Performance of Power Efficient LDPC Coded OFDM over AWGN Channel

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Abstract: **OFDM with Quadrature amplitude modulation (QAM) technique can be used for high speed optical applications. As the order of modulation increases, the bit error rate (BER) increases. Forward Error correction (FEC) coding like LDPC coding is generally used to Improve BER performance. LDPC provides large minimum distance and also the power efficiency of the LDPC code increases significantly with the code length. Here we have given a theoretical review on the design of encoder and decoder for the Block codes followed by the fundamentals of various modified Block codes. Finally using a long Irregular LDPC code, it is shown that LDPC coded OFDM provides very low bit error rate compared to OFDM without coding case with a gain in transmitter power and thus making the link power efficient.**

Keywords: orthogonal frequency division multiplexing (OFDM); Block Codes; Turbo codes; Low density parity check coding (LDPC); Additive White Gaussian Noise (AWGN) channel

I. INTRODUCTION

OFDM provides an effective and low complexity means for eliminating intersymbol interference for transmission over frequency selective fading channels. It mitigates the severe multipath propagation effects that causes massive data errors and loss of signal in the microwave and UHF spectrum. In OFDM the subcarrier frequencies are chosen in such a manner that the signals are mathematically orthogonal over one OFDM symbol period [1]. During the past decades, channel coding has been used extensively in most digital transmission systems, from those requiring only error detection, to those needing very high coding gains, like deep-space links. In past, optical communication systems have ignored channel coding, until it became clear that it could be a powerful, yet inexpensive, tool to add margins against line impairments such as amplified spontaneous emission (ASE) noise, channel cross talk, nonlinear pulse distortion, and fiber aging-induced losses[2]. Nowadays, channel coding is a standard practice in many optical communication links. Turbo codes were proposed for the first time in 1993 by Berrou et al. [3] that showed astonishing performance: a rate-1/2 turbo code together with a binary PSK (BPSK) modulation in an AWGNchannel showed coding gains very close (0.5 dB) to theShannon capacity limits

The huge interest raised by turbo codes led researchers in the field to resurrect and improve the LDPC codes [4].

Turbo and LDPC codes have revolutionized coding theory and their use in optical communication, is still under heavy investigation and discussion. The second part of the paper discusses theoretically the complete Block diagram of OFDM in baseband and pass band domain.[7,9]. In the third part we have discussed the advantages of using error control coding followed by the discussion on various types of modified linear Block codes and the LDPC code and its types[5,6,8,9,10]. In the fourth part, the performance of LDPC coded OFDM over AWGN channel is analyzed through hard decision and soft decision decoding so as to prove it as a suitable candidate for high speed optical applications. The fifth part lists the conclusion.

The similar type of work might have been done by someone else too, but the analysis methodology is totally new up to the best of our knowledge and data available to us.

II. OFDM SYSTEM MODEL

The block diagram of OFDM Transceiver in baseband and pass band domain [9] is given in Fig.1. An OFDM system treats the source symbols in the frequency-domain. These symbols are used as the inputs to an IFFT block that brings the signal into the time domain. The IFFT takes in M symbols at a time, where M is the number of subcarriers in the system. Each of these M input symbols has a symbol period of T seconds. It may be recalled that the basis functions for an IFFT are M orthogonal sinusoids. These sinusoids each have a different frequency and the lowest frequency is DC. Each input symbol acts like a complex weight for the corresponding sinusoidal basis function. Since the input symbols are complex, the value of the symbol determines both the amplitude and phase of the sinusoid for that subcarrier. The IFFT output is the summation of all M sinusoids. The block of M output samples from the IFFT make up a single OFDM symbol. Figure. 2(a-b) below shows the OFDM symbol for different order QAM.The length of the OFDM symbol is MT where T is the IFFT input symbol period mentioned above.

After some additional processing, the time-domain signal that results from the IFFT is transmitted across the channel. At the receiver, an FFT block is used to process the received signal and bring it into the frequency domain. Ideally, the FFT output will be the original symbols that were sent to the IFFT at the transmitter.

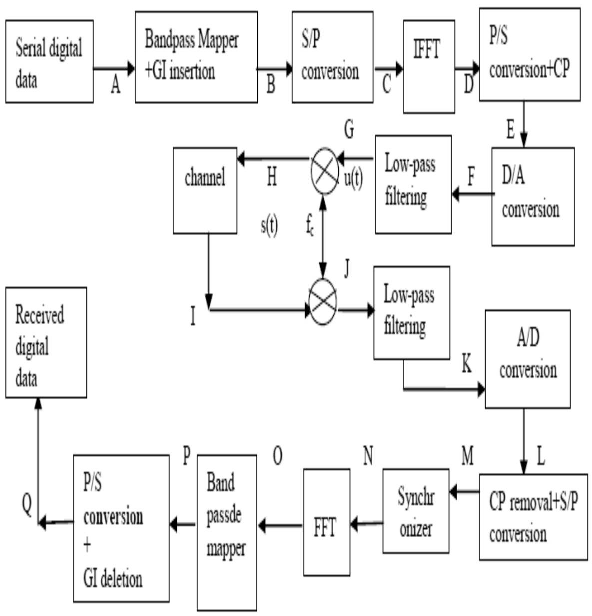


Figure 1. OFDM Transciver in baseband and pass band domain

In the Figure.1 the total block diagram shows OFDM implementation in Pass band. If the Block between nodes G to H is removed and G connects to channel block directly and then if block between I and J is removed and I directly connects to the block (Low pass filtering), we get the baseband implementation of OFDM.The Pass band magnitude spectrum and the power spectrum are shown in Figure. 3(a-b).The time response of the signal u(t) is given in the Figure.4 with upper half showing in-phase component and lower half showing the quadrature components. The OFDM is implemented in baseband domain with different order QAM. The BER of this is shown below in Figure.5. It shows that as the modulation order increases the BER increases. Also since the OFDM symbols have different peak levels with QAM , equalization is needed at receiver side and which will be executed by introducing one equalizer block between FFT block and Band pass Demapper in Figure.1. This will be implemented in near future which will further improve the BER. The Peak to average power ratio of OFDM signal also increases with the increase in no. of subcarriers. But, the spectral efficiency increases with higher order modulation. The use of error control coding reduces this BER and also the PAPR and make the link Power efficient [9]. The next section highlights the advantages of this with the example of Binary symmetric channel.

III. Error Control Coding

A. *Role of coding*

Consider a binary symmetric channel .Where '0' will become'0' with probability '1-P' and '1' will become '1' with '1-P'probability.So, error probability is 'P'. The implied assumptionis " each bit is independent of whatever happens to other bits" .The ideal value for 'P' is '0'.So, the optimization in the transmitter side requires a lot of power in transmitter to drive 'P' ideally to '0'.So, the drawback here is, we need much more signal power than the theoretical case to reduce 'P'. So, the efficient solution is to use error control codes which reduces 'P' in an indirect way. The kind of SNR we need in a noisy channel, to make this P close to zero, depends on the transmitting constellation. The Block diagram of a Digital communication system with error control coding is given in Figure.6.Values of parameters in the block are:

- Message $\in \{0,1\}^k$
- Encoder is an one-to-one mapping function
- Codeword $\in \{0,1\}^n$
- Set of all messages $=\{0,1\}^k$
- Set of all Codeword's $\neq \{0,1\}^n$ (since encoder is one-to-one mapping)
- Set of all Codeword's $\subset \{0,1\}^n$
- Received word $\in \{0,1\}^n$
- All received words are possible according to channel modeling. That means if one vector is being sent, all vectors in $\{0,1\}^n$ can be received with possible probability. So, all the possibilities with non- zero probabilities are in the receiver side.
- The decoder will not do the inverse operation of encoder, since the message to codeword mapping is one -to -one and codeword to received word mapping is one -to -many.
- So, the objective in designing a decoder is to minimize the error making of decoder.
- That means the objective of decoder design is to minimize P_r(decoded message$\neq$ transmitted message)
- So, Receiver/decoder mapping is many- to- one.

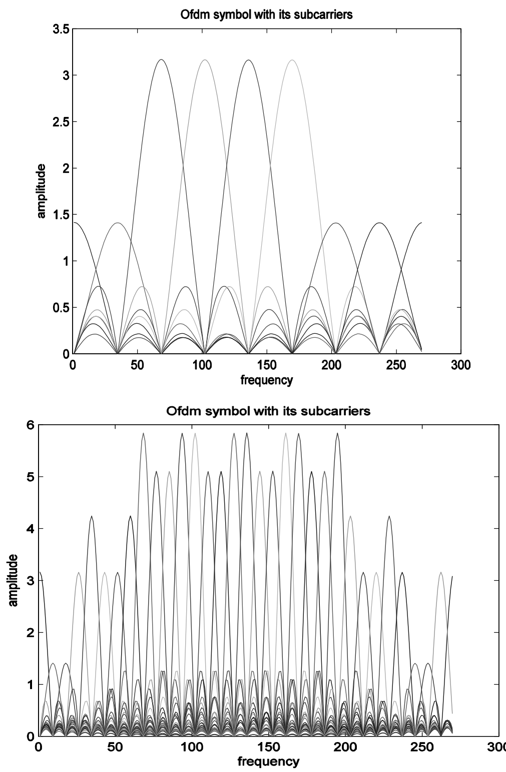


Fig.2 (a) OFDM symbol with 8QAM
Fig2. (b) OFDM symbol with 32QAM

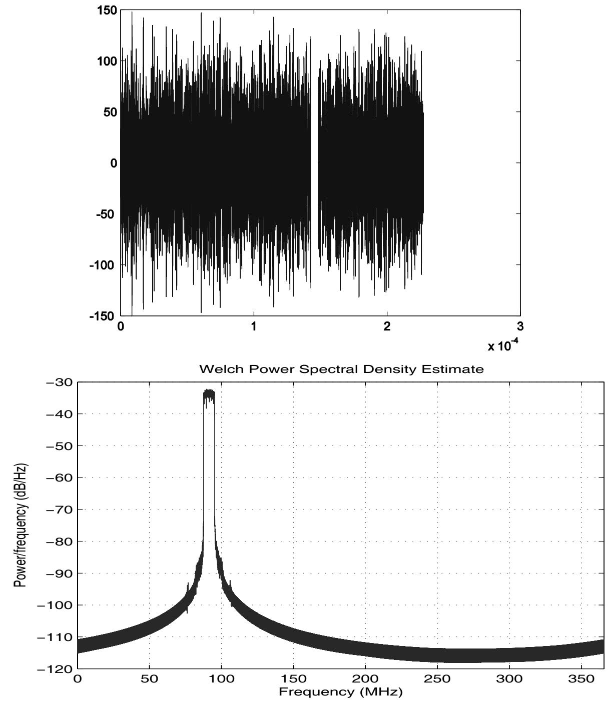


Fig.3 (a) Pass band magnitude spectrum
Fig.3(b) Pass band power spectrum

B. *Design considerations of encoder and decoder*

- We have 2^k codewords to be selected from 2^npossibilities.
- When there are so many possibilities of designing an encoder and we are picking one ,then the design problem becomes non-trivial.
- So, selection of encoder means selection of map from message to codeword, which is arbitrary.
- Since only codeword is transmitted, therefore selection of list of code words is important than the mapping from message to codeword.
- Our aim is to design an encoder which will minimize the probability of error.
- For good coding choose the list of code words first and then design an encoder for the selected list of code words and finally design a decoder for the chosen list of code words.
- Hence, for good encoder and decoder design, the probability of error must be very less and the design penalties are ;Complexity,Delay and Rate(k/n).The complexity has gone through the roof since now we are sending more no of bits with coding , instead of single bit as in uncoded scheme for BSC.
- Here the delay is a crucial factor since before decoding one bit, we are waiting for all the n bits.
- So, even if we are transmitting at a very high rate, then also delay will create problem. Changing the time constants we can handle delay. It is not an impossible issue and it can be dealt with.
- From communication point of view , previously we are using channel once to attempt to communicate one bit, but now in Figure-6 the channel is used n times to attempt to communicate k bits. So, the rate (k/n) is a crucial penalty. So, Rate is nothing but the no. of bits sent in one channel use.
- Now, using coding the gain factor of the system will include probability of error and transmit power.
- With coding the probability of error will go down faster than the case of without coding.
- With coding there is a gain in transmitter power at same error rate. Using coding we can decrease the transmit power and thus the cost of transmitter will bring down and thus it makes the link power efficient.

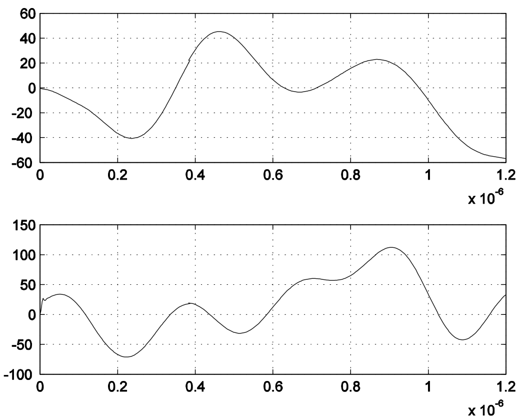


Fig. 4 Time response of u(t)

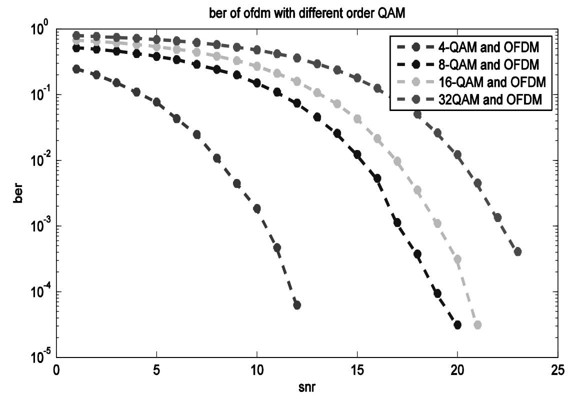


Fig.5 BER of OFDM with 4,8,16 and 32 QAM(without coding

C. Discussion of linear block codes

Any linear block code is represented as (n, k) where Linear block codes have the property of linearity, i.e. the sum of any two code words is also a codeword, and they are applied to the source bits in blocks, hence the name linear block codes. 'n' is the length of the codeword, in symbols and 'k' is the number of source symbols that will be used for encoding . The information sequence is divided into message blocks of k-information bits as ; u= $(u_0,u_1,u_2,\ldots\ldots,u_{k-1})$.

The encoder maps each block of k-information bits to an n-bit codeword as; v= $(v_0, v1, v2\ldots\ldots, v_{n-1})$.

The encoder for a block code is memory less. The set of code words of length n is called the binary block code. The codeword sequence, in general, can be non-binary, but we only consider binary codes since they are the most commonly used in practice [10]. The decoder produces an estimation($\hat{u}$)of the information sequence based on received data sequence . Equivalently the decoder can also estimate ($\hat{v}$) of the code sequence and then using inverse encoder mapping, it will find u^ corresponding to v^ .The decoding rule is an assignment of an estimate v^ to each of the received sequence r. The average probability of error is given by:

$$P(E)=P(\hat{v}\neq v)=\sum_{r}P(\hat{v}\neq v/r)\,P(r)$$

Choose $\hat{v}$ such that *P (v^≠v/r)* is minimized for each r. Minimizing *P (v ≠ v/r)* is equivalent to maximizing *P(v ^ =v/r)*

for each r.For each r, computeP(v/r) = P(r⁄(v).P(v)/P(r) for every v and choose v that maximizes P(v/r). This is Bay's rule.

Maximizing P(v/r) is same as maximizing P(r/v).P(v), since P(r) doesn't depend on v. A maximum a- posteriori probability (MAP) decoder chooses $\hat{v}$ such that P(v/r) is maximized. If all code words are equally likely, then maximizing P(v/r) is same as maximizing P(r/v).

A maximum likelihood (ML) decoder chooses $\hat{v}$ such that P(r/v) is maximized. Since log x is a monotone increasing function of x, maximizing P(r/v) is equivalent to maximizing log P(r/v). For a codeword of length 'n' transmitted on a BSC channel with crossover probability 'p', what will be the ML decoding rule then? This is discussed under points below. Hamming distance d (r,v) between r and v is the number of positions for which $r_i \neq v_i$.

$$\log P(r/v)=\left|d(r,v)\log\left(\frac{p}{1-p}\right)+n\log(1-p)\right)$$

Where $\log\left(\frac{p}{1-p}\right)$ <0 for p<1/2 and **nlog(1-p)** is aconstant.

Now, the ML decoding rule says that for each, choose v ˆ as the codeword v which minimizes the hamming distance d (r,v).

D . Modified Linear Block Codes:

The modification techniques can be applied to any type of block codes like reed solomon coding, Low density parity check coding etc [10, 11].

1. *Punctured codes:*

A (n,k) linear block code is punctured by deleting its **λ** parity bits and it is denoted as **(n- λ, k)** linear block code. It is shown below. Here a (8,4) block code is punctured into a (7,4) linear block code.

2. *Extended Codes:*

A (n,k) linear block code is extended by adding additional λ parity bits and it is denoted as (n+λ, k) linear block code. It is shown below. Here a (7,4) code is extended to (8,4) code.

3. *Shortened Codes:*

A (n,k) linear block code is shortened by deleting its λ information bits and it is denoted as (n-λ, k-λ) linear block codes. Here an (8,4) code is shortened to (7,3) code as below.

4. *Lengthened Codes:*

A (n,k) linear block code is lengthened by adding additional λ information bits and it is denoted as (n+λ, k+λ) linear block codes. Here an (7,3) code is lengthened to (8,4) code.

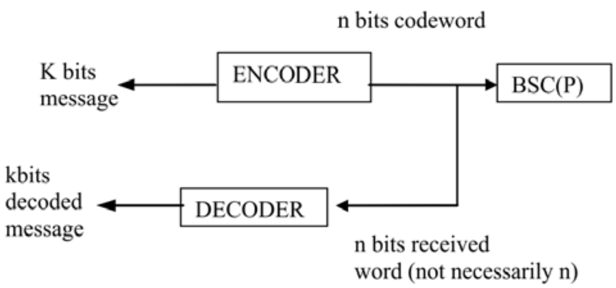


Fig.6 Block Diagram of Digital Communication system with error control coding

E. LDPC code and its types

Low-density parity-check (LDPC) codes are linear block codes specified by a parity check matrix H containing mostly 0's and only a small number of 1's.A regular (n,w_c ,w_r) LDPC code is a code of block length n with a m × n parity check matrix where each column contains a small fixed number, $w_c \geq 3$, of 1's and each row contains a small fixed number, $w_r \geq w_c$ of 1's.Low-density implies that $w_c <<$ m and $w_r <<$ n.Number of ones in the parity check matrix H = $w_c \cdot n = w_r \cdot m$.where $m \geq (n - k)$ =) $R = k/n \geq 1 - (w_c/w_r)$, and thus $w_c < w_r$.

1. *Gallager's construction of LDPC (regular) codes:*

Let, n be the transmitted block-length of an information sequence of length k. m is the number of parity check equations. Construct a m × n matrix with w_c 1' s per column and w_r 1' s per row[4]. Divide a m × n matrix into w_c.m/w_c.n sub-matrices, each containing a single 1 in each column. The first of these sub-matrices contains all 1"s in descending order. The other sub-matrices are merely column permutations of the first sub-matrix.one example is shown in fig.8(a).

2. *Irregular LDPC codes:*

For an irregular low-density parity-check code the degrees of each set of nodes are chosen according to some distribution.

In the construction of irregular LDPC code, the First step involves Selecting a profile that describes the desired number of columns of each weight and the desired number of rows of each weight. Second step includes a Construction method, i.e. algorithm for putting edges between the vertices in a way that satisfies the constraints. The edges are placed "completely at random" subject to the profile constraints. It is shown in fig.8(b).

3. *Decoding process in LDPC code:*

Hard decision decoding involves Bit-flipping algorithm and The soft decision decoding involves Sum-product and Min sum algorithms[10,11]. Here we will discuss about bit flipping algorithm below:

- The set of bits contained in a parity-check equation constitutes a parity check set.
- Parity check set tree is a representation of parity check set in a tree structure. An arbitrary bit d is represented by the node of the base of the tree. Each line rising from this node represents one of the parity-check sets containing d.

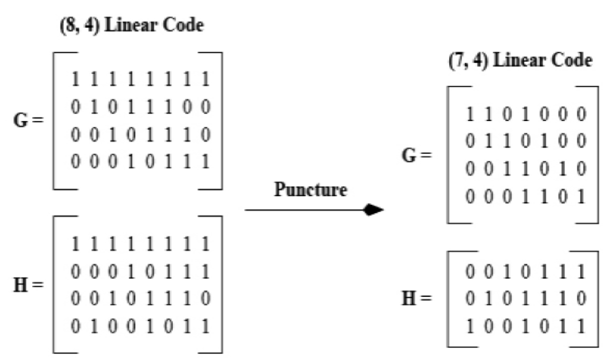


Fig.7(a) Punctured (8,4)block code

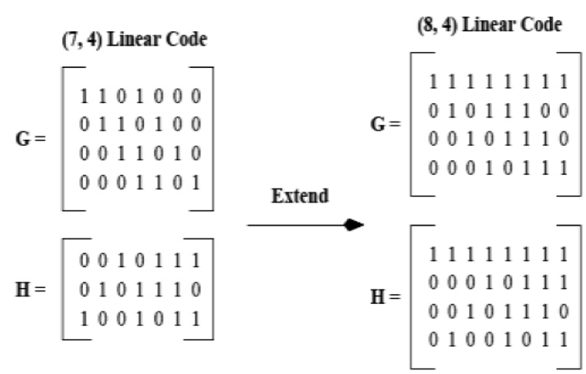


Fig.7(b) Extended (7,4) block code

- The other nodes bits in these parity-check sets are represented by the nodes on the first tier of the tree.The lines rising from tier 1 to tier 2 of the tree represent the other parity-check sets containing the bits on tier 1.The nodes on tier 2 represent the other bits in those parity-check sets.

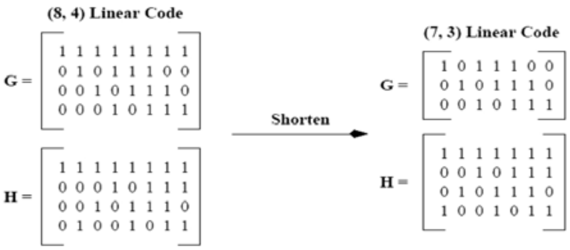


Fig.7(c) shortened (8,4) block code

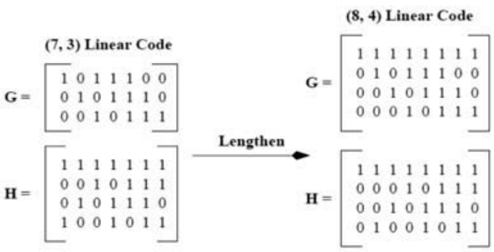


Fig.7(d) Lengthened (7,3) block code

$$H = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

Fig.8 (a) Parity check matrix of a regular (12,3,6)LDPC code

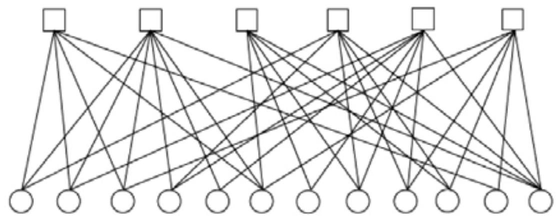
IV. PERFORMANCE OF LDFC CODED OFDM OVER AWGN CHANNEL

This system is simulated by introducing the LDPC encoder block at label „A‟ of Figure(1) and LDPC decoder block at label „Q‟. Here the simulation is implemented using matlab coding. The ldpc code is an irregular LDPC code with parity check matrix(32400,64800). Parity-check matrix of the LDPC code is stored as a sparse logical matrix. The system was simulated for OFDM with 16,32and 64QAM . Columns 32401 to 64800 are a lower triangular matrix. Only the elements on its main diagonal and the sub diagonal immediately below are 1's. The LDPC decoder is of Hard decision type while the band pass decoder is of approximate llr (log likelihood ratio) type. The information is binary in nature. TheFigure.9 shows the BER plot for these three cases. We have seen that there is a reduction in BER with coding with less transmitted power, making the link power efficient.

V. CONCLUSIONS

Using higher order QAM, the peak value of subcarriers in OFDM symbol increases. So, using higher order modulation requires PAPR reduction. For high speed applications, since higher order QAM is to be used, we need good Channel equalization techniques for equalizing the varying peaks.

Using the LDPC coding, the BER has reduced significantly compared to without coding case of OFDM simulation using higher order QAMs.The LDPC coded OFDM has very low ber at low SNR‟s and at high SNR‟s it is also low.



$$\lambda(x) = \tfrac{1}{4}x + \tfrac{7}{12}x^2 + \tfrac{1}{12}x^3 + \tfrac{1}{12}x^4$$
$$\rho(x) = \tfrac{1}{3}x^4 + \tfrac{1}{3}x^5 + \tfrac{1}{3}x^6$$

Fig.8(b) based on graphical structure, where λ(x) and ρ(x) are Column and row distributions

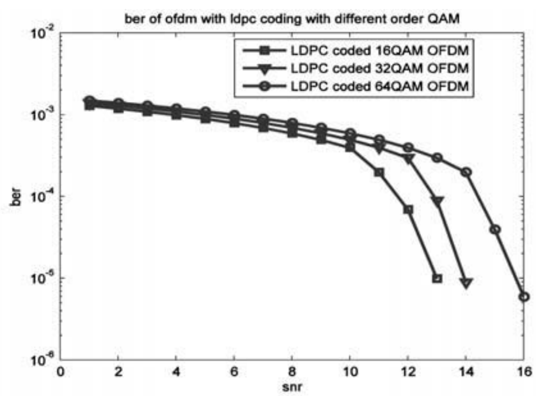


Fig.9 BER of LDPC coded OFDM for 16, 32 and 64 QAM

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