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Boundary Collocation Technique for Heat Conduction in Eccentric Systems

Manoj K. Moharana

Assistant Professor, Department of Mechanical Engineering
National Institute of Technology Rourkela, Rourkela, Odisha, 769008
Email: moharanam@nitrkl.ac.in

Abstract

Heat conduction through eccentric systems with circular inner surface and regular polygonal outer surface has been analyzed. Constant wall temperature at the inner surface and convective heat transfer from the outer surface is considered. A two dimensional semi analytical method using boundary collocation technique is used in which collocation points are considered on the outer periphery of the eccentric systems. The temperature contour in the solid domain and heat transfer to the ambient are predicted. The geometry of the critical and crossover condition of insulation around a heated circular cylinder is also determined when regular polygonal shaped insulation is provided eccentrically.

1. Introduction

Circular tubes are widely used in heat exchangers and different heat transfer applications. Sometimes the wall thickness of the circular tube varies in the angular direction due to manufacturing defect; thus forms an eccentric annulus. In some applications a tube with inner circular and outer regular polygonal shape are also used. Sometimes the inner circular surface has some eccentricity with respect to the centre of the circumscribing circle of the outer surface. Sometimes insulations are provided over a heated circular pipe eccentrically or arise due to manufacturing defect; thus creating an eccentric system as shown schematically in Figure 1. When the number of regular outer sides is very high, it leads to an eccentric annulus.

When a heat transfer domain is closed by two surfaces and is maintained at constant but some difference between them, heat transfers from the boundary at higher temperature to the other boundary. In such situation the concept of conduction shape factor is used for calculating heat transfer between the two isothermal surfaces which is purely function of the geometry only. Many studies do exist in literature dealing with conduction shape factor in a variety of geometric configurations [1-6] including eccentric systems [7-8]. The conduction shape factor for an eccentric annulus with isothermal boundaries is given by [9]. Kolodziej and Strek [6] studied conduction heat transfer in regular polygonal cross-sections and calculated conduction shape factor. Moharana and Das [7] analyzed heat conduction through regular polygonal tubes with eccentric circular hole using a one dimensional approximate method called "sector method" where heat transfers from the inner isothermal surface to the outer isothermal surface.

Practically, isothermal inner surface and convective outer surface is more appropriate condition compared to both isothermal surfaces. To the best of the knowledge of the author, this condition is not studied for the geometries as shown in Figure 1 where as some other geometries are studied under these conditions. Moharana and Das [8] studied heat conduction through eccentric annulus using three different techniques; one dimensional approximate method, two dimensional collocation technique and perturbation technique, and determined the geometry of the critical as well as crossover perimeter of insulation around a circular cylinder when the insulation is provided eccentrically. Moharana and Das [10] studied heat conduction through circular tube having oval, lenticular and teardrop shaped outer surface.

In this background, a two-dimensional semi-analytical boundary collocation technique is used for the heat conduction analysis in eccentric systems having circular inner surface and regular polygonal outer surfaces as shown in Figure 1. The inner surface is at constant temperature and heat transfer from the outer surface to the ambient fluid is by convection. This situation resemble with most practical heat transfer situations involving geometries as shown in Figure 1.

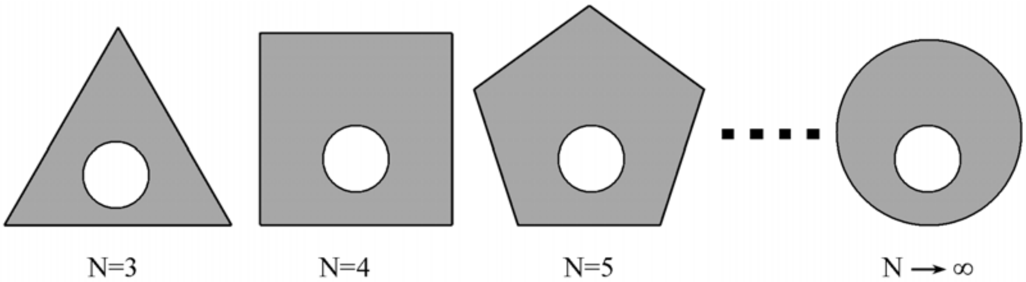

Figure 1. Schematic of regular polygonal tubes with an eccentric circular hole.

2. Mathematical Analysis

A two dimensional semi analytical boundary collocation method is used for the heat conduction analysis of the domain of the problem. The boundary collocation method can be summarized as the method that consists in using the exact solution to the governing differential equation of the problem and satisfying the given boundary conditions at a finite number of discrete points along the boundary [6]. Many applications of boundary collocation method in the field of continuous mechanics of solid structures can be found in [11,12]. Limited use of boundary collocation method for study of fluid flow [13-15] and heat transfer [6,8,10] do exist in the literature. The heat conduction analysis has been carried out with the following assumptions:

a) Thermal conductivity of the solid is constant.
b) Internal heat generation is zero.
c) Steady state heat conduction through the eccentric domain.
d) The eccentric circular inner surface is at constant temperature.
e) Heat transfer from the outer surface to the ambient fluid is by means of convection only.

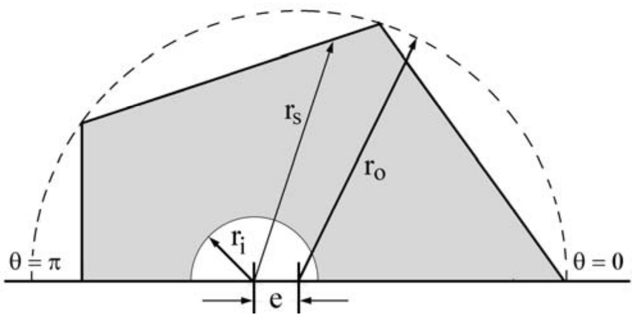

Figure 2. Schematic diagram of the symmetrical module of a regular pentagonal tube having an eccentric circular hole.

Figure 2 shows a regular pentagonal tube having an eccentric circular hole. Considering geometric symmetry, only half of this eccentric tube is considered in this analysis between θ = 0, and θ = π in the counter clock wise direction. Following non dimensional parameters are introduced

$$\Theta = \frac{T - T_\infty}{T_i - T_\infty}, \qquad R = \frac{r}{r_i}, \qquad \overline{R} = \frac{r_s(\theta)}{r_i}, \qquad \overline{E} = \frac{e}{r_i} \tag{1}$$

Here T_i is the temperature of the inner isothermal surface and T_∞ is the ambient fluid temperature. The governing equation in non dimensional form will be

$$\frac{\partial^2\Theta}{\partial R^2} + \frac{1}{R}\frac{\partial\Theta}{\partial R} + \frac{1}{R^2}\frac{\partial^2\Theta}{\partial\theta^2} = 0 \tag{2}$$

The general solution of Eq. (2) can be expressed as [6]:

$$\Theta(R,\theta) = A + B\ln R + C\theta + D\theta\ln R + \sum_{k=1}^{\infty}\left(A_k R^{\lambda_k} + B_k R^{-\lambda_k}\right)\cos(\lambda_k\theta) + \sum_{k=1}^{\infty}\left(C_k R^{\lambda_k} + D_k R^{-\lambda_k}\right)\sin(\lambda_k\theta) \tag{3}$$

where A, B, C, D, A_k, B_k, C_k, D_k, λ_k are unknown constants. The boundary conditions are

$$\frac{\partial \Theta}{\partial \theta} = 0 \quad \text{for} \quad \theta = 0 \tag{4a}$$

$$\frac{\partial \Theta}{\partial \theta} = 0 \quad \text{for} \quad \theta = \pi \tag{4b}$$

$$\Theta = 1 \quad \text{for} \quad R = 1 \tag{4c}$$

$$\frac{\partial \Theta}{\partial R} + Bi^{*}\Theta = 0 \quad \text{for} \quad R = \bar{R} \tag{4d}$$

where $Bi^{*} = h \cdot r_i / k_t$, h is the heat transfer coefficient from the outer surface, k_t is the thermal conductivity of the solid tube. Equation (4a) will be fulfilled if C = D = C_k = D_k = 0 for k = 1, 2, 3… Secondly, using Eq. (4b) one can find $\lambda_k = k$. Now, Eq. (3) will have the form

$$\Theta(R,\theta) = A + B\ln R + \sum_{k=1}^{\infty}\left(A_k R^{k} + B_k R^{-k}\right)\cos(k\theta) \tag{5}$$

Using Eq. (4c) in Eq. (5), A = 1, and $B_k = -A_k$. Now Eq. (5) will be

$$\Theta(R,\theta) = 1 + B\ln R + \sum_{k=1}^{\infty} A_k\left(R^{k} - R^{-k}\right)\cos(k\theta) \tag{6}$$

Introducing B = Y_1, $A_1 = Y_2$, $A_2 = Y_3$ and so on, Eq. (6) will be

$$\Theta(R,\theta) = 1 + Y_1 \ln R + \sum_{k=2}^{\infty} Y_k\left(R^{(k-1)} - R^{-(k-1)}\right)\cos\left((k-1)\theta\right) \tag{7}$$

Thus, Eq. (7) is the temperature profile in the domain of the problem where Y_1, Y_2, Y_3... are unknown constants. Now Eq. (4d) is considered at finite number of collocation points (say M) on the outer periphery of the eccentric system and subsequently used in Eq. (7). Secondly, the infinite series in Eq. (7) is also terminated after first M number of terms. This result in

$$Y_1\left[\frac{1}{\bar{R}} + Bi^{*}\ln\bar{R}\right] + \sum_{k=2}^{M} Y_k\left[\frac{(k-1)}{\bar{R}}\left(\bar{R}^{(k-1)} + \bar{R}^{-(k-1)}\right) + Bi^{*}\left(\bar{R}^{(k-1)} - \bar{R}^{-(k-1)}\right)\right]\cos[(k-1)\theta] = -Bi^{*} \tag{8}$$

where the unknowns Y_1, Y_2, Y_3...Y_M need to find from the above set of equation. Now, the final solution of the governing equation will have the form

$$\Theta(R,\theta) = 1 + Y_1 \ln R + \sum_{k=2}^{M} Y_k (R^{(k-1)} - R^{-(k-1)})\cos[(k-1)\theta] \tag{9}$$

Thus, Eq. (9) describes the two dimensional temperature variations within the eccentric domain. This expression is valid for a regular polygonal tube (as shown in Figure 1) having any number of sides. Only one requires finding an expression for $\bar{R}$ as a function of angle θ for all geometry to be used.

3. Results and Discussion

For the present analysis the value of dimensionless heat transfer $\bar{Q}$ is equal to $-Y_1$ where

$$\bar{Q} = \frac{Q}{2\pi k_t (T_i - T_\infty)} \tag{10}$$

where $\bar{Q}$ is the heat transfer, k is the conductivity of the circular tube. Thus, only the logarithmic term in Eq. (9) decide the value of $\bar{Q}$. Equation (9) with the help of Eq. (8) can be used to find the isotherms in any eccentric system and subsequently the heat transfer can also be calculated. For the purpose of validation of the present analysis a circular tube is considered. A regular polygon with infinite number of sides leads to a circle. Consider an eccentric system having inner circular hole of radius r_i and having an eccentricity e with respect to the centre of the circumscribed circle of the outer regular polygon having N number of sides (see Figure 1). When $e\to 0$, and $N\to\infty$, it leads to a circular tube. For a circular tube Aziz [16] calculated the dimensionless heat transfer rate given by

$$\overline{Q} = \frac{1}{\ln\left(\frac{r_o}{r_i}\right) + \frac{1}{\left(\frac{r_o}{r_i}\right)Bi^*}} \tag{11}$$

where Bi* is equal to hr_i/k_t and h is the heat transfer coefficient of the ambient fluid which is at a temperature of T_∞. The dimensionless heat transfer $\overline{Q}$ varying with r_o/r_i are shown in Figure 3 which indicates that the predictions from the present analysis are matching perfectly with that of Aziz [16]. When $Bi^{*-1} > 1$, with increasing r_o/r_i, the heat transfer $\overline{Q}$ increases; beyond a certain value (called critical condition), it again starts decreasing. The critical and the crossover limit for insulation wrap around a circular cylinder are also shown in Figure 3. When $Bi^{*-1} \le 1$, with increase in r_o/r_i, the heat transfer $\overline{Q}$ always decreases as maximum heat transfer takes place when the cylinder is bare.

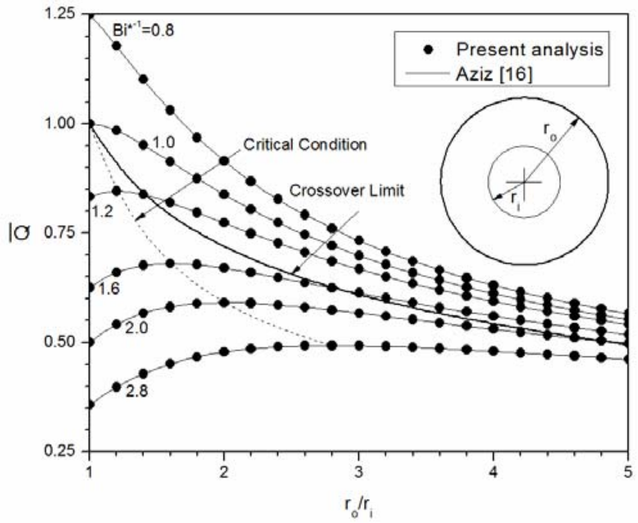

Figure 3. Heat dissipation in a circular tube as a function of radius ratio.

Next, heat conduction through eccentric systems with circular inner surface and regular polygonal outer surface are considered. For such an eccentric system the possible geometric configuration is guided by

$$\left(\frac{r_o}{r_i}\right)_{min} = \frac{1+\left(\frac{e}{r_i}\right)}{\cos\left(\frac{\pi}{N}\right)} \tag{12}$$

where N is the number of sides of the regular polygonal outer surface. For an eccentric annulus, Eq. (12) will have the form

$$\left(\frac{r_o}{r_i}\right)_{min} = 1+\left(\frac{e}{r_i}\right) \tag{13}$$

Figure 4 shows the temperature contour in dimensionless form as given in Eq. (1), in a square and a regular pentagonal tube with eccentric circular inner surface having e/r_i = 1.0 and r_o/r_i = 4.0, Bi^* = 1.0, where r_o is the radius of the circumscribed circle of the regular polygon, and e is the eccentricity of the inner circular surface with respect to the centre of the circumscribed circle. The inner surface is at constant temperature T_i i.e. $\Theta = 1$, and convective heat loss taking place from the outer surface.

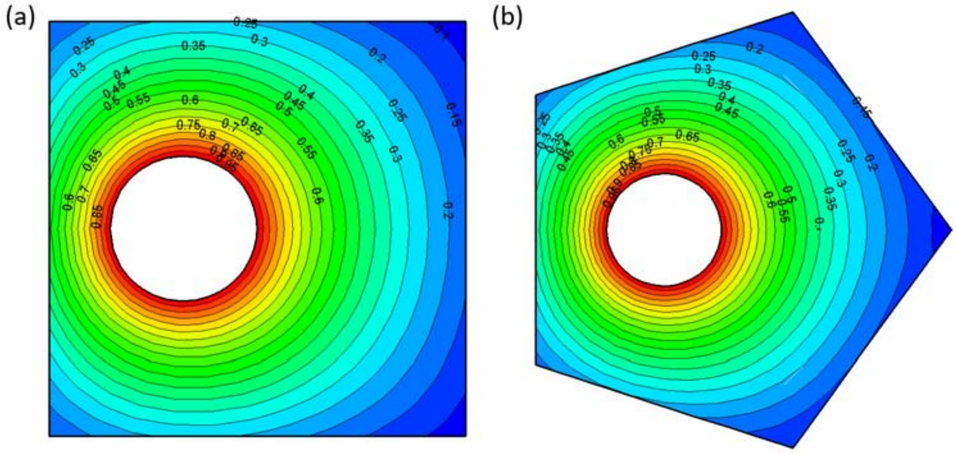

Figure 4. Temperature contour in a regular polygonal tube with eccentric circular inner surface (a) Square (b) Pentagonal.

The present analysis of heat conduction in eccentric systems could be important in many applications such as heat transfer through an insulation eccentrically placed on a circular cylinder. Figure 5 shows variation of dimensionless heat dissipation $\overline{Q}$ in an eccentric annulus where $\overline{Q}$ is varying with radius ratio (r_o/r_i) for circular eccentric insulation systems around a heated cylinder/pipe (see inset in Figure 5) of radius r_i and $e/r_i = 0.2$. For this eccentric insulation system, the minimum value of r_o/r_i possible is 1.2, which is shown in Figure 5 by a vertical dotted line.

When the eccentric configuration is such that r_o/r_i is minimum possible, the heat dissipation value shifts to a higher or lower value compared to the bare system. The value of heat transfer from the bare system is shown by horizontal lines in the interval $1 < (r_o/r_i) < (r_o/r_i)_{min}$. With increasing value of r_o/r_i, the value of $\overline{Q}$ gradually (i) increases and then decreases beyond the critical value or (ii) decreases continuously with increasing radius ratio (r_o/r_i) depending on the value of Bi^{*-1}, i.e., the value of $\overline{Q}$ at $(r_o/r_i)_{min}$ compared to $\overline{Q}$ of the bare system is a function of Bi^{*-1}.

At lower values of Bi^{*-1}, addition of minimum eccentric insulation $(r_o/r_i)_{min}$ with given eccentricity e/r_i, heat dissipation always reduce compared to bare system. But at higher value of Bi^{*-1}, addition of minimum eccentric insulation increases heat dissipation compared to heat dissipation from the bare system. The value of Bi^{*-1} at which addition of minimum insulation does not affect the heat dissipation is approximately at $Bi^{*-1} = 1.16$. This is not shown in Figure 5. This case arise when the crossover perimeter is equal to $(r_o/r_i)_{min}$. However, the overall variation of heat dissipation trend in Figure 5 is similar to that of Figure 3.

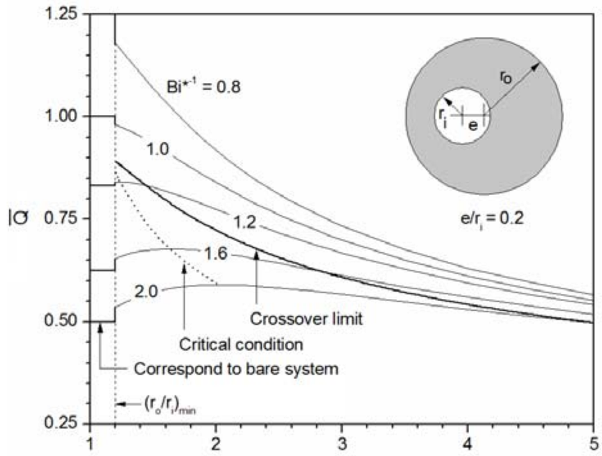

Figure 5. Variation of heat dissipation with radius ratio in an eccentric system having e/r_i 0.2

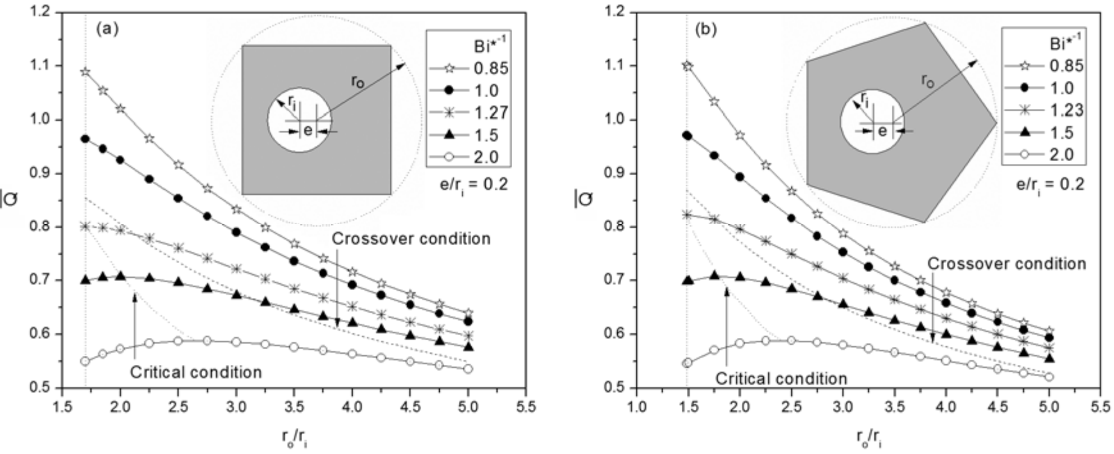

Figure 6. Variation of heat dissipation with radius ratio in eccentric regular polygonal duct (a) Square (b) Pentagon.

Figure 6 shows the variation of heat dissipation in two eccentric geometries (see the inset diagrams) where the outer surface is a regular polygon (square and pentagon respectively in Figure 6(a) and 6(b)). The inner circular surface (radius r_i) is having an eccentricity e (e/r_i = 0.2) with respect to the centre of the circumscribed circle (of radius r_o) of the outer regular polygonal surface. As per Eq. (12), the value of $(r_o/r_i)_{min}$ corresponding to Figure 6(a) and 6(b) are 1.697 and 1.483 respectively (shown by vertical dotted line). Here the shifting of the inner circular surface away from the centre of the circumscribed circle is considered to its left as shown in the inset diagrams in Figure 6 (a) and 6 (b).

When eccentricity arises, the effective wall thickness on one side comparatively decreases and on the other side it increases. Thus, the wall thermal resistance on one side decreases and on the other side it increases. The net thermal resistance decides increase or decrease in the net heat transfer from the eccentric system compared to concentric condition. Secondly, for fixed eccentricity, the heat transfer $\bar{Q}$ will vary with varying r_o/r_i. The trend of variation of $\bar{Q}$ with varying radius ratio (r_o/r_i) is similar to that observed in Figure 5. At lower values of Bi^{*-1}, $\bar{Q}$ decreases with increasing value of radius ratio (r_o/r_i). At higher value of Bi^{*-1}, on increasing value of r_o/r_i, $\bar{Q}$ first increases, reaches to its peak value and then starts decreasing. The geometric condition at this peak value of $\bar{Q}$ is the critical condition which is function of e/r_i and r_o/r_i. It is to note that r_o depends on number of sides (N) of the regular polygonal outer surface and the length of each side of this regular polygon. In Figure 6 (a) it is found that when $Bi^{*-1} \leq 1.27$, heat transfer $\bar{Q}$ is maximum at $(r_o/r_i)_{min}$ i.e. r_o/r_i = 1.697 and on further increasing radius ratio, $\bar{Q}$ decreases gradually. When $Bi^{*-1} > 1.27$, there exist critical condition at $r_o/r_i > 1.697$. Similarly in Figure 6 (b) the corresponding Bi^{*-1} is found to be 1.23.

Secondly, the crossover condition can also be observed in Figure 6 (a) and 6 (b) with respect to a bare circular hot cylinder of radius r_i, for which the heat transfer from its surface to the ambient fluid is fixed and is equal to Bi*. At the crossover condition, the heat transfer from the eccentric insulation system is equal to the value of heat transfer from the bare system. As r_o/r_i increases further beyond this crossover condition, value of $\bar{Q}$ decreases continuously but the curve gets flattened. Thus, the heat transfer effectively reduces only when r_o/r_i is increased beyond the crossover condition. In a similar fashion to Figure 6, both critical and crossover condition can be obtained for the eccentric geometries as given in Figure 1 with any number of regular sides.

4. Summary and Conclusion

Heat conduction through regular polygonal tubes having eccentric circular inner surface has been analyzed using two dimensional semi analytical boundary collocation technique. Constant wall temperature at the inner surface and convective heat transfer from the outer surface is considered. The temperature contour and heat transfer are predicted. A direct comparison with a circular tube validated the results. The present analysis has been applied to determine the geometry of the critical as well as crossover condition of insulation around a heated circular cylinder when regular polygonal shaped insulation is provided eccentrically.

5. References

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