

# An analysis of strain in chip breaking using slip-line field theory with adhesion friction at chip/tool interface

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## Abstract

A slip-line field model for orthogonal cutting with step-type chip breaker assuming adhesion friction at chip/tool interface is developed using Kudo's basic slip-line field. An alternative method is suggested for estimation of breaking strain in the chip. The model proposed predicts that with decrease in distance of chip breaker from the cutting edge of the tool, the breaking strain and shear strain in the secondary deformation zone increase while the total plastic strain decreases. The breaking of the chip is found to be solely dependent on the breaking strain, and not on 'material damage' or the specific cutting energy. The chip radius of curvature, cutting ratio, range of position of chip breaker for effective chip breaking are computed. The calculated results are found to be in general agreement with experimental measurements.

*Keywords:* Metal machining; Step-type chip breaker; Slip-line solutions; Strain; Adhesion friction

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## 1. Introduction

Chip control is essential to ensure reliable operation in automated machining systems. Effective chip control requires predictability of chip form/chip breakability for a given set of input machining conditions. However, it is difficult to achieve this with a high degree of accuracy due to a lack of suitable predictive theories or applicable methods to quantify chip breakability in machining. This is attributed to the complex mechanism of chip formation under various combinations of machining conditions with numerous interacting process parameters involved.

Considerable work has been carried out in the past on the analysis of chip breaking performance and prediction of chip breakability. Henriksen [1–3] and Okushima et al. [4] found that the degree of chip breaking is dependent on the feed rate or undeformed chip thickness and on the radius of chip curvature imposed by the action of the chip former. It was reported that the chip breaking increased as the radius

of chip curvature decreased or as the uncut chip thickness or feed rate is increased. It was also pointed out that effective chip breaking could only be achieved over a limited range of chip curvature. Experimental investigation using a step-type chip breaker has been reported by Nakayama [5,6], Trim and Boothroyd [7] and Subramanian and Bhattacharya [8].

A criterion for chip breaking based on chip strain analysis was first presented by Nakayama [9]. He showed that the chip breaks when the strain on the chip surface exceeds the fracture strain of the chip material. For medium carbon steel this strain was reported to be equal to or greater than 0.05. Takayama et al. [10] and Jawahir [11], however, found the corresponding values of breaking strain to be 0.046–0.052 and 0.036–0.048, respectively. A hybrid algorithm for predicting chip form/chip breakability has also been proposed by Fang et al. [12].

A 'material damage-based model' for predicting chip breakability was presented by Athavale and Strenkowski [13]. According to these authors two chip breaking criteria must be considered: the first dealing with estimation of material damage during chip formation (shearing of the material in the primary shear plane and subsequent deformation in the secondary shear zone) and the second relating to the initial chip geometry, chip curl radius and subsequent bending by the

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## Nomenclature

$F_b$	force exerted by chip breaker on the chip, acting radially towards the centre of chip curvature
$F_1$	force acting on the boundary slip-line AB, perpendicular to $F_b$
$F_2$	force acting on the boundary slip-line BC, parallel to $F_b$
$H$	height of the chip breaker
HTR	$\frac{H}{t_0}$
$k$	yield stress in shear
$l_n$	contact length of the chip/tool interface
$M$	moment exerted by the slip-lines AB and BC
$n$	index of stress distribution or constant based on material properties of the tool and work piece combinations
$p_c$	hydrostatic pressure at point C
$R_{chip}$	outer radius of the chip formed by the chip breaker
$R_0$	outer radius of the chip from hodograph
$t_0$	uncut chip thickness, i.e. feed (in case of orthogonal cutting)
$t_{chip}$	chip thickness
$W$	distance of the chip breaker from the principal cutting edge of tool
WTR	$\frac{W}{t_0}$

## Greek letters

$\varepsilon_t$	total plastic strain
$\varepsilon_p$	primary shear strain along the shear plane ABD
$\varepsilon_s$	secondary shear strain in secondary deformation zone
$\varepsilon_b$	breaking strain
$\gamma$	orthogonal rake angle of cutting tool
$\eta, \beta, \theta$	slip-line field angles
$\mu$	coefficient of friction
$\rho$	scale parameter representing the geometrical scale of the field
$\sigma_n$	normal stress
$\tau$	shear stress
$\zeta$	cutting ratio = $\frac{t_{chip}}{t_0}$

chip former. They also proposed a material damage criterion using the hole growth model proposed by McClintock et al. [14]. An energy approach to chip breaking while machining with grooved tool inserts has been suggested by Grzesik and Kwiatkowska [15]. These authors correlated the specific cutting energy consumed during machining with different types of chip forms.

Theoretical studies to evaluate performance of chip breakers using slip-line field theory has also been reported by Shi and Ramalingam [16], Fang and Jawahir [17] for groove type

chip breakers, by Dewhurst [18] for ramp type chip breakers and by Maithy and Das [19,20] for step-type chip breakers. However, no attempt has been made to date to correlate the strain and strain energy calculated from the slip-line field analysis with effectiveness of chip breaking.

In the present paper, a slip-line field analysis is carried out for pure orthogonal cutting using a cutting tool with a parallel step-type smooth chip breaker. The slip-line field studied is that proposed earlier by Kudo [21]. Adhesion friction is assumed at the chip/tool interface. The total shear strain, shear strain in the secondary shear zone, breaking strain and radius of chip curvature has been estimated for different positions of the chip breaker. The bounds on breaking strain has been evaluated for effective chip breaking. The theoretical results are also compared with those attained from experiments.

## 2. Methodology

The slip-line field due to Kudo [21] for metal machining with step-type chip breaker involving chip curl is shown in Fig. 1(a). The work material undergoes plastic deformation along the primary shear line consisting of the curves AB and BD, which are  $\beta$ -lines. The chip boundary is indicated by the curves AB and BC, where BC is a concave  $\alpha$ -line. The

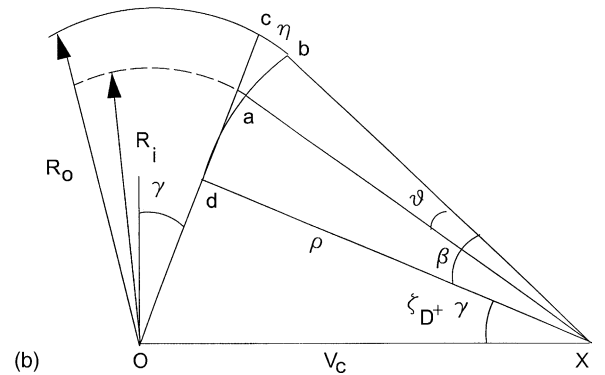
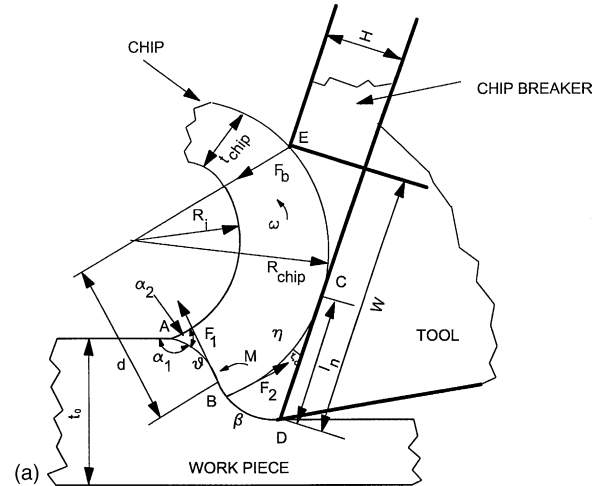


Fig. 1. (a) Kudo's slip-line field with step-type chip breaker. (b) Hodograph.

secondary deformation zone is shown by the area enclosed by the curves BC, BD and the rake surface CD. It is assumed that rake face friction is governed by the adhesion friction law suggested by Maekawa et al. [22]. This may be written as:

$$\tau = k \left[ 1 - \exp \left( - \left( \frac{\mu \sigma_n}{k} \right)^n \right) \right]^{\frac{1}{n}} \quad (1)$$

where  $\tau$  is the shear stress,  $k$  the yield stress in shear of the work material,  $\sigma_n$  the normal stress,  $\mu$  the low stress level coefficient of friction and  $n$  is a constant that depends on tool/work-piece combination.

After crossing the deformation region, the material forms a curled chip of constant curvature due to rigid body rotation. A step-type chip breaker is placed in front of this chip, which reduces its radius of curvature, by imposing a force on it and thus helps in breaking.

Referring to the hodograph as shown in Fig. 1(b) it is seen that the material suffers a velocity discontinuity of magnitude  $\rho$  on crossing the primary shear-line. Hence, velocity along the slip-line DBA is indicated by the circular arc db in the hodograph, similarly the velocity along slip-line BC is shown by the hodograph curve be. Since the chip is rotating rigidly with angular velocity  $\omega$ , the images of lines BA and BC appear in the hodograph, but rotated through  $90^\circ$  in the direction of  $\omega$  multiplied by the scale factor  $\rho$ . Thus, the curves ab and be in the hodograph are geometrically similar to the curves AB and BC in the slip-line field, respectively. Hence, slip-line curve BA is also a circular arc of radius  $\frac{\rho}{\omega}$ .

It is readily seen that the column vector  $\bar{\sigma}$  for the radius of curvature of the slip-line CB is calculated from the relationship:

$$\bar{\sigma} = \left( \frac{\rho}{\omega} \right) \text{CL} \bar{c} \quad (2)$$

where CL is the linear operator defined by Dewhurst [23] that constructs the field between the circular arc db and the tool face (Fig. 1(b)), consistent with the adhesion friction condition given by Eq. (1) and  $\bar{c}$  is a column vector representing the unit circle. Slip-line curve BD is similarly calculated from CB using the corresponding operator. Hence, forces and moments on the chip boundary could be calculated.

Referring to Fig. 1(a), it may be seen that the field has three degrees of freedom: angular range  $\eta$  and  $\theta$  of slip lines CB and AB, respectively, and the hydrostatic pressure  $p_c$  at point C. Three boundary conditions also exist, from which these three field variables can be determined. With reference to Fig. 1(a), if the forces on the chip boundary ABC are resolved parallel and perpendicular to the chip breaker force  $F_b$ , equilibrium of the chip requires that

$$F_2 = F_b \quad (3)$$

$$F_1 = 0 \quad (4)$$

$$M + F_b d = 0 \quad (5)$$

Further, the radius of curvature of the chip calculated from the slip-line field must be equal to that imposed by the chip breaker. Hence

$$R_0 - R_{\text{chip}} = 0 \quad (6)$$

where

$$R_{\text{chip}} = \frac{(W - l_n)^2}{2H} + \frac{H}{2} \quad (7)$$

Eqs. (4)–(6) were expressed as functions of the field variables  $\eta$ ,  $\theta$  and  $p_c$  resulting in three non-linear algebraic equations. These equations were solved by an algorithm developed by Powell [24] to minimise the sum of the squares of the residuals. In all cases the levels of accuracy was less than  $10^{-10}$ .

In this manner solutions were generated and machining parameters were computed for a tool with rake angle  $\gamma = 10^\circ$ ,  $\mu = 2.0$  and  $n = 1.5$ . The programme incorporated flatness and mass flux checks as reported in references [19,20]. It also contained checks to ensure that the rigid vertices at A are not overstressed by applying Hill's criteria [25].

### 3. Calculation of strains

The shear strains induced in the material for any given geometry were calculated from the corresponding slip-line field configuration using the method suggested by Atkins et al. [26]. For computing the shear strain  $\varepsilon_p$  for the primary shear line ABD, it was discretised into 15 straight elemental regions. For each element the average normal component of the velocity was obtained from the hodograph and the shear strain was calculated as the ratio of the magnitude of the velocity discontinuity to the normal velocity. On summing up the shear strains for all elements the total 'material damage' for the primary shear line was estimated.

For computing the shear strain for the secondary deformation zone BCD, it was divided into 15 streamlines. For each stream line the shear strain was computed in the manner explained in the appendix. The total strain was calculated by summing up the strains for each streamline.

Breaking strain  $\varepsilon_b$  was evaluated using the relation:

$$\varepsilon_b = \ln \left[ \frac{R_{\text{chip}}}{R_{\text{chip}} - \frac{t_{\text{chip}}}{2}} \right] \quad (8)$$

where  $R_{\text{chip}}$  is outer radius of the chip and  $t_{\text{chip}}$  is chip thickness. For the above calculations it was assumed that the neutral plane passes through the middle of the chip section.

### 4. Results and discussion

The results of computation from the present slip-line field analysis are shown in Figs. 2–9 for a tool with orthogonal rake angle  $\gamma = 10^\circ$  and rake face friction parameters  $\mu = 2.0$  and  $n = 1.5$ . The non-dimensional chip breaker parameter HTR and WTR in these plots refer to the ratio of height  $H$

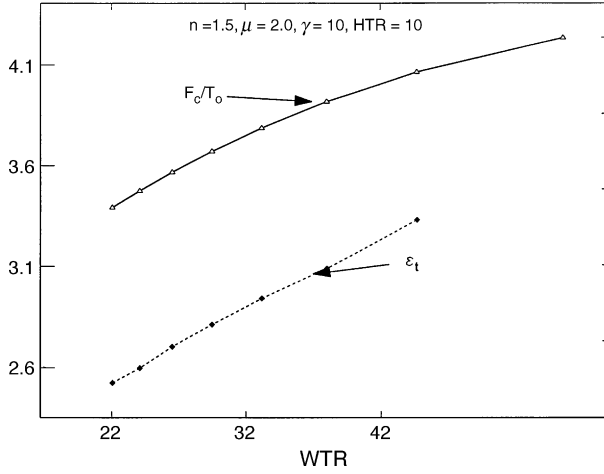


Fig. 2. The variation of specific cutting energy and total strain with WTR (ratio of chip breaker distance to uncut chip thickness).

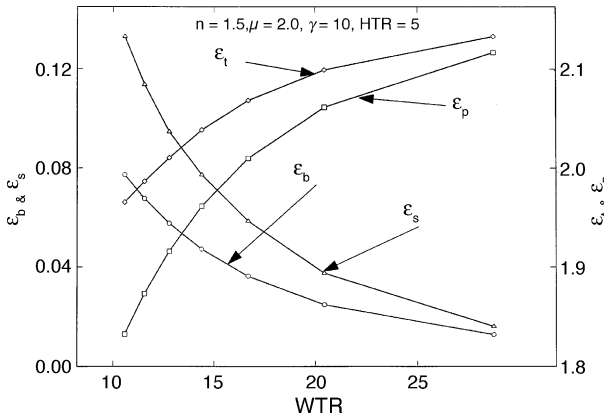


Fig. 3. The variation of primary shear strain, secondary shear strain, breaking strain and total strain with WTR.

and distance  $W$  of the chip breaker to uncut chip thickness  $t_0$  (Fig. 1(a)).

Referring to Fig. 2 it may be seen that both specific cutting energy ( $F_C/t_0$ ) and the total strain  $\epsilon_t$  suffered by the material

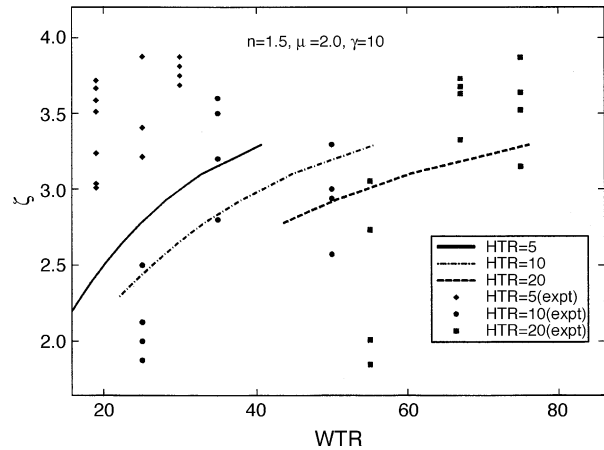


Fig. 4. The theoretical variation of cutting ratio with WTR and its comparison with experimental results.

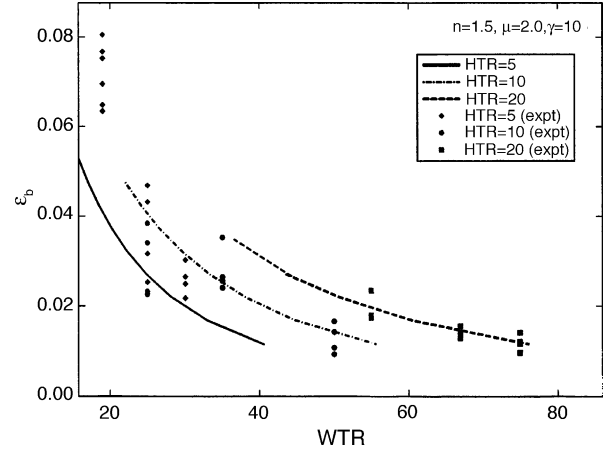


Fig. 5. The theoretical variation of breaking strain with WTR and its comparison with experimental values.

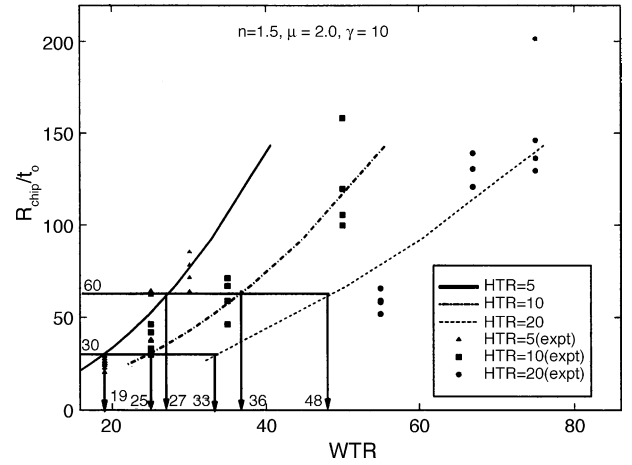


Fig. 6. The theoretical variation of normalized radius of curvature with WTR and its comparison with experimental values.

in the chip formation process increase as WTR increases. The breaking strain  $\epsilon_b$  and the strain  $\epsilon_s$  induced in the material in the secondary shear zone BCD, however, are found to decrease with WTR (Fig. 3). It may further be observed with

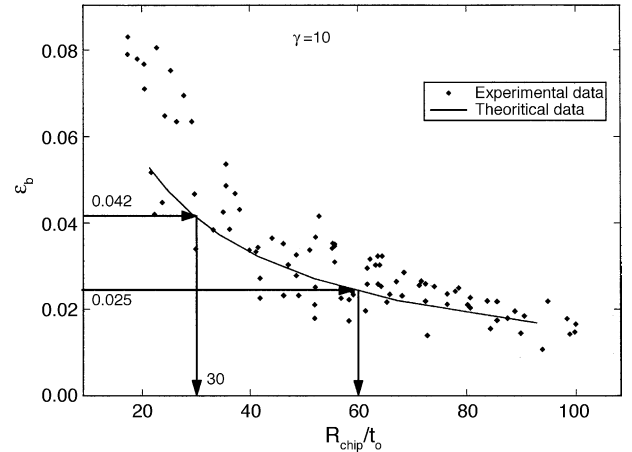


Fig. 7. The theoretical variation of breaking strain with normalized radius of curvature and its comparison with experimental data.

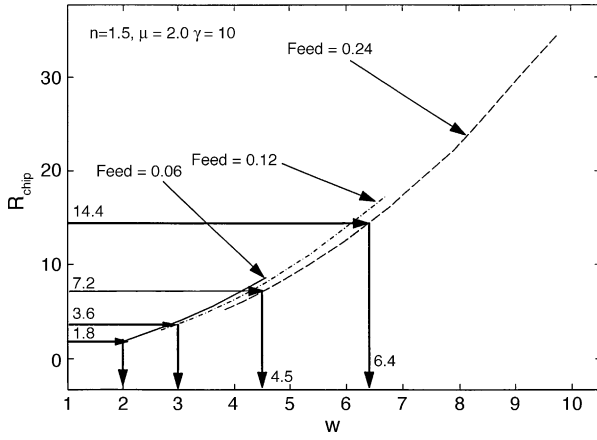


Fig. 8. The variation of chip radius of curvature with chip breaker distance for various feeds.

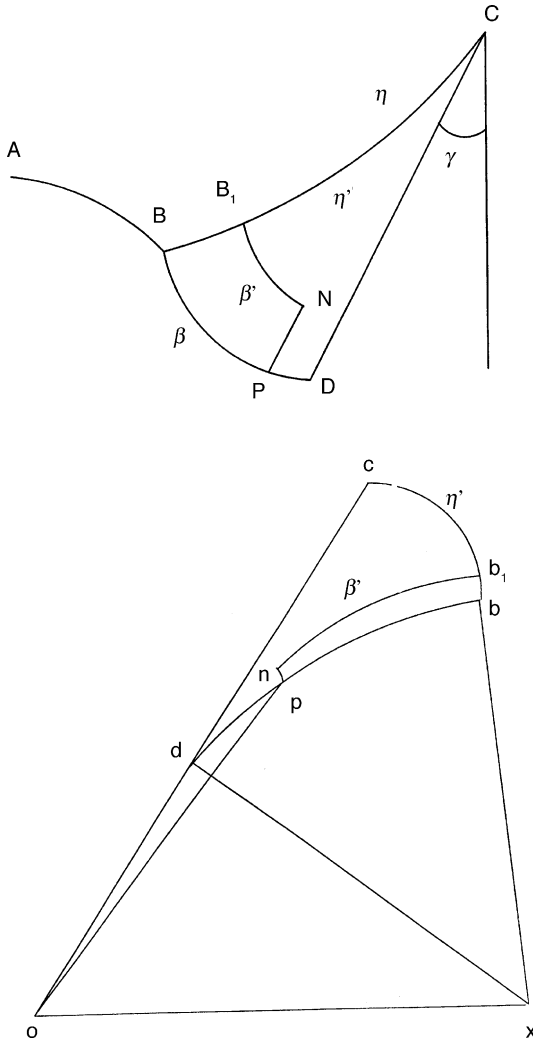


Fig. 9. Calculation of shear strain in the secondary deformation zone. (a) Slip-line field. (b) Hodograph.

reference to Fig. 4 that the chip thickness ratio  $\zeta$  increases as WTR increases. Since in metal machining an increased value of  $\zeta$  is always associated with an increased shear strain, these results are consistent with those shown in Fig. 2. The results also agree with the findings of Dewhurst [18] for a ramp type chip breaker. It may also be seen that  $\varepsilon_s$  constitutes only about 10% of the total strain: the bulk of the ‘damage’ being experienced by the material when it crosses the primary shear line.

It, therefore, appears that a chip breaking criterion based on specific cutting energy as proposed by Grzesik and Kwiatkowska [15] nor that based on total ‘material damage’ proposed by Athavale and Strenkowski [13] can be taken as a criterion to assess effectiveness of chip breaking at least within the assumption of rigid-perfectly plastic material behaviour. It is a well-known experimental observation that for any given value of feed as the chip breaker moves away from the tool tip, the effectiveness of chip breaking decreases. The present theoretical analysis suggest that both  $(F_C/t_0)$  and  $\varepsilon_t$  increases with WTR even though the chip breaker becomes less effective. Hence, it appears that the bending strain  $\varepsilon_b$  is the most critical parameter in chip breaking and when this attains a certain threshold value there is likelihood of chip fracture.

The variation of breaking strain  $\varepsilon_b$  with WTR is shown in Fig. 5 for values of HTR equal to 5, 10 and 20. The figure indicates that for the same value of feed, as the chip breaker comes closer to the cutting edge, the breaking strain increases. The normalised radius of curvature  $R_{chip}/t_0$  under such situation, however, decreases (Fig. 6) indicating that the chip curls more tightly with increased tendency to fracture. The variation of breaking strain with  $R_{chip}/t_0$  is shown in Fig. 7. This variation is found to be almost independent of HTR.

Using fuzzy logic technique, Fang et al. [12] had suggested classification of chips. These authors reported that in case of orthogonal cutting, spiral and circular chips are considered as ‘excellent’ and ‘good’ from chip breakability point of view, when their radius of curvature is ‘about’ 6 mm. Hence, the chips with fractional turn and radius of curvature of ‘about’ 6 mm are termed as ‘effectively broken chips’. The chips with radius of curvature ‘much less than’ or ‘much greater than’ 6 mm were considered as ‘over broken’ and ‘under broken’ chips, respectively.

In the present case it was observed that ‘effective breaking’ was obtained while machining with feed values of 0.10 and 0.20 mm/rev for the chip breaker position of 3–4.5 mm. The normalised radii of curvature for these feed values were about 30 and 60, respectively. Referring to Fig. 7 it may be seen that the breaking strains  $\varepsilon_b$ , for these cases are found to be between 0.025 and 0.042. These values are marginally lower than those reported by Nakayama [6]. It must be mentioned that a chip after losing contact with chip breaker undergoes elastic recovery (Worthington [27]). As a result the measured radius of curvature of chips is always higher than those obtained from slip-line field analysis. This phenomenon along with the simplified formula used for calculation of breaking strain



Table 1

Range of positions of chip breaker for effective chip breaking

HTR	Feed, $t_0 = H/HTR$ (mm/rev)	Effective WTR range	Value of $W = WTR \times t_0$ (mm)	
			Min	Max
5	$1.2/5 = 0.24$	19–27	4.56	6.48
10	$1.2/10 = 0.12$	25–36	3.0	4.32
20	$1.2/20 = 0.06$	33–48	1.98	2.88

(Eq. (8)) may be the reason for small discrepancy between the present values and those reported by Nakayama.

For any given height of the chip breaker, the effective range of chip breaker distance in practical metal machining situation can be obtained from plots similar to those shown in Fig. 6. Referring to this figure it may be seen that for any given value of HTR the effective WTR values lie between points with ordinate values between 30 and 60. For a constant chip breaker height of 1.2 mm as in the present case the effective range of chip breaker distance  $W$  for HTR values of 5, 10 and 20 is shown in Table 1. The radii of chip curvature corresponding to these feeds may be seen with reference to Fig. 8. It is evident from the graph, that even though chip curl radius and  $W$  vary widely, the breaking strain remains substantially constant.

## 5. Conclusion

In the present study, slip-line field analysis for orthogonal machining using step-type chip breaker has been carried out assuming adhesion friction at chip/tool interface. It is shown that the chip breaking criterion based neither on specific cutting energy nor that based on material damage can be taken as adequate criterion for chip breaking.

The total strain induced in the material on crossing the primary shear line and the shear zone has been estimated for various cutting conditions. The breaking strain has also been calculated by assuming the neutral axis to remain at the middle of chip cross-section.

It is seen that the breaking strain in the chip is the most important factor on which chip breaking depends. A method has been suggested for determining chip breaker distance for any given feed and chip breaker height for effective chip breaking.

The theoretical results agree reasonably well with those obtained from orthogonal cutting tests.

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## Appendix A

The method for determination of secondary shear strain in the secondary deformation zone is as follows:

1. Secondary deformation zone is the region formed by slip-lines CB, BD and tool face CD.
2. For any point P on  $\beta$ -slip-line BD (see Fig. 9(a)) the corresponding point 'p' on the hodograph curve bd (Fig. 9(b)) is located.
3. Absolute velocity of point P is obtained from hodograph, which is equal to 'op'.
4. As the tool advances, the work material suffers a shock along BD and moves along the line 'op' which is parallel to the velocity at point P. Through this point P a tangent PN, parallel to 'op' is drawn.
5. After a time interval  $\Delta t$ , material reaches point N. Coordinates of point N are calculated from the equations given below:

$$X_N = X_P + V_X \Delta t \quad (9)$$

$$Y_N = Y_P + V_Y \Delta t \quad (10)$$

where  $V_X$  and  $V_Y$  are horizontal and vertical components of velocity 'op'.

6. To determine the angular coordinates  $\eta'$  and  $\beta'$  of N within the secondary deformation zone an initial guess for the field angles was made and the coordinates  $X'_N$  and  $Y'_N$  of N were estimated from  $\eta'$  and  $\beta'$ .
7. The exact values of  $\eta'$  and  $\beta'$  were then determined by solution to the two algebraic equations

$$X_N - X'_N = 0 \quad (11)$$

$$Y_N - Y'_N = 0 \quad (12)$$

using Powell's algorithm [24].

8. The strain suffered by the material on movement from point P to N is calculated by the method suggested by Atkins et al. [26].
9. The point corresponding to N of slip-line is located in hodograph and the whole process is repeated until the material reaches the  $\alpha$ -line CB of secondary zone.

In this way the geometry of the streamline and the corresponding strain are calculated.

The procedure is repeated for several streamlines to get fairly accurate estimate of secondary shear strain.

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