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On the generation of entropy in a counterflow heat exchanger

S. Sarangi and K. Chowdhury

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Second law analysis in process design has received considerable attention in recent years.[1] Thermal and chemical processes have been considered by many authors in terms of one of two entities: exergy (available energy) and irreversibility (entropy production). Irreversibility analysis of heat exchangers, designed to transfer a specified amount of heat between the fluid streams, has been described by McClintock.[2] Bejan[3] has recently introduced the concept of designing heat exchangers for specified irreversibility rather than specified amount of heat transferred. This technique has been used by several authors in the field of cryogenic engineering.[4-7] Bejan[3] has considered two sources of irreversibility: heat transfer across a finite temperature difference and pressure drop due to fluid friction. Recently Chowdhury and Sarangi[8,9] used irreversibility analysis to predict the optimum thermal conductivity of the separating wall in a concentric tube counterflow heat exchanger. They have accounted for the entropy generation due to axial conduction in the wall, along with that due to lateral heat transfer and fluid friction.

Entropy generation in a heat exchanger

The number of entropy production units N_s may be derived[3] as

$$N_s = \ln[1 + \epsilon(R-1)] + \frac{1}{\nu}\ln[1 - \nu\epsilon(R-1)/R] \quad (1)$$

where R is inlet temperature ratio T_2/T_1, ν is capacity rate ratio C_{min}/C_{max}, ϵ is heat exchanger efficiency, and N_s is number of entropy production units, $= \dot{S}/C_{min}$, where $\dot{S}$ is rate of entropy generation. In reference 3, N_s is defined as S/C_{max}.

The following are the assumptions in the derivation of (1): one, the contribution of fluid friction to entropy generation is negligible; two, both ends of the heat exchanger are adiabatic; and three, there is no heat transfer to the surroundings from the outer wall of the heat exchanger. By differentiating (1) it may be seen that N_s is maximum for an efficiency

$$\epsilon = 1/(1+\nu)$$

The authors are from the Advanced Centre for Cryogenic Engineering, Indian Institute of Technology, Kharagpur 721 302, India. Paper received 22 September 1981.

and

$$(N_s)_{max} = \frac{1+\nu}{\nu}\ln\frac{R+\nu}{1+\nu} - \frac{\ln R}{\nu} \quad (2)$$

For some particular cases simpler forms of (1) and (2) may be derived for ready reference.

(1) Balanced flow: $\nu = 1$

$$N_s = \ln[1+\epsilon(R-1)] + \ln[1-\epsilon(R-1)/R]$$
$$= \ln[1 + \frac{(R-1)^2}{R}\epsilon(1-\epsilon)]$$

and

$$(N_s)_{max} = \ln\frac{(R+1)^2}{4R} \text{ at } \epsilon = 0.5 \quad (3)$$

(2) $\nu \to 0$

Such a case may arise if one of the channels carries a liquid or a condensing vapour, the other carrying a gas at ordinary pressures.

$$N_s = \ln[1+\epsilon(R-1)] - \epsilon\frac{(R-1)}{R}$$

and (4)

$$(N_s)_{max} = \ln R - \frac{(R-1)}{R} \text{ at } \epsilon = 1$$

(3) $\epsilon \to 0$

$$N_s = \frac{(R-1)^2}{R}\epsilon \quad (5)$$

(4) $\epsilon = 1$

$$N_s = \ln R + \frac{1}{\nu}\ln\left[1 - \nu\frac{(R-1)}{R}\right] \quad (6)$$

which equals 0 for $\nu = 1$ and,

$$\ln R - \frac{(R-1)}{R} \text{ for } \nu = 0$$

Computed results for a temperature ratio $R = 4$ have been plotted in Fig. 1 against efficiency ϵ and capacity rate ratio ν. The locus of maximum entropy generation point may be expressed by the relation:

$$N_s = \ln[1+\epsilon(R-1)] + \frac{\epsilon}{1-\epsilon} \ln \frac{1+\epsilon(R-1)}{R} \quad (7)$$

Nearly ideal heat exchanger with nearly balanced capacity rate

For nearly ideal heat exchangers ($1-\epsilon \ll 1$), (1) may be expanded in a Taylor's series around $\epsilon = 1$ to give

$$N_s = \ln R + \frac{1}{\nu} \ln \left[1 - \frac{\nu(R-1)}{R}\right] + \frac{\nu(R-1)^2(1-\epsilon)}{R(\nu+R-\nu R)} \quad (8)$$

which may be expressed as

$$N_s = N_{s,\,\text{imbalance}} + N_{s,\,\text{inefficiency}} \quad (9)$$

where

$$N_{s,\,\text{imbalance}} = \ln R + \frac{1}{\nu} \ln \left[1 - \nu \frac{(R-1)}{R}\right] \quad (9a)$$

and

$$N_{s,\,\text{inefficiency}} = \frac{\nu(R-1)^2(1-\epsilon)}{R(\nu+R-\nu R)} \quad (9b)$$

For nearly balanced capacity rate, the coefficient of $(1-\epsilon)$ in (9b) may be evaluated at $\nu = 1$ for first order approximation. Then,

$$N_{s,\,\text{inefficiency}} = \frac{(R-1)^2}{R}(1-\epsilon) \quad (10)$$

In reference 3 the term $(1-\epsilon)$ has been approximated by its value under balanced flow condition, $1/N_{tu}$. Because of the strong dependence of efficiency on capacity rate ratio, this becomes a poor approximation at even minor imbalance, resulting in significant change of design parameters.

The expression[10] for $(1-\epsilon)$ may be approximated to first order by the following expression.

$$1-\epsilon = \left[\frac{2-\nu}{N_{tu}} - \frac{1-\nu}{2}\right] \cdot H\left(\frac{2}{N_{tu}} - 1 + \nu\right)$$

where

$$H(x) = 1 \text{ for } x \geqslant 0 \text{ and } H(x) = 0 \text{ for } x < 0$$

This may be substituted in (10) to yield

$$N_{s,\,\text{ineffency}} \simeq \frac{(R-1)^2}{R}\left[\frac{2-\nu}{N_{tu}} - \frac{1-\nu}{2}\right] H\left(\frac{2}{N_{tu}} - 1 + \nu\right)$$

$$\simeq \left[-\frac{(R-1)^2(1-\nu)}{2R} + \frac{(R-1)^2(2-\nu)}{R}\right.$$

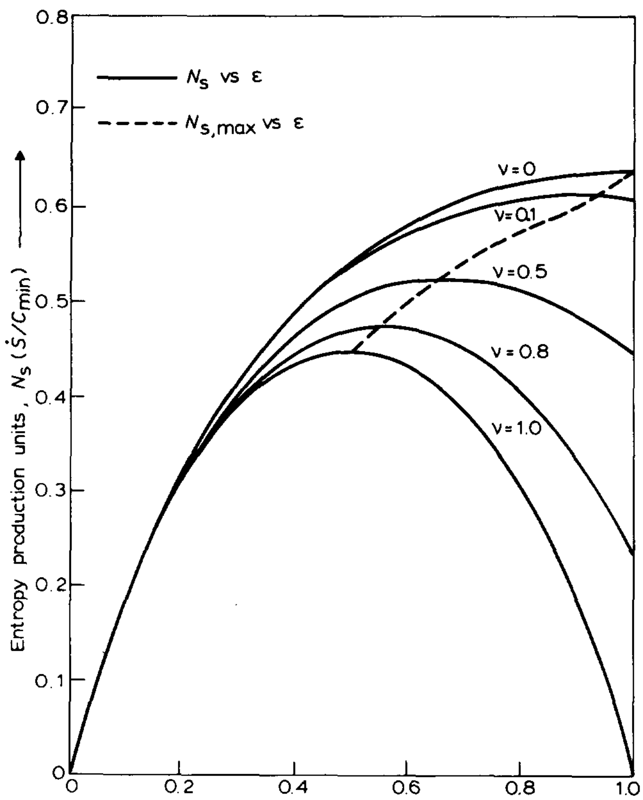

Fig. 1 Computed results for temperature ratio $R = 4$ plotted against efficiency ϵ and capacity rate ratio, ν

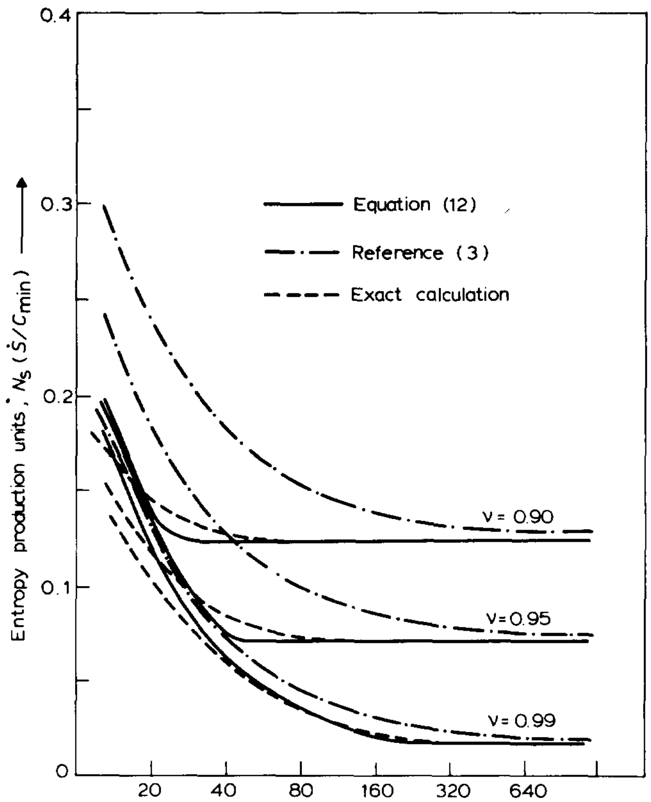

Fig. 2 Comparison of exact calculation results obtained using equation (12) and reference 3

$$\left(\frac{1}{N_{tu,1}} + \frac{1}{N_{tu,2}}\right)\Bigg] \times H\left(\frac{2}{N_{tu}} - 1 + \nu\right) \qquad (11)$$

Hence

$$N_s = N_{s,\,\text{imbalance}} + \frac{(R-1)^2}{R}\left[\frac{2-\nu}{N_{tu}} - \frac{1-\nu}{2}\right] H\left(\frac{2}{N_{tu}} - 1 + \nu\right) \qquad (12)$$

Results of (12) have been compared with exact calculation and results of Reference 3 in Fig. 2 for a temperature ratio of 4. It may be observed that the new expression gives a much closer approximation and also can be easily incorporated into the new design procedure of Bejan.

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