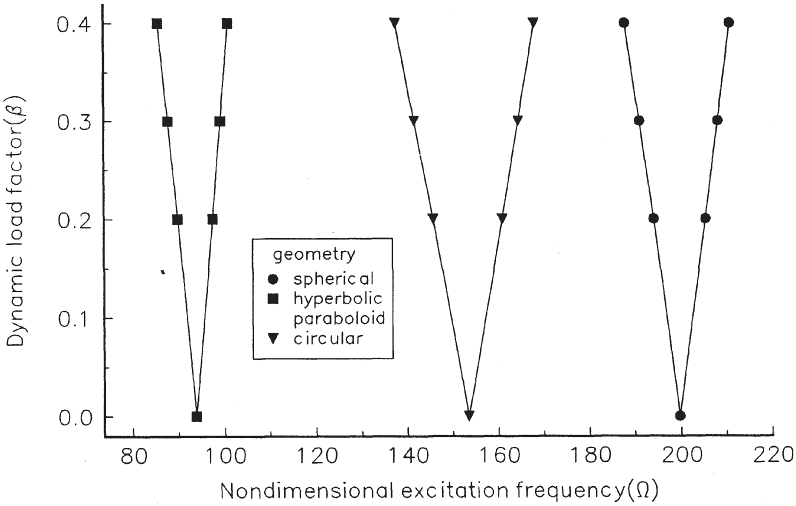

Figure 8. *Effect of curvature on instability region of different curved panels for* a/b = *1,* b/h = *250,* b/R_y = *0.25,* a/R_x = *0, 0.25, –0.25,* α = *0.0.*

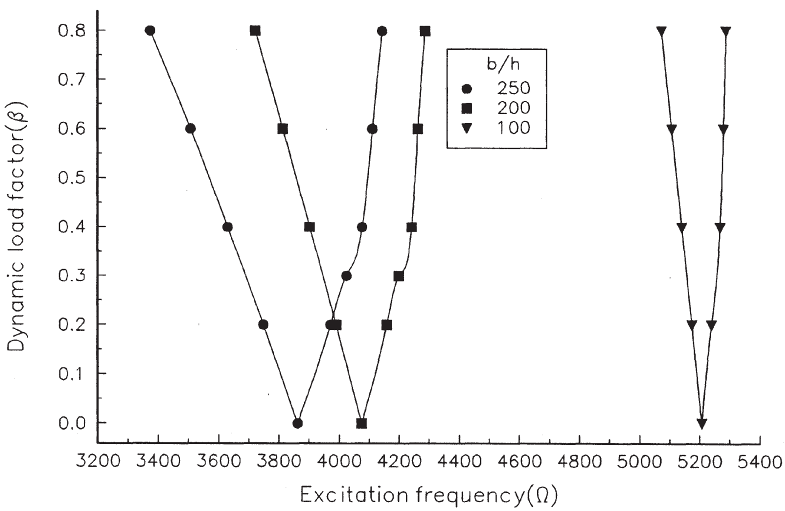

Figure 9. *Effect of side to thickness ratio on instability region of the curved panel for* a/b = *1,* b/h = *100, 200, 250,* a/R_x = b/R_y = *0.25,* α = *0.2.*

effect of side to thickness ratio on instability regions is shown in Figure 9. The onset of dynamic instability regions occur later for lower *b/h* ratio and the width of the instability regions increased with increase of *b/h* ratio. The investigation has been extended to instability behaviour under various non-uniform loadings. For this, various partial and concentrated edge loadings are considered. The instability behaviour of curved panels under non-uniform loading seems to be quite different to that of under uniform loading. The onset of dynamic stability regions occurs earlier with increase of percentage of loaded edge length. Figure 10 shows that the instability occurs later for a small patch of loading (*c/b* = 0.2) as compared to a higher load band width of *c/b* = 0.8. Even the onset of instability occurs later for the partial loading from both ends (Figure 11) than that of partial loading from one end. This may be due to the constraint at the edges. The effects of positions of loadings for single (Figure 12) and double pair of concentrated loadings (Figure 13) are studied. The instability behaviour is also affected by the positions of concentrated loading. As can be seen, the instability in general occurs at lower excitation frequencies with increase of distance from the edges (*c/b*). The curved panel with a small patch of loading behaves in a similar manner to that of a panel subjected to a pair of concentrated loading near the edges. The panel with a double pair of concentrated loading near the edges shows highest stiffness among all the loading cases considered.

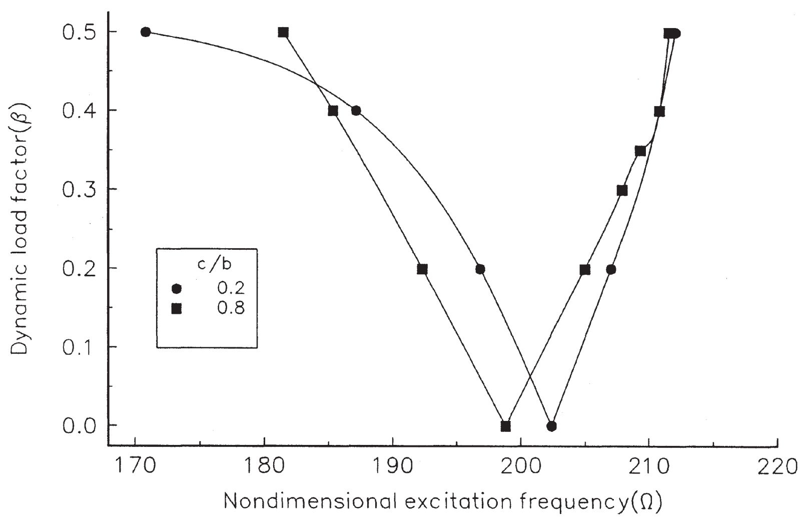

Figure 10. *Effect of percentage of loaded edge length on instability region of a spherical panel for loading at one end,* a/b = *1,* $a/R_x = b/R_y$ *= 0.25,* c/b *= 0.2 and 0.8,* α *= 0.2.*

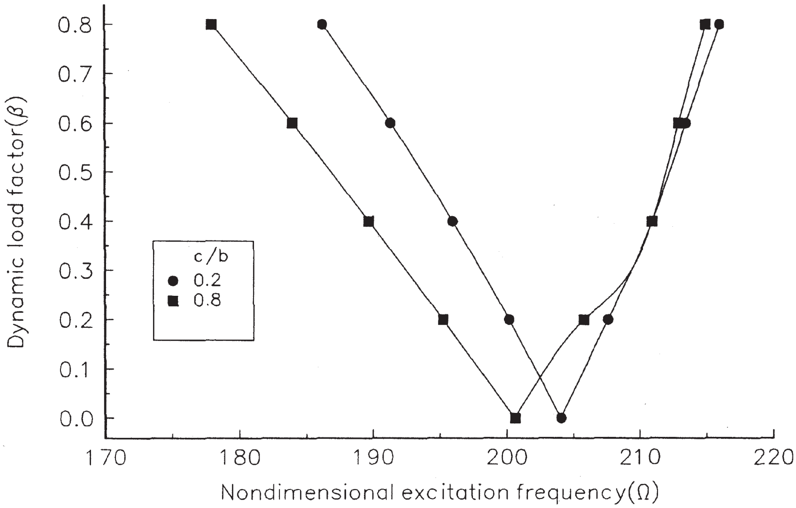

Figure 11. *Effect of percentage of loaded edge length on instability region of a spherical panel for loading from both ends,* a/b = *1,* $a/R_x = b/R_y$ = *0.25,* c/b = *0.2 and 0.8,* α = *0.2.*

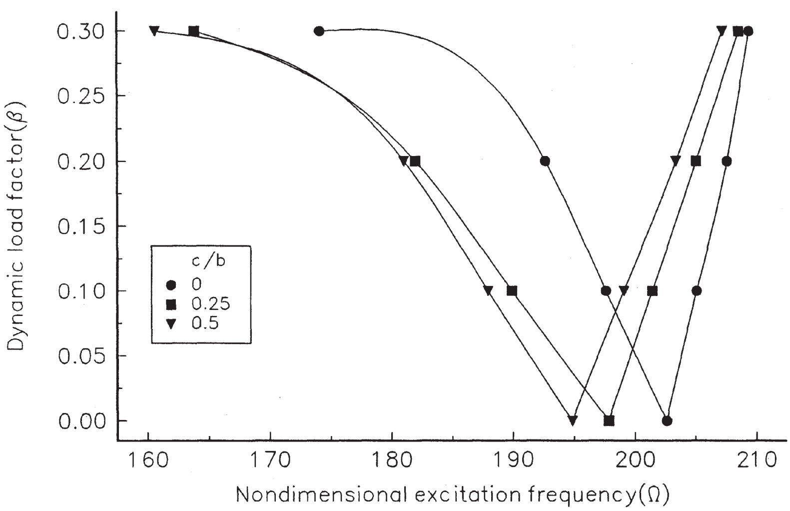

Figure 12. *Effect of position of concentrated load on instability region of a spherical shell for single pair of loads,* a/b = *1,* $a/R_x = b/R_y$ = *0.25,* c/b = *0, 0.25 and 0.5,* α = *0.2.*

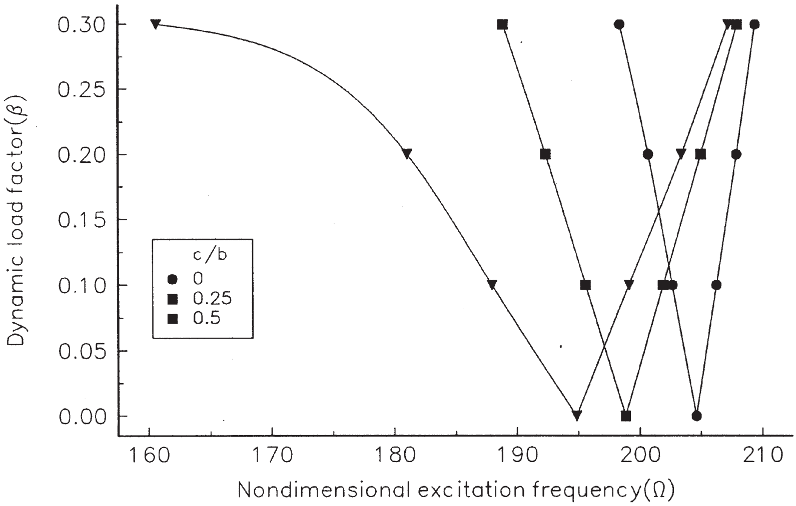

Figure 13. *Effect of position of concentrated load on instability region of a spherical shell for double pair of loads,* a/b = *1,* $a/R_x = b/R_y$ = *0.25,* c/b = *0, 0.25 and 0.5,* α = *0.2.*

CONCLUSIONS

The results of the stability studies of the shells can be summarised as follows:

1. The laminated composite shells become more stiff with more number of layers.
2. Due to static component, the instability regions tend to shift to lower frequencies with wide instability regions showing destabilizing effect on the dynamic stability behaviour of the curved panel.
3. The instability begins at higher excitation frequencies having wider instability zones with increase of degree of orthotropy.
4. The instability regions have been influenced due to the restraint provided at the edges.
5. The instability regions start at higher frequencies with lower shallowness ratio.
6. The ply orientation of 45° seems to be a preferential orientation for uniaxially loaded antisymmetric angle-ply doubly curved panels for the ranges of geometry and material properties considered.
7. The curved panels show more stiffness due to increase of curvatures. The hyperbolic paraboloid panels behave in between a singly curved cylindrical panel and a doubly curved spherical shell.
8. The onset of instability occurs at lower excitation frequencies with higher *b/h* ratio, but with wider instability region.

9. The onset of instability occurs at a higher excitation frequencies for loadings having small bandwidth and for concentrated loads near the edges but at lower frequencies for loading of large bandwidth and for concentrated loads away from the edges.

SYMBOLS

a, b	dimensions of shell
R_x, R_y	radii of curvature of shell
c	load bandwidth/distance of concentrated load from edge
E_{11}, E_{22}	Young's modulus
G_{12}, G_{13}, G_{23}	shear modulus
$[K]$	stiffness matrix
$[K_g]$	geometric stiffness matrix
$[M]$	mass matrix
$\{q\}$	vector of generalized coordinates
w	deflection of mid-plane of doubly curved panel
ν_{12}, ν_{21}	Poisson's ratio
θ_x, θ_y	rotations about axes
ρ	mass density
σ_x, σ_y, τ_{xy}	initial in-plane stresses
Ω, ω	frequency of forcing function and transverse vibration
α, β	static and dynamic load factors
P_{cr}	critical buckling load

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