Axial conduction is a major source of inefficiency in a compact counterflow heat exchanger. Any attempt to reduce axial conduction by using material of low thermal conductivity for the separating wall results in increased resistance to lateral heat flow, thereby reducing the overall thermal efficiency of the heat exchanger. The governing equations including axial conduction and lateral resistance due to the separating wall have been solved and an expression, for the overall efficiency of the heat exchanger has been derived in terms of relevant nondimensional parameters. Computed results have been presented which give the optimum thermal conductivity of the wall material.

Effect of finite thermal conductivity of the separating wall on the performance of counterflow heat exchangers

K. Chowdhury and S. Sarangi

Key words: cryogenics, heat exchanger, thermal conductivity, performance

| Nomenclature | β* | intermediate variable  
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<tr>
<td>β1, β2, βc</td>
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<td>( \beta_1 + \beta_2 + 4 \beta_c )</td>
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The performance deterioration in counterflow heat exchangers due to axial conduction has been investigated by several authors. Based on the original formulation by Landau and Hlinka, Kroeger has computed the overall inefficiency of heat exchangers for a wide range of flow parameters. But in none of these papers has the resistance of the separating wall to lateral heat transfer been explicitly considered. Any attempt to reduce axial conduction by using material of low thermal conductivity for the separating wall results in increased resistance to lateral heat flow, thereby reducing the overall thermal efficiency of the heat exchanger.

Since a wide range of materials ranging from copper and aluminium to plastics are being used in heat exchanger design, it is time to analyse the heat transfer process including both axial conduction and lateral resistance by

\[
\text{k}_{\text{fluid}} = \frac{2}{Pe} \left[ 1 + \frac{C_{\text{tube}}}{C_{\text{shell}}} \right]
\]

where Pe is the Peclet Number of the fluid in the (inner) tube and C the heat capacity rate of the fluid (mass flow rate x specific heat). The second law analysis, while predicting the optimum thermal conductivity of the wall, is unable to help in computing the thermal efficiency for a given set of flow parameters. In this paper, we derive an expression for the efficiency of heat exchangers considering the separating wall. Chowdhury and Sarangi have recently derived an expression for the optimum thermal conductivity of the wall in a concentric tube heat exchanger on the basis of minimum-entropy-generation principle:

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Fig. 1 Longitudinal section of the wall and a schematic temperature profile

both axial conduction and lateral resistance due to the separating wall in terms of relevant nondimensional parameters.

Theory

The following assumptions are usually valid for concentric tube counterflow heat exchangers used in cryogenic systems: The wall thickness is small compared to tube diameter. (Wall thickness is determined by the pressure difference between the two sides); both ends of the heat exchanger are adiabatic; and there is no heat leak from the surroundings. A longitudinal section of a portion of the heat-exchanger wall is shown in Fig. 1, explaining the nondimensional temperatures ($\theta$) and the heat flow rates ($q$) at different locations.

The energy conservation equations for the two fluid streams and the axial heat transfer equation may be expressed in dimensionless form as:

$$\begin{align*}
\frac{d}{dx} + \beta_1 \theta_1 - \beta_1 \theta_{cl} &= 0 \\
\frac{d}{dx} - \beta_2 \theta_2 + \beta_2 \theta_{c2} &= 0
\end{align*}$$

with boundary conditions:

$$\begin{align*}
x &= 0: \theta_1 = 1 \text{ and } \frac{d\theta_c}{dx} = 0 \\
x &= 1: \theta_2 = 0 \text{ and } \frac{d\theta_c}{dx} = 0
\end{align*}$$

The solution of (2) - (6) gives the temperature profiles in the two fluid streams and the wall. The overall inefficiency ($1 - \epsilon$) is then given by the expression

$$1 - \epsilon = \frac{\frac{H_1 - G_1}{r_1} \exp(-r_1) \frac{H_2 - G_2}{r_2} \exp(-r_2) \frac{H_3 - G_3}{r_3} \exp(-r_3)}{1 - \frac{1}{\beta_1} - \frac{1}{2\beta_2}} + \sum_{j=1}^{3} G_j B_{j\epsilon} \exp(-r_j)$$

where $r_j$, $G_j$ and $H_j$ are functions of the parameters $\beta_1$, $\beta_2$, $\beta_3$, $\nu$ and $\lambda$. The solution procedure and definition of $r_j$, $G_j$ and $H_j$ are given in Appendix 1.

For the special case of balanced flow ($\nu = 1$) condition, some of the terms in (7) become indeterminate. An alternative derivation is given in Appendix 2 resulting in the expression

$$1 - \epsilon = \beta_{oc} + B_{le} \left(1 - \frac{1}{\beta_1} - \frac{1}{2\beta_2}\right) + \sum_{j=1}^{3} G_j B_{j\epsilon} \exp(-r_j)$$

Conclusion

For tube-in-tube and shell-and-tube heat exchangers $\epsilon$ may be expressed in terms of $\lambda$ and a geometrical parameter $l/t$ as $\beta_{oc} = \lambda/(l/t)^2$. Then the efficiency $\epsilon$, and hence the effective $N_{tu}$, are functions of five parameters $\beta_1$, $\beta_2$, $l/t$, $\lambda$ and $\nu$. Fig. 2 (a-c) show the dependence of $N_{tu}$ (effective) on $\lambda$, $l/t$, $\beta_1$ and $\nu$ for the case of balanced design ($\beta_1 = \beta_2$). Kroeger's $^3$ have shown that variation of $\beta_1/\beta_2$ has a negligible effect on the overall inefficiency of the heat exchanger. The optimum value of $\lambda$ and hence of $k_{wall}$ agree exactly with (1). The symmetry of the $N_{tu\text{, eff}}$ vs $\ln \lambda$ curve suggests that a material should be chosen for the wall to keep the ratio $k_{wall}/k_{opt}$ (or $k_{opt}/k_{wall}$, whichever is greater than unity) the lowest. The computed results also confirm that Kroeger's $^3$ results can be used to predict efficiency if the lateral resistance of the wall is incorporated into the total $N_{tu}$. In most practical cases the difference between Kroeger's results and exact calculation is negligible.

Authors

The authors are from the Advanced Centre for Cryogenic
Appendix 1

Since axial conduction is small compared to the total lateral heat flow, the following may be assumed.

$$\theta_c = (\theta_{c1} + \theta_{c2})/2$$

and

$$\dot{q} = (\dot{q}_{1} + \dot{q}_{2})/2$$

or

$$\beta_1 (\theta_1 - \theta_{c1}) + \frac{\beta_2}{\nu} (\theta_{c2} - \theta_2)$$

$$= 2 \beta_c (\theta_{c1} - \theta_{c2})$$

(A2)

Solving (A1) and (A2)

$$\theta_{c1} = \gamma_1 \theta_1 - \gamma_2 \theta_2 + \gamma_{c1} \theta_c$$

and

$$\theta_{c2} = -\gamma_1 \theta_1 + \gamma_2 \theta_2 + \gamma_{c1} \theta_c$$

where,

$$\gamma_1 = \beta_1/\nu$$

$$\gamma_2 = \beta_2/\nu v^*$$

$$\gamma_{c1} = (2\beta_1 + 4\beta_c)/\beta^*$$

$$\gamma_{c2} = (2\beta_2 + 4\nu \beta_c)/\nu v^*$$

$$\beta^* = \beta_1 + \beta_2/\nu + 4\beta_c$$

The governing equations (2) – (4), then transform to:

$$\frac{d}{dx} (\beta_1 - \beta_1 \gamma_1) \theta_1 + \beta_1 \gamma_2 \theta_2 - \beta_1 \gamma_{c2} \theta_c = 0$$

(A4)

$$\frac{d}{dx} (\beta_2 - \beta_2 \gamma_2) \theta_2 - \beta_2 \gamma_1 \theta_1 + \beta_2 \gamma_{c1} \theta_c = 0$$

(A5)

$$- \frac{d}{dx} \frac{\theta_1}{\lambda} + \frac{d}{dx} \frac{\theta_2}{\lambda} + \frac{d^2 \theta_c}{dx^2} = 0$$

(A6)

Following Kroeger, the solution may be sought in the form:
\[ \theta_k = A_{0k} + \sum_{j=1}^{3} A_{jk} \exp(-r_j x); \quad k = 1, 2 \text{ and } c \]

(A7)

where \( r_j \) is the non-zero roots of the characteristic equation:

\[
\begin{vmatrix}
-r + \beta_1 (1 - \gamma_1) & -\beta_1 \gamma_2 & -\beta_1 \gamma c \\
-\beta_2 \gamma_1 & -r - \beta_2 (1 - \gamma_2) & -\beta_2 \gamma c \\
r & -r & \lambda r^2
\end{vmatrix} = 0
\]

(A8)

On substituting (A7) into the modified governing equations (A4) – (A6) and equating coefficients of \( \exp(-r_j x) \),

\[
A_{01} = A_{02} = A_{0c}
\]

(A9)

and

\[
A_{1j} = G_j A_{jc}; j = 1, 2, 3
\]

(A10)

The constants \( A_{jc} \) may be determined by substituting (A9) into the boundary conditions (5) and (6).

\[
A_{0c} = \frac{1}{P} \sum_{Q} \frac{H_1 - G_1}{r_1} \exp(-r_1) \left[ \exp(-r_2) - \exp(-r_3) \right]
\]

(A11)

and

\[
A_{1c} = -\frac{1}{P} \frac{1}{r_1} \left[ \exp(-r_2) - \exp(-r_3) \right]
\]

(A12)

where \( \Sigma \) represents a cyclic sum over the subscripts 1, 2 and 3 and \( A_{2c} \) and \( A_{3c} \) obtained by a cyclic permutation of suffixes in (A12). In (A11) and (A12),

\[
P = \sum_{Q} \frac{H_1}{r_1} \left[ \exp(-r_1) - G_1 \right] \left[ \exp(-r_2) - \exp(-r_3) \right]
\]

(A13)

Inefficiency of the heat exchanger,

\[
1 - \epsilon = \theta_{1e} = [\theta_1]_{x=1}
\]

\[
= A_{01} + \sum_{j=1}^{3} A_{1j} \exp(-r_j)
\]

\[
= A_{0c} + \sum_{j=1}^{3} G_j A_{jc} \exp(-r_j)
\]

\[
= A_{0c} - \frac{1}{P} \sum_{Q} \frac{G_1}{r_1} \exp(-r_1) \left[ \exp(-r_2) - \exp(-r_3) \right]
\]

Using (A11),

\[
1 - \epsilon = \frac{1}{P} \sum_{Q} \frac{H_1 - G_1}{r_1} \exp(-r_1) \left[ \exp(-r_2) - \exp(-r_3) \right]
\]

(A14)

Equation (A14) may be expressed in the form of determinants to yield (7).
$$B_{1c} = \frac{1}{Q} \left[ r_3 (\exp(-r_3) - \exp(-r_3)) - [G_2 - H_2 \exp(-r_2)] r_3 [1 - \exp(-r_3)]\right]$$ (A16) + [G_2 - H_2 \exp(-r_2)] r_3 [1 - \exp(-r_3)]

$$B_{2c} = \frac{r_3}{Q} [1 - \exp(-r_3)]$$

$$B_{3c} = -\frac{r_3}{Q} [1 - \exp(-r_3)]$$

Inefficiency of the heat exchanger:

$$1 - \epsilon = \left[ \theta_1 \right]_{x=1}$$

$$= B_{01} + B_{11} + \sum_{j=2}^{\infty} B_{1j} \exp(-r_j)$$

$$= B_{0c} + B_{1c} \left( 1 - \frac{1}{\beta_1} - \frac{1}{2\beta_c} \right)$$

$$+ \sum_{j=3}^{\infty} G_j B_{jc} \exp(-r_j)$$ (A18)