Mixed Convection from an Exponentially Stretching Surface in Non-Darcy Porous Medium in the Presence of Soret and Dufour Effects

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Outline

1. Basic concepts
   - Mixed convection
   - Stretching Surface
   - Soret and Dufour effects
2. Mathematical formulation of the proposed problem
3. Results and Discussion

Mathematical Formulation

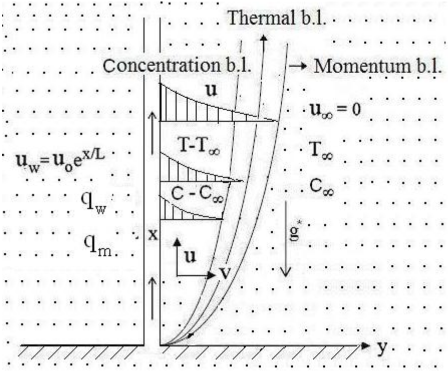

Figure: *Physical model and coordinate system*

Assumptions

1. The flow is in steady state.
2. The flow is two-dimensional and laminar.
3. The fluid is incompressible and Viscous.
4. The porous medium have constant physical properties and are in local thermodynamical equilibrium.
5. The flow is moderate, so pressure drop is proportional to the linear combination of fluid velocity and the square of the velocity.

In addition, the Soret and Dufour effects are considered.

Governing Equations

Employing boundary layer assumptions and using Boussinesq approximations, the governing equations for the viscous fluid are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{1}{\varepsilon^2}\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \frac{\nu}{\varepsilon}\frac{\partial^2 u}{\partial y^2} + g^*\left(\beta_T(T - T_\infty) + \beta_C(C - C_\infty)\right) - \frac{\nu}{K_p}u - \frac{b}{K_p}u^2 \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{DK_T}{C_s C_p}\frac{\partial^2 C}{\partial y^2} \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} + \frac{DK_T}{T_m}\frac{\partial^2 T}{\partial y^2} \tag{4}$$

Associated B.C.

The boundary conditions are

$$u = u_w(x), v = 0, -k\left(\frac{\partial T}{\partial y}\right)_w = q_w e^{x/L}, -D\left(\frac{\partial C}{\partial y}\right)_w = q_m e^{x/L} at y = 0 \tag{5a}$$

$$u = 0, T = T_\infty, C = C_\infty \quad as \quad y \to \infty \tag{5b}$$

where b is the Forchheimer constant, K_p is the permeability, ϵ is the porosity, β_T is the coefficient of thermal expansion, β_C is the coefficient of solutal expansions, α is the thermal diffusivity, D is the solutal diffusivity of the medium, C_p is the specific heat capacity, C_s is the concentration susceptibility, T_m is the mean fluid temperature and K_T is the thermal diffusion ratio, q_w and q_m are parameters of the temperature concentration distribution in the stretching surface.

Similarity Transformations

In view of the continuity equation (1), defining the stream function ψ such that

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x}. \tag{6}$$

Sub.(6) in (2)-(4) and then using the following transformations

$$\left.\begin{aligned} &\eta = \left(\frac{Re}{2}\right)^{1/2} \frac{y}{L} e^{x/2L}, \; \psi = \sqrt{2}\nu Re^{1/2} e^{x/2L} f(\eta), \\ &T(x,y) = T_\infty + \frac{\sqrt{2} q_w L}{k} Re^{-1/2} e^{x/2L} \theta(\eta), \\ &C(x,y) = C_\infty + \frac{\sqrt{2} q_m L}{D} Re^{-1/2} e^{x/2L} \phi(\eta), \end{aligned}\right\} \tag{7}$$

Reduced Governing Equations

the governing equations become

$$\frac{1}{\varepsilon}f''' + \frac{1}{\varepsilon^2}\left(ff'' - 2f'^2\right) + 2Ri \ \ e^{-3X/2}\left(\theta + \mathcal{B}\phi\right) - \frac{2}{Da.Re}e^{-X}f' - \frac{2.Fs}{Da}f'^2 = 0 \quad (8)$$

$$\frac{1}{Pr}\theta'' + f\theta' - f'\theta + D_f\phi'' = 0 \quad (9)$$

$$\frac{1}{Sc}\phi'' + f\phi' - f'\phi + S_r\theta'' = 0 \quad (10)$$

Where L is the characteristic length of the plate, $X = \frac{x}{L}$ is the X-location.

Reduced Governing Equations

$Gr = \dfrac{g^* \beta_T q_w L^4}{k\nu^2}$ is the thermal Grashof number,

$Re = \dfrac{u_0 L}{\nu}$ is the Reynolds number,

$Ri = \dfrac{Gr}{Re^{5/2}}$ is the mixed convection parameter,

$\mathcal{B} = \dfrac{\beta_C q_m k}{\beta_T q_w D}$ is the buoyancy ratio,

$Da = \dfrac{K_p}{L^2}$ is the Darcy number,

$Fs = \dfrac{b}{L}$ is the Forchheimer number,

$D_f = \dfrac{DK_T}{C_s C_p \nu} \dfrac{q_m k}{q_w D}$ is the Dufour number,

$S_r = \dfrac{DK_T}{T_m \nu} \dfrac{q_w D}{q_m k}$ is the Soret number.

Reduced Boundary Conditions

Boundary conditions (5) in terms of f, θ and ϕ become

$$\eta = 0: \; f(0) = 0, \; f'(0) = 1, \; \theta'(0) = -1, \; \phi'(0) = -1, \tag{11a}$$

$$\eta \to \infty: \; f'(\infty) = 0, \; \theta(\infty) = 0, \; \phi(\infty) = 0. \tag{11b}$$

The non-dimensional skin friction C_f, the local Nusselt number Nu_x and local Sherwood number Sh_x are given by

$$C_f \sqrt{Re_x} = \sqrt{2X} \quad f''(0), \tag{12a}$$

$$\frac{Nu_x}{\sqrt{Re_x}} = \frac{X}{\sqrt{2}} e^{X/2} \frac{1}{\theta(0)}, \tag{12b}$$

$$\frac{Sh_x}{\sqrt{Re_x}} = \frac{X}{\sqrt{2}} e^{X/2} \frac{1}{\phi(0)}, \tag{12c}$$

Numerical Method

The system of non-linear ordinary differential equations (8) - (10) together with the boundary conditions (11) are locally similar and solved numerically. In the present study we have adopted the following default parameter values for the numerical computations: $Pr = 1.0$, $Sc = 0.22$, $Re = 200$, $Da = 0.1$, $X = 0.5$, $Ri = 1.0$, $\varepsilon = 0.6$, $\mathcal{B} = 0.5$ and the maximum value of η at ∞ is 15.

Table: *Effects of skin friction, heat and mass transfer rates for varying values of Fs, Sr and Df .*

Fs	S_r	D_f	$f''(0)$	$\frac{1}{\theta(0)}$	$\frac{1}{\phi(0)}$
0.0	2.0	0.03	-1.62634	0.88415	0.22078
0.3	2.0	0.03	-2.23773	0.81095	0.19973
0.7	2.0	0.03	-2.86109	0.74607	0.18212
1.0	2.0	0.03	-3.25201	0.71014	0.17284
0.5	2.0	0.03	-2.10975	0.93162	0.27515
0.5	1.6	0.0375	-2.12270	0.92509	0.28786
0.5	1.2	0.05	-2.13548	0.91654	0.30203
0.5	1.0	0.06	-2.14168	0.91082	0.30977
0.5	0.8	0.075	-2.14757	0.90325	0.31803
0.5	0.5	0.12	-2.15490	0.88391	0.33171
0.5	0.2	0.3	-2.15280	0.82185	0.34879
0.5	0.1	0.6	-2.13548	0.74015	0.35831

Conclusions

In this paper, the Soret and Dufour effects on a steady mixed convection from an exponentially stretching surface in a viscous fluid saturated non-Darcy porous medium has been analyzed.

- An increase in the Forchheimer number Fs, increase heat and mass transfer rates and the local skin friction factor.
- The local mass transfer rate increases whereas local heat transfer rate decrease with increase in the value of the Dufour number(or simultaneous decrease in the Soret number). The skin-friction coefficient increases and then decreases with increase in the value of the Dufour number (or simultaneous decrease in the Soret number).