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# The effect of variable specific heat of the working fluids on the performance of counterflow heat exchangers

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The variation of fluid properties such as specific heat must be accounted for in the design of heat exchangers. Kays and London[1] have suggested the use of a mean fluid temperature for evaluation of specific heat where the absolute temperature variation is limited to a factor of two. In cryogenic practice, however, the absolute temperature may vary over a much wider range.

The specific heat $c_p$ of hydrogen at ordinary pressures changes from nearly 7R/2 at room temperature to 5R/2 below 80 K due to freezing of the rotational degrees of freedom. In the case of supercritical hydrogen, the specific heat varies by a factor of 4 in the 300–80 K temperature range.[2]

The effectiveness of a counterflow heat exchanger is given as[1]

$$\varepsilon = \frac{1-\exp(-N_{tu}(1-\nu))}{1-\nu\exp(-N_{tu}(1-\nu))} \quad \text{for } \nu < 1$$
$$= \frac{N_{tu}}{N_{tu}+1} \quad \text{for } \nu = 1 \tag{1}$$

where $\varepsilon$ is the effectiveness, $N_{tu}$ the number of heat transfer units, and $\nu$ = the heat capacity rate ratio.

If the specific heats of the fluids are dependent on temperature, both the parameters $N_{tu}$ and $\nu$ cannot be defined for the total heat exchanger and hence (1) cannot be used directly to predict effectiveness. In such cases, the total exchanger may be considered[1] to be made up of a number of segments operating in series, each of these segments being small enough to ignore the intra-segment variation of fluid properties.

$N_{tu}$ and $\nu$ are determined for each individual segment and are substituted in (1) to give the effectiveness and the fluid exit and inlet temperatures for that segment. Successive application of these steps in a suitable iterative scheme yields the overall effectiveness of the exchanger.

In contrast to the constant specific heat model, the variable specific heat problems require an extended numerical procedure. It is necessary to determine if a variable specific heat system can be described in terms of a characteristic $N_{tu}$ and the constant specific heat model.

The $N_{tu}$ of a differential element d$x$ may be written as:

$$dN_{tu} = \frac{UA\,dx}{L(\dot{m}c_p)_{min}} \tag{2}$$

where $U$ is the overall heat transfer coefficient, $A$ the total heat transfer area, $L$ the overall length, and $(\dot{m}c_p)_{min}$ the minimum heat capacity flow rate. (2) may be integrated over the length $L$ to give

$$N_{tu} = \frac{UA}{L\dot{m}} \int_0^L \frac{dx}{c_p(x)} = \frac{UA}{\dot{m}\bar{c}_p} \tag{3}$$

$\dot{m}$ being the flow rate of the fluid with smaller capacity rate $\dot{m}c_p$ and $\bar{c}_p$ a characteristic specific heat. Thus,

$$\frac{1}{\bar{c}_p} = \frac{1}{L}\int_0^L \frac{dx}{c_p(x)}$$
$$\simeq \frac{1}{L}\int_{T_C}^{T_H} \frac{1}{c_p(T)}\left(\frac{dT}{dx}\right)^{-1} dT \tag{4}$$

$T_C$ and $T_H$ being the inlet temperatures of the cold and hot fluid streams respectively. If a linear temperature profile is assumed throughout the exchanger,

$$\frac{dT}{dx} = \frac{T_H - T_C}{L} \quad \text{and}$$
$$\frac{1}{\bar{c}_p} = \frac{1}{T_H - T_C}\int_{T_C}^{T_H} \frac{dT}{c_p(T)} = \frac{1}{c_{p,hm}} \tag{5}$$

$c_{p,hm}$ is the harmonic mean specific heat over the temperature range $T_C - T_H$ and is a candidate for use in (3) for determination of $N_{tu}$, which may be used in (1) to determine the effectiveness. There are, however, two soruces of error: the fluid temperatures at the ends are different from $T_C$ and $T_H$ due to finite effectiveness, and the temperature profile is not strictly linear even under balanced flow due to variable specific heat. In

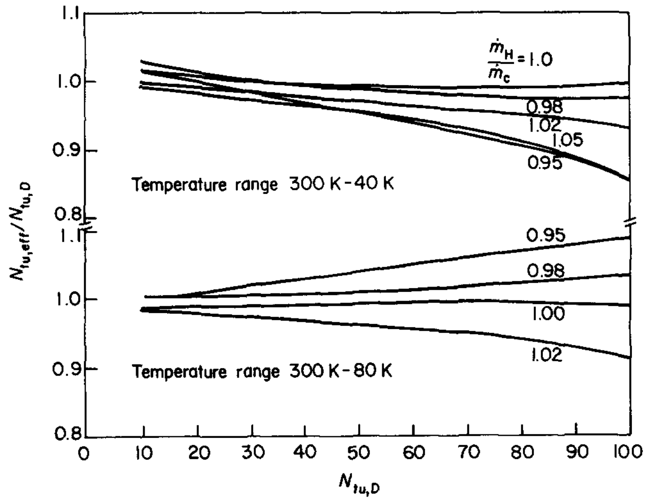


Fig. 1 Comparison of effective $N_{tu,}$ (with variable specific heat) with design $N_{tu,}$ based on harmonic mean specific heat. Working fluid is normal hydrogen at a pressure of 2 MPa

spite of these approximations, $c_{p,\,hm}$ appears to be a very good estimate of the average specific heat to be used in a constant specific heat model.

In order to test this hypothesis we have numerically computed the effectiveness of heat exchangers with $N_{tu,\,D}$, the design $N_{tu}$ in the range 10 to 100 and capacity rate ratio $\nu$ in the range 0.95 to 1.05. Normal hydrogen at a pressure of 2 MPa (supercritical fluid) has been used in both streams. The effectiveness $\varepsilon$ has been reduced to an effective $N_{tu}$ by (1) with $N_{tu,\,eff}$ substituting for $N_{tu}$. Fig. 1 gives a plot of $N_{tu,\,eff}$ vs $\nu$ and $N_{tu,\,D}$ in two temperature ranges 300–80 K and 300–40 K. It is observed that the harmonic mean specific heat gives good results for balanced flow heat exchangers ($\nu = 1$), but shows significant deviation for unbalanced flow $\nu < 1$ particularly at high $N_{tu,\,D}$.

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