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STABILITY OF LAMINATED COMPOSITE PRETWISTED CANTILEVER PANELS

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ABSTRACT: The present study deals with the stability analysis of angle ply laminated composite twisted panels using Finite element method. Here eight-noded isoparametric quadratic shell element is used to develop the finite element procedure. To investigate the vibration and stability behaviour of twisted panels, the effect of various geometrical parameters like angle of twist, aspect ratio, lamination parameters, shallowness ratio are studied. The influence of various parameters on the vibration and stability characteristics of twisted panels have been examined. Numerical results are presented to show the effects of pre-twist angles, geometry and lamination details on the stability characteristics of twisted plates.

INTRODUCTION

Cantilevered twisted plates are having extensive use in engineering applications such as compressor or turbine blades, aircraft/Marine propeller, and helicopters . Due to its wide range of application in practical field, it requires to understand the nature of deformation, vibration and stability behaviour of twisted plates. It is being observed that these structures subjected to in plane loading may fail due to insufficient stability due to slenderness rather than the structure subjected to high stresses. Composite materials are being increasingly used in automotive, marine and especially aerospace applications, primarily because of the large values of specific strength and these can be tailored through the variation of fibre orientation and stacking sequence to obtain an efficient design. The advent of advanced fibre-reinforced composite materials such as graphite/epoxy , glass/epoxy and kevlar/epoxy with their high potential weight savings has resulted in a significant increase in the use of laminated plates and shells. Laminated composite plates are often fabricated in angle-ply sequences.

An excellent survey of the earlier works in the free vibration of turbo-machinery blades has been carried out by Leissa [1] and again updated by Leissa [2] through 1986. The stresses and deformation of pre-twisted and tapered blades using Finite element method by Ramamurti and Sreenivasamurthy [3]. The vast majority of earlier researches treat the blades as beams; one dimensional case. Such a idealization is highly inaccurate for the blade with small aspect ratio. Shallow shell theory and Ritz method are employed by Leissa, Lee and Wang [4] to determine the frequencies of turbo-machinery blades with twist for different degree of shallowness and thickness. A modeling method for the vibration analysis of pre-twisted blades with a concentrated mass is presented by Yoo, Kwak and Chung [5]. However, there are few studies on free vibration of composite pre-twisted plates. Vibration studies for laminated composite twisted cantilever plates is made by Qatu and Leissa [6] using Ritz method. In this context the first known natural frequency mode shapes of the laminated composite cantilever twisted plates are presented. Although extensive free vibration frequencies and mode shapes are studied, the results were however confined to symmetric laminates only. The vibration and damping analysis of pre-twisted composite blades is studied by Nabi and Ganesan [7] using a three nodded shell element. He, Lim and Kitipornchai [8] have studied the free vibration of symmetric as well as anti-symmetric laminates explaining the limit of linear twisting curvature. Hu and Tsuiji [9] have investigated on the free vibration analysis of twisted cylindrical thin panels by the Raleigh-Ritz method. The free vibration analysis of laminated composite pre-twisted cantilever plates are studied by Karmakar and Sinha [10] using finite element method. Parhi, Bhattacharyya and Sinha [11] has studied on the dynamic analysis of multiple delaminated composite twisted plates. The stability of untwisted laminated composite plate is studied by many investigators [12-14]. The various numerical stability studies on composite plates increases the interest regarding

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the extensive study on vibration and buckling of twisted composite plates . But the behaviour of twisted plates subjected to in-plane loads is less understood.

In the present investigation, the stability characteristics of laminated composite pre-twisted cantilever panels subjected to in-plane loads are studied. The vibration of twisted plates with curvature (camber) is studied considering shear deformation for completeness. The effects of angles of pre-twist, aspect ratio, and lamination parameters of pre-twisted panels on the vibration and buckling loads are studied.

THEORY AND FORMULATION

The basic configuration of the problem considered here is a pre twisted cantilever plate subjected to in-plane edge loading, as shown in Figure 1.

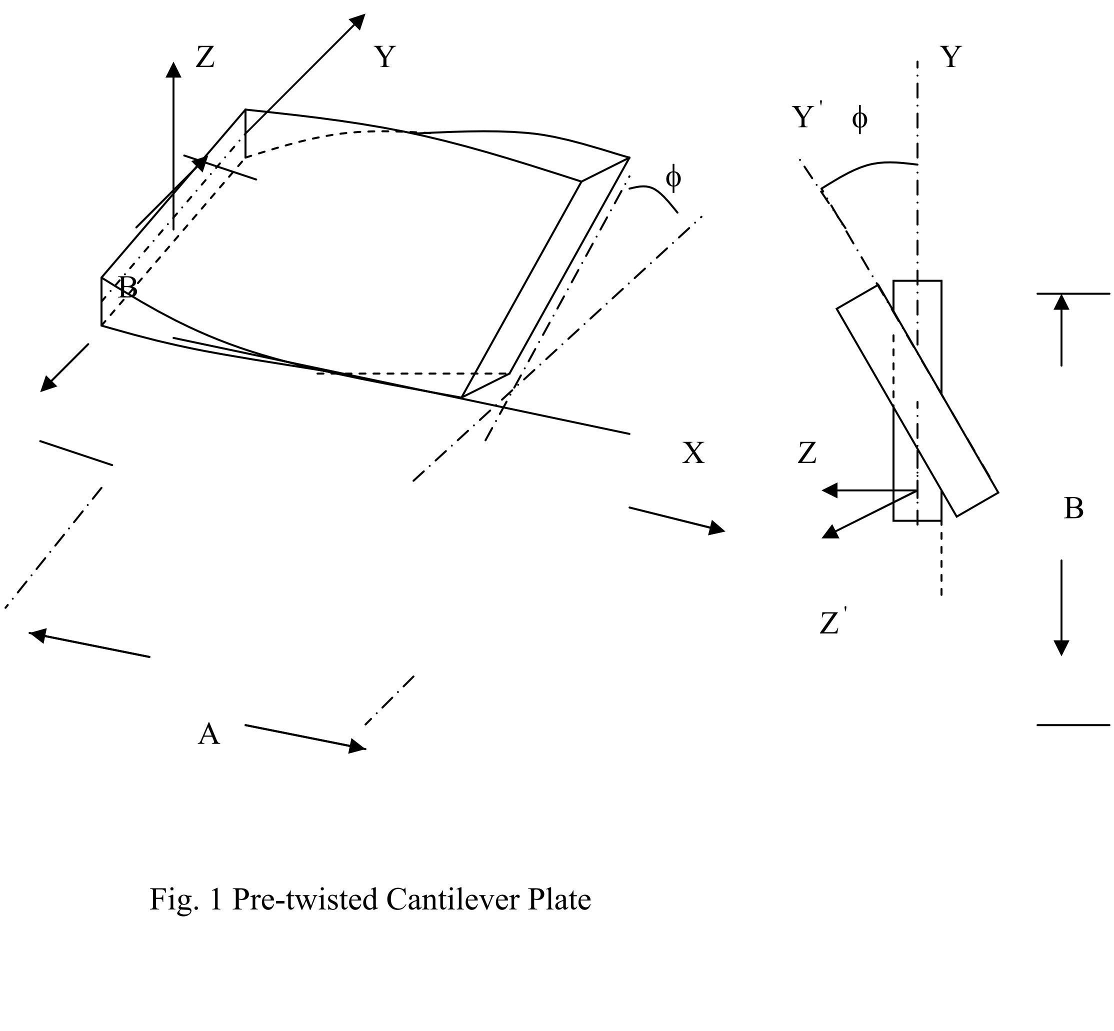

Fig. 1 Pre-twisted Cantilever Plate

The equation of equilibrium for a free vibration of a structure subjected to in plane loads can be expressed in the form:

$$[M]\{\ddot{q}\}+[[K_e]-\lambda[K_g]]\{q\}=0 \qquad (1)$$

An eight nodded curved isoparametric element is employed in the present analysis with five degrees of freedom u, v, w, θ_x and θ_y per node. First order shear deformation theory is employed and the displacement field assumes that mid-plane normal remains straight before and after deformation, but not necessarily normal after deformation, so that

$$\bar{u}(x,y,z)=u(x,y)+z\theta_x(x,y)$$
$$\bar{v}(x,y,z)=v(x,y)+z\theta_y(x,y)$$
$$\bar{w}(x,y,z)=w(x,y)$$

where, $\bar{u}$, $\bar{v}$, $\bar{w}$ and *u, v* and *w* are displacement components in the x, y, z directions at any point and at the mid surface respectively. The conventional constitutive relationships for the laminated twisted plate becomes:

$$\begin{Bmatrix} N_i \\ M_i \\ Q_i \end{Bmatrix} = \begin{bmatrix} A_{ij}....B_{ij}....0 \\ B_{ij}....D_{ij}....0 \\ 0......0....S_{ij} \end{bmatrix} \begin{Bmatrix} \varepsilon_j \\ k_j \\ \gamma_m \end{Bmatrix}$$

RESULTS AND DISCUSSION

Numerical results are presented for laminated composite untwisted $\phi=0^0$ and pre-twisted cantilever plates for ($\phi=15^0,30^0,45^0$). The following boundary conditions are used for the clamped edge of the angle ply laminated composite twisted plates.

Clamped edge: u=v=w=θx=θy=0 at any edge.

Convergence Study

The convergence studies are made for non-dimensional fundamental frequencies of vibration of laminated composite twisted cantilever plates for different thickness ratios and are shown in Table 1. From the above convergence study, 10 ×10 mesh has been employed to idealise the panel in the subsequent analysis.

Table 1 Convergence of non-dimensional fundamental frequencies of vibration of composite twisted cantilever plates with $45^0/-45^0/45^0$ **lamination.**

Non dimensional frequency, $\varpi = \omega a^2 \sqrt{(\rho / E_{11} h^2)}$

$a/b = 1, E_{11} = 138GPa, E_{22} = 8.96Gpa, G_{12} = 7.1GPa, \nu_{12} = 0.3$

Mesh	Non-dimensional fundamental frequencies of free vibration for different thickness ratio and angles of twist					
	a/h=100			a/h=20		
	0	15	30	0	15	30
4×4	0.4615	0.5300	0.5165	0.4570	0.4744	0.4770
8×8	0.4596	0.5261	0.5123	0.4546	0.4719	0.4745
10×10	0.4592	0.5256	0.5118	0.4541	0.4714	0.4741
Qatu&Leissa [6]	0.4613	0.5286	0.5149	0.4611	0.4790	0.4831

Comparison with previous studies

The present finite element formulation is validated for free vibration analysis as shown in Table 2, by comparing the fundamental frequency of vibration of graphite /epoxy [θ, -θ,θ] pre-twisted ($\phi = 15^0$) cantilever plates with that of Qatu & Leissa [6] using Ritz method and He, Lim and Kitipornchai [8]. To validate the formulation further, the buckling results of untwisted ($\phi = 0^0$) singly curved angle ply panels, are compared (Table 3) with Moita *et al*. [15].The above studies indicate good agreement between the present study and those from the literature.

Table 2 Comparison of non-dimensional fundamental frequencies of vibration of graphite epoxy pre-twisted cantilever [θ, -θ, θ] plates. , ϕ =15^0

Non dimensional frequency, $\varpi = \omega a^2 \sqrt{(\rho / E_{11} h^2)}$

$a/b = 1, E_{11} = 138GPa, E_{22} = 8.96Gpa, G_{12} = 7.1GPa, \nu_{12} = 0.3$

b/h	Reference	Non-dimensional fundamental frequencies of free vibration for different ply orientations(θ)						
		0	15	30	45	60	75	90
100	Ref [6]	1.0035	0.9296	0.7465	0.5286	0.3545	0.2723	0.2555
	Ref [8]	1.0034	0.92938	0.74573	0.52724	0.35344	0.27208	0.25544
	Present FEM	1.00295	0.92798	0.74381	0.52560	0.35278	0.27200	0.25543
20	Ref [6]	1.0031	0.8981	0.6899	0.4790	0.3343	0.2695	0.2554
	Ref [8]	1.0031	0.89791	0.68926	0.47810	0.33374	0.26934	0.25540
	Present FEM	0.99107	0.87025	0.67939	0.47143	0.33074	0.26786	0.25506

Table 3 Comparison of buckling loads for the thin un-twisted angle-ply cylindrical panel with symmetric lay-up[0^0/-α^0/+α^0/-90^0]$_s$

$E_{11} = 181GPa, E_{22} = 10.3Gpa, G_{12} = G_{23} = G_{13} = 7.17GPa, \nu_{12} = 0.28$

h=1.0mm, R/h=150, L/R=1.0, ϕ=0

Non dimensional buckling load, $\lambda = N_x R / E_{11} h^2$

α	b/L=1.309		b/L=1.047		b/L=0.786	
	Present	Moita *et al*.[10]	Present	Moita *et al*.[10]	Present	Moita *et al*.[10]
0	0.123	0.122	0.121	0.121	0.121	0.129
15	0.150	0.147	0.147	0.147	0.160	0.159
30	0.193	0.192	0.191	0.190	0.205	0.205
45	0.220	0.220	0.217	0.211	0.232	0.232
60	0.213	0.214	0.209	0.206	0.230	0.230
75	0.180	0.179	0.175	0.174	0.194	0.193
90	0.155	0.155	0.148	0.147	0.165	0.163

Numerical Results:-

After validation , free vibration results are extended for laminated composite pre twisted cantilever curved panels (plates with cambers) with addition of curvature. The studies are then extended to stability studies on composite pre-twisted cantilever plates. The geometrical properties of the twisted panels are

a=b=500mm, h=5mm, ρ=1580 kg/ m^3

The material properties of the graphite epoxy panels are

$E_{11} = 138GPa, E_{22} = 8.96Gpa, G_{12} = 7.1GPa, \nu_{12} = 0.3$

Non-dimensional frequency $= \varpi = \omega a^2 \sqrt{(\rho / E_{11} h^2)}$

Non-dimensional buckling Load $= \lambda = N_x b^2 / E_{22} h^3$

The free vibration frequencies of four lowest modes of vibration of composite pre twisted cantilever curved panel (twisted plate with camber) are presented in Table 4 and compared with frequencies of twisted plate without curvature in Table 2 by Qatu and Leissa [6] . As indicated, the free vibration frequencies of four lowest modes of vibration increase with addition of curvature. There is significant increase in non-dimensional frequencies of higher modes. The preferential ply orientation seems to be 15^0 for all modes of vibration for a twisting angle of 30^0. Then the studies are extended further to examine in detail the effects of various parameters on stability of composite, pre-twisted cantilever plates. The buckling loads are computed for square laminated twisted plates of b/h=100 for different angle of twist and ply orientation as shown in the Table 5. The buckling loads tend to decrease with increase of lamination angle from 0^0 to 90^0 for untwisted plates (except for 90^0). But for all the cantilever twisted plates, for this lamination, (θ,-θ,θ), 0^0 seems to be the preferential ply orientation for maximum non-dimensional buckling loads. There is general trend of decrease of buckling loads with decrease of angle of twist. However, the untwisted cantilever plates with 15^0, 30^0,45^0 and 60^0 ply orientation shows different trends. So the buckling behaviour of twisted plates are quite different from the untwisted plates.

Table 4. Non-dimensional free vibration frequencies of laminated composite pre-twisted cantilever plates with curvature (camber)

a/b=1, b/h=100, $b/R_y = 0.25$

Non dimensional frequency, $\varpi = \omega a^2 \sqrt{(\rho / E_{11} h^2)}$

$E_{11} = 138GPa, E_{22} = 8.96Gpa, G_{12} = 7.1GPa, \nu_{12} = 0.3$

ϕ	θ	Non-dimensional free vibration frequencies			
		Mode 1	Mode 2	Mode 3	Mode 4
0	0	1.5497	1.9063	3.0167	5.3533
	15	1.5536	2.0500	3.3357	5.6167
	30	1.5428	1.9879	3.6336	5.5220
	45	1.3180	1.6531	3.8333	5.2046
	60	1.0340	1.2979	3.8543	4.8463
	75	0.8758	1.0232	3.4562	4.3328
	90	0.8228	0.9100	3.1928	3.9425
15	0	1.5497	1.9063	3.0167	5.3533
	15	1.5526	2.011	3.1253	5.5242
	30	1.5428	1.9880	3.6337	5.5220
	45	1.3180	1.6531	3.8333	5.2046
	60	1.0339	1.2978	3.8543	4.8463

	75	0.8757	1.0232	3.4563	4.3328
	90	0.8228	0.9100	3.1928	3.9425
30	0	1.015	5.565	8.991	9.786
	15	1.022	5.637	9.036	10.072
	30	0.895	4.903	8.337	9.829
	45	0.689	3.752	7.272	8.686
	60	0.4818	2.637	6.360	7.530
	75	0.3421	1.937	5.310	6.246
	90	0.288	1.664	4.618	5.848

Then the buckling studies are extended to a laminated composite twisted plates with camber (b/Ry=0.25). Comparing Table 5 and Table 6, the buckling loads have significantly increased with introduction of curvature in the panel. As shown in the Table 6, the buckling loads vary little with introduction of small angle of twist $\phi=15^0$. However the buckling load decreases significantly when the angle of twist increases from 15^0 to 30^0.

Table 5. Variation of buckling load for different angle of twist for three layer (θ,-θ,θ), graphite /epoxy twisted plates,

b/h= 100, a/b=1 , E_{11}=138 GPa,E_{22}=8.96GPa, G_{12}=7.1Gpa, ν_{12} =0.30

Angle of Twist(ϕ) in Degree	Ply orientation (θ) in Degree						
	0	15	30	45	60	75	90
0	3.179	2.162	1.139	0.573	0.315	0.224	0.260
15	3.019	2.562	1.540	0.782	0.354	0.219	0.195
30	2.544	2.195	1.412	0.703	0.315	0.187	0.164

Table-6 Buckling load for three layer graphite /epoxy twisted plates with camber

b/h= 100, a/b=1, b/R_y=0.25 , (θ,-θ,θ),

E_{11}=138 GPa,E_{22}=8.96GPa, G_{12}=7.1Gpa, ν_{12} =0.30

ϕ in Degree	Ply orientation (θ) in Degree						
	0	15	30	45	60	75	90
0	7.826	6.525	5.884	4.858	3.222	2.366	2.122
15	7.827	6.525	5.884	4.858	3.22	2.366	2.122
30	2.775	2.828	2.135	1.235	0.589	0.299	0.215

The buckling loads are then computed for a thick (b/h=20) pre twisted cantilever plates as shown in Table 7. The non-dimensional buckling loads for thick twisted plates (b/h=20) are less in comparision with thin pre twisted plates (b/h=100). Unlike the thin plates, the buckling load decreases with increase in angle of twist and 0^0 seems to be preferential ply orientation for all categories of thick twisted plates. Then the study is extended more practically oriented turbine blades with aspect ratio a/b=3 (Table-8). The buckling loads tend to decrease significantly with increase of aspect ratio . However for this geometry of pretwisted cantilever plates, the buckling load tends to decrease with increase of angle of twist and 0^0 seems to preferential ply orientation for all plates .

Table-7 Buckling load for three layer graphite /epoxy twisted plates

b/h= 20, a/b=1 , (θ,-θ,θ), E1=138 GPa,E2=8.96GPa,G12=7.1Gpa, ν 12 =0.30

ϕ in Degree	Ply orientation (θ) in Degree						
	0	15	30	45	60	75	90
0	2.998	2.226	1.275	0.647	0.322	0.235	0.199
15	2.979	2.045	1.270	0.605	0.310	0.214	0.195
30	2.504	1.889	1.233	0.584	0.278	0.182	0.164
45	1.768	1.435	0.929	0.450	0.207	0.130	0.116

Table 8 Buckling load for three layer graphite /epoxy twisted plates

b/h= 100, a/b=3 , (θ,-θ,θ), E1=138 GPa,E2=8.96GPa,G12=7.1Gpa, ν 12 =0.30

ϕ in Degree	Ply orientation (θ) in Degree						
	0	15	30	45	60	75	90
0	0.352	0.324	0.103	0.053	0.032	0.245	0.0228
15	0.114	0.106	0.1464	0.062	0.033	0.023	0.017
30	0.282	0.239	0.136	0.005	0.029	0.020	0.018
45	0.200	0.171	0.103	0.046	0.022	0.014	0.013

CONCLUSION

The results of the studies of the buckling and vibration behaviour of laminated composite pre-twisted cantilever plates can be summarised as follows.

1) The frequencies of vibration increase with introduction of camber(curvature) in laminated composite pre twisted cantilever plates.

2) There is a general trend of decrease of buckling load with increase of lamination angle and 0^{0} seems to be preferential ply orientation for maximum buckling load.
3) The buckling loads increase with decrease of angle of twist unlike the untwisted plates .
4) The buckling loads have significantly increased with introduction of curvature in the panel.
5) The preferential ply orientation is 0^{0} for all category of thick twisted plates.
6) The stability resistance decreases significantly with increase of aspect ratio.

From the above studies it can be concluded that the vibration and stability behaviour of the composite pre twisted cantilever plates is greatly influenced by the geometry, ply orientation and angle of twist . This can be used to the adventure of tailoring during design of laminated composite pretwisted cantilever plates.

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