Mixed Convection from an Exponentially Stretching Surface in Non-Darcy Porous Medium in the Presence of Soret and Dufour Effects

Dr. Ch. RamReddy

Department of Mathematics, National Institute of Technology, Rourkela-769008, Odisha.
Outline

1. Basic concepts
   - Mixed convection
   - Stretching Surface
   - Soret and Dufour effects

2. Mathematical formulation of the proposed problem

3. Results and Discussion
Mathematical Formulation

Figure: *Physical model and coordinate system*
Assumptions

1. The flow is in steady state.
2. The flow is two-dimensional and laminar.
3. The fluid is incompressible and viscous.
4. The porous medium have constant physical properties and are in local thermodynamical equilibrium.
5. The flow is moderate, so pressure drop is proportional to the linear combination of fluid velocity and the square of the velocity.

In addition, the Soret and Dufour effects are considered.
Governing Equations

Employing boundary layer assumptions and using Boussinesq approximations, the governing equations for the viscous fluid are given by

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)
\]

\[
\frac{1}{\varepsilon^2} \left( \frac{u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \frac{\nu}{\varepsilon} \frac{\partial^2 u}{\partial y^2} + g^* \left( \beta_T (T - T_\infty) + \beta_C (C - C_\infty) \right) - \frac{\nu}{K_p} u - \frac{b}{K_p} u^2 \quad (2)
\]

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{D K_T}{C_s C_p} \frac{\partial^2 C}{\partial y^2} \quad (3)
\]

\[
u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} + \frac{D K_T}{T_m} \frac{\partial^2 T}{\partial y^2} \quad (4)
\]
Associated B.C.

The boundary conditions are

\[
\begin{align*}
    u &= u_w(x), \quad v = 0, \quad -k \left( \frac{\partial T}{\partial y} \right)_w = q_w e^{x/L}, \quad -D \left( \frac{\partial C}{\partial y} \right)_w = q_m e^{x/L} \quad \text{at} \quad y = 0 \\
    u &= 0, \quad T = T_\infty, \quad C = C_\infty \quad \text{as} \quad y \to \infty
\end{align*}
\]

(5a) (5b)

where \(b\) is the Forchheimer constant, \(K_p\) is the permeability, \(\epsilon\) is the porosity, \(\beta_T\) is the coefficient of thermal expansion, \(\beta_C\) is the coefficient of solutal expansions, \(\alpha\) is the thermal diffusivity, \(D\) is the solutal diffusivity of the medium, \(C_p\) is the specific heat capacity, \(C_s\) is the concentration susceptibility, \(T_m\) is the mean fluid temperature and \(K_T\) is the thermal diffusion ratio, \(q_w\) and \(q_m\) are parameters of the temperature concentration distribution in the stretching surface.
Similarity Transformations

In view of the continuity equation (1), defining the stream function $\psi$ such that

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x}. \quad (6)$$

Sub.(6) in (2)-(4) and then using the following transformations

$$\eta = \left(\frac{Re}{2}\right)^{1/2} \frac{y}{L} e^{x/2L}, \quad \psi = \sqrt{2}\nu Re^{1/2} e^{x/2L} f(\eta),$$

$$T(x, y) = T_\infty + \frac{\sqrt{2}q_w L}{k} Re^{-1/2} e^{x/2L} \theta(\eta),$$

$$C(x, y) = C_\infty + \frac{\sqrt{2}q_m L}{D} Re^{-1/2} e^{x/2L} \phi(\eta),$$

(7)
Reduced Governing Equations

the governing equations become

\[
\frac{1}{\varepsilon} f''' + \frac{1}{\varepsilon^2} \left(ff'' - 2f'^2\right) + 2 Ri \, e^{-3X/2} \left(\theta + B\phi\right)
- \frac{2}{Da \cdot Re} e^{-X} f' - \frac{2.Fs}{Da} f'^2 = 0 \quad (8)
\]

\[
\frac{1}{Pr} \theta'' + f \theta' - f' \theta + D_f \phi'' = 0 \quad (9)
\]

\[
\frac{1}{Sc} \phi'' + f \phi' - f' \phi + S_r \theta'' = 0 \quad (10)
\]

Where \( L \) is the characteristic length of the plate, \( X = \frac{X}{L} \) is the \( X \)-location.
Reduced Governing Equations

\[ Gr = \frac{g^* \beta_T q_w L^4}{k \nu^2} \]  is the thermal Grashof number,

\[ Re = \frac{u_0 L}{\nu} \]  is the Reynolds number,

\[ Ri = \frac{Gr}{Re^{5/2}} \]  is the mixed convection parameter,

\[ \mathcal{B} = \frac{\beta_c q_m k}{\beta_T q_w D} \]  is the buoyancy ratio,

\[ Da = \frac{K_p}{L^2} \]  is the Darcy number,

\[ Fs = \frac{b}{L} \]  is the Forchheimer number,

\[ D_f = \frac{D K_T q_m k}{C_s C_p \nu q_w D} \]  is the Dufour number,

\[ Sr = \frac{D K_T q_w D}{T_m \nu q_m k} \]  is the Soret number.
Reduced Boundary Conditions

Boundary conditions (5) in terms of $f$, $\theta$ and $\phi$ become

\[
\eta = 0 : \quad f(0) = 0, \quad f'(0) = 1, \quad \theta'(0) = -1, \quad \phi'(0) = -1, \tag{11a}
\]

\[
\eta \to \infty : \quad f'(\infty) = 0, \quad \theta(\infty) = 0, \quad \phi(\infty) = 0. \tag{11b}
\]

The non-dimensional skin friction $C_f$, the local Nusselt number $Nu_x$ and local Sherwood number $Sh_x$ are given by

\[
C_f \sqrt{Re_x} = \sqrt{2X} \quad f''(0), \tag{12a}
\]

\[
\frac{Nu_x}{\sqrt{Re_x}} = \frac{X}{\sqrt{2}} e^{X/2} \frac{1}{\sqrt{\theta(0)}}, \tag{12b}
\]

\[
\frac{Sh_x}{\sqrt{Re_x}} = \frac{X}{\sqrt{2}} e^{X/2} \frac{1}{\sqrt{\phi(0)}}, \tag{12c}
\]
Numerical Method

The system of non-linear ordinary differential equations (8) - (10) together with the boundary conditions (11) are locally similar and solved numerically. In the present study we have adopted the following default parameter values for the numerical computations: $Pr = 1.0$, $Sc = 0.22$, $Re = 200$, $Da = 0.1$, $X = 0.5$, $Ri = 1.0$, $\varepsilon = 0.6$, $B = 0.5$ and the maximum value of $\eta$ at $\infty$ is 15.
**Table:** Effects of skin friction, heat and mass transfer rates for varying values of $Fs$, $Sr$ and $Df$.

<table>
<thead>
<tr>
<th>$Fs$</th>
<th>$Sr$</th>
<th>$Df$</th>
<th>$f''(0)$</th>
<th>$\frac{1}{\theta(0)}$</th>
<th>$\frac{1}{\phi(0)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>2.0</td>
<td>0.03</td>
<td>-1.62634</td>
<td>0.88415</td>
<td>0.22078</td>
</tr>
<tr>
<td>0.3</td>
<td>2.0</td>
<td>0.03</td>
<td>-2.23773</td>
<td>0.81095</td>
<td>0.19973</td>
</tr>
<tr>
<td>0.7</td>
<td>2.0</td>
<td>0.03</td>
<td>-2.86109</td>
<td>0.74607</td>
<td>0.18212</td>
</tr>
<tr>
<td>1.0</td>
<td>2.0</td>
<td>0.03</td>
<td>-3.25201</td>
<td>0.71014</td>
<td>0.17284</td>
</tr>
<tr>
<td>0.5</td>
<td>2.0</td>
<td>0.03</td>
<td>-2.10975</td>
<td>0.93162</td>
<td>0.27515</td>
</tr>
<tr>
<td>0.5</td>
<td>1.6</td>
<td>0.0375</td>
<td>-2.12270</td>
<td>0.92509</td>
<td>0.28786</td>
</tr>
<tr>
<td>0.5</td>
<td>1.2</td>
<td>0.05</td>
<td>-2.13548</td>
<td>0.91654</td>
<td>0.30203</td>
</tr>
<tr>
<td>0.5</td>
<td>1.0</td>
<td>0.06</td>
<td>-2.14168</td>
<td>0.91082</td>
<td>0.30977</td>
</tr>
<tr>
<td>0.5</td>
<td>0.8</td>
<td>0.075</td>
<td>-2.14757</td>
<td>0.90325</td>
<td>0.31803</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>0.12</td>
<td>-2.15490</td>
<td>0.88391</td>
<td>0.33171</td>
</tr>
<tr>
<td>0.5</td>
<td>0.2</td>
<td>0.3</td>
<td>-2.15280</td>
<td>0.82185</td>
<td>0.34879</td>
</tr>
<tr>
<td>0.5</td>
<td>0.1</td>
<td>0.6</td>
<td>-2.13548</td>
<td>0.74015</td>
<td>0.35831</td>
</tr>
</tbody>
</table>
Conclusions

In this paper, the Soret and Dufour effects on a steady mixed convection from an exponentially stretching surface in a viscous fluid saturated non-Darcy porous medium has been analyzed.

- An increase in the Forchheimer number \( F_s \), increase heat and mass transfer rates and the local skin friction factor.

- The local mass transfer rate increases whereas local heat transfer rate decrease with increase in the value of the Dufour number (or simultaneous decrease in the Soret number). The skin-friction coefficient increases and then decreases with increase in the value of the Dufour number (or simultaneous decrease in the Soret number).