A. K. Satapathy
e-mail: aksatapathy@rec.ori.nic.in

R. K. Sahoo

Department of Mechanical Engineering,
Regional Engineering College,
ROURKELA-769 008,
Orissa, India

Rewetting of an Infinite Slab With Uniform Heating Under Quasi-Steady Conditions

The two-dimensional quasi-steady conduction equation governing conduction controlled rewetting of an infinite slab, with one side flooded and the other side subjected to a constant heat flux, has been solved by Wiener-Hopf technique. The solution yields the quench front temperature as a function of various model parameters such as Peclet number, Biot number and dimensionless heat flux. Also, the critical (dryout) heat flux is obtained by setting the Peclet number equal to zero, which gives the minimum heat flux required to prevent the hot surface being rewetted. [DOI: 10.1115/1.1484111]

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1 Introduction

The process of quenching of hot surfaces is of practical importance in nuclear and metallurgical industries. For instance, in the event of a postulated loss-of-coolant accident (LOCA) in water cooled reactors, the clad surface of the fuel elements may reach very high temperature because the stored energy in the fuel cannot be removed adequately by the surrounding steam. In order to bring the reactor to a cooled shutdown condition, an emergency core cooling system is activated to reflood the core. The time delay in re-establishing the effective cooling may result in a cladding temperature rise, significantly above the saturation temperature. If the cladding temperature rises above the rewetting temperature, a stable vapor blanket will prevent the immediate return to liquid-solid contact. Rewetting is the re-establishment of liquid contact with a hot cladding surface and, thereby, bringing it to an acceptable temperature. Also, quenching phenomenon is of considerable practical interest in many other applications such as steam generators, evaporators, cryogenic systems and metallurgical processing. The cooling process during quenching is characterized by the formation of a wet patch on the hot surface, which eventually develops into a steadily moving quench front. As the quench front moves along the hot surface, two regions can be identified: a dry region ahead of the quench front and a wet region behind the quench front. The upstream end of the solid (wet region) is cooled by convection to the contacting liquid, while its downstream end (dry region) is cooled by heat transfer to a mixture of vapor and entrained liquid droplets, called precursory cooling.

The rewetting model for a two-dimensional two-region heat transfer with a step change in heat transfer coefficient at the quench front has been solved for a single slab [1–3] or for a composite slab [4]. In the single slab model the unwetted side is considered to be adiabatic, whereas in case of a composite slab a three layer composite is considered to simulate the fuel and the cladding separated by a gas filled gap between them. The solution method commonly employed is Wiener-Hopf technique. The two-dimensional rewetting model for a single slab with a uniform heat flux and precursory cooling has been solved by an approximate integral method [5]. The one-dimensional rewetting model with a uniform heat flux has been solved for a smooth plate [6] and for both smooth and grooved plates [7], considering the dry region to be adiabatic.

The analysis of rewetting of a hot surface subjected to a boundary heat flux and the dryout induced by the heat flux is of specific interest while considering the decay heating of a nuclear fuel [5] or in the design of heat pipes for thermal radiators [6,7]. Chan and Zhang [7] observed that the existence of heat flux on the wall poses an unsteady state solution for the heat conduction equation, even after the equation is transformed to the Lagrangian coordinate moving with the quench front. In this respect, they also considered the rewetting velocity as well as the plate temperature (at far ahead of the quench front) to be time variant. In the present paper, however, precursory cooling in the dry region has been included in the boundary condition in order to consider the quasi-steady state conduction equation. Further, reported literature on analytical investigations indicates that Wiener-Hopf solution for the rewetting model with a boundary heat flux does not exist. In the present analysis, Wiener-Hopf technique has been employed because of its accuracy and computational simplicity. Besides, the advantage of using the Wiener-Hopf technique may be recognized in case of handling discontinuous boundary conditions, where the singularity due to the discontinuity can be readily resolved by decomposing an appropriate kernel function in the complex Fourier domain.

In the present study, the physical model consists of an infinitely extended vertical slab with one side flooded and the other side subjected to a uniform heat flux. The model assumes constant but different heat transfer coefficients for the wet and dry regions on the flooded side. The two-dimensional quasi-steady conduction equation governing the conduction-controlled rewetting of the infinite slab has been solved by Wiener-Hopf technique. The present solution involves the exact decomposition of the kernel function, so that the solution may be valid for all range of values of the parameters used in the model. The solution has been compared with other analytical solutions and depicted in the graphical form.

2 Mathematical Model

The two-dimensional transient heat conduction equation for the slab is

$$k\left(\frac{\partial^2 T}{\partial X^2}+\frac{\partial^2 T}{\partial Y^2}\right)=\rho C\,\frac{\partial T}{\partial t} \quad 0<X<\delta \quad 0<Y<L \quad L\rightarrow\infty \quad (1)$$

where L is the length of the slab and δ is the thickness of the slab. The density, specific heat and thermal conductivity of the slab material are ρ, C, and k respectively. The origin of the coordinate frame is at left-bottom corner of the slab. To convert this transient equation into a quasi-steady state equation, the following transformation is used:

$$\overline{x}=X \quad \overline{y}=Y-ut$$

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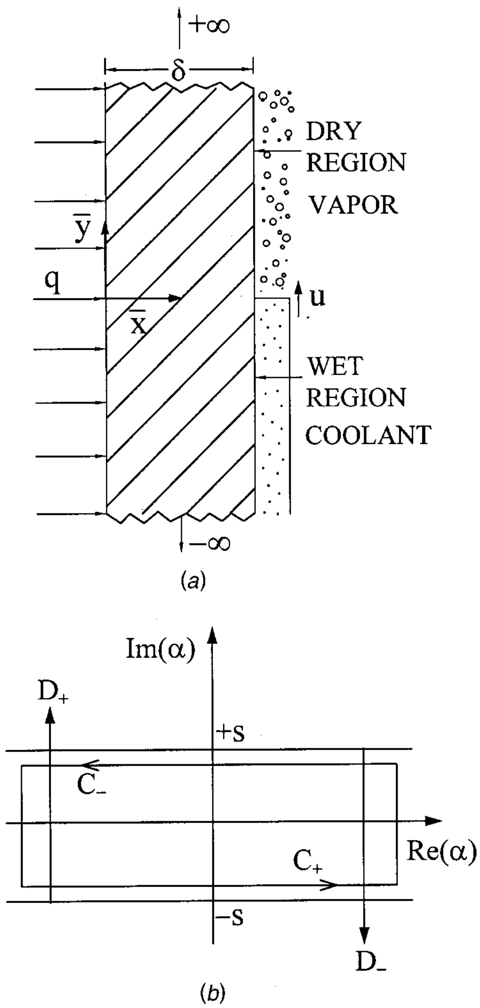

Fig. 1 (*a*) Physical domain of infinite slab; and (*b*) common strip of analyticity in the complex Fourier plane.

where u is the constant quench front velocity and $\bar{x}$, $\bar{y}$ are normal and axial coordinates respectively (Fig. 1(*a*)). Experiments have shown that, if the slab is long enough compared to the penetration depth to heat transfer field, the temperature distribution around the heat source/sink soon becomes independent of time. That is, an observer stationed at the origin of the moving $(\bar{x},\bar{y})$ coordinate system fails to notice any appreciable change in the temperature distribution around him as the front moves on. This is identified as the apparent steady state or quasi-steady state condition. Thus the transformed heat conduction equation in a coordinate system moving with the quench front is

$$\frac{\partial^2 T}{\partial \bar{x}^2}+\frac{\partial^2 T}{\partial \bar{y}^2}+\frac{\rho C u}{k}\frac{\partial T}{\partial \bar{y}}=0 \quad 0<\bar{x}<\delta \quad -\infty<\bar{y}<\infty \tag{2}$$

The above equation is the governing partial differential equation in quasi-steady state for the slab, in which $\partial T/\partial t=0$ in the moving coordinate system.

In conduction-controlled rewetting analysis, it is believed that conduction of heat along the slab from the dry region to wet region is the dominant mechanism of heat removal ahead of the quench front, which results in a lowering of the surface temperature immediately downstream of the quench front and causes the quench front to progress further. Since only axial conduction is considered, the effect of coolant mass flux, coolant inlet subcooling and its pressure gradient etc. are not accounted for explicitly, but implicitly in terms of wet region heat transfer coefficient, which is incorporated in the boundary condition. In the present analysis, the heat transfer coefficient h_1 is assumed to be constant over the entire wet region. The coolant temperature is taken to be equal to its saturation temperature T_s. On the dry side of the slab, the wall is cooled by the surrounding vapor. The heat transfer coefficient accounting for both convective and radiative cooling effects on the dry side is assumed equal to h_2, a constant, which is smaller than h_1. The temperature of the surrounding vapor is assumed equal to T_w, which can be interpreted as the initial temperature of the hot surface without a boundary heat flux. This is justified because the vapor in dry region would be superheated due to the existence of imposed surface heat flux on the wall. The rewetting (quench front) temperature is denoted by T_0.

Following Yao [5], it may be envisaged that the temperature field is sufficiently flat in the axial direction at infinity. Consequently, the first and second derivatives of temperature in $\bar{y}$-direction can be neglected at far upstream of the quench front (at $\bar{y}\to-\infty$) as well as at far downstream of the quench front (at $\bar{y}\to+\infty$). The above two assumptions are adequate to prescribe the temperature at infinity ($\bar{y}\to\pm\infty$). The far-field boundary conditions then become

$$\begin{aligned} T&=T_s+\frac{q}{k}(\delta-\bar{x})+\frac{q}{h_1} \quad \bar{y}\to-\infty \\ T&=T_w+\frac{q}{k}(\delta-\bar{x})+\frac{q}{h_2} \quad \bar{y}\to+\infty \end{aligned} \tag{3}$$

The conventional rewetting models (without a boundary heat flux) usually assume the vapor temperature in the dry region equal to its saturation temperature so that it would be used as a sink temperature. In the presence of a boundary heat flux, however, it is well justified to assume the vapor temperature equal to the initial wall temperature because the vapor would be superheated owing to the existence of the boundary heat flux. The surface temperature of the slab at far ahead of the quench front (at $\bar{x}=\delta$, $\bar{y}\to+\infty$) can be calculated (Eq. (3)) to be equal to (T_w+q/h_2). In situations when $T_0\geqslant T_w$, the temperature of dry region wall $T(\delta,\bar{y})$ will be above T_w and, hence, the wall will be cooled by the vapor. On the other hand, for $T_0<T_w$, only a finite part of the dry region wall immediately ahead of the quench front will be less than T_w, whereas for the remaining part it will be more than T_w over an infinite length. This implies that the former part of the dry region wall of a finite length will be heated by the vapor while the latter part of an infinite length will be cooled by the vapor and the overall effect is to cool the dry region wall. Thus, on the whole, in both the situations the vapor temperature would behave as the sink temperature. Moreover, the boundary conditions in Eq. (3) suggest that precursory cooling in the dry region cannot be neglected in the case of existence of boundary heat flux on the wall. Equation (2) can be expressed in the following dimensionless form

$$\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}+\mathrm{Pe}\frac{\partial\theta}{\partial y}=0 \quad 0<x<1 \quad -\infty<y<\infty \tag{4}$$

The associated boundary conditions are

$$\begin{aligned} &\frac{\partial\theta}{\partial x}+Q=0 \quad \text{at } x=0 \quad -\infty<y<\infty \\ &\frac{\partial\theta}{\partial x}+B_1\theta=0 \quad \text{at } x=1 \quad y<0 \\ &\frac{\partial\theta}{\partial x}+B_2(\theta-1)=0 \quad \text{at } x=1 \quad y>0 \\ &\theta=\frac{Q}{B_1}+Q(1-x) \quad \text{at } y\to-\infty \\ &\theta=1+\frac{Q}{B_2}+Q(1-x) \quad \text{at } y\to+\infty \\ &\theta=\theta_0 \quad \text{at } x=1 \quad y=0 \end{aligned} \tag{5}$$

The non-dimensional variables used above are

$$x=\frac{\overline{x}}{\delta},\quad y=\frac{\overline{y}}{\delta},\quad \theta=\frac{T-T_s}{T_w-T_s},\quad B_1=\frac{h_1\delta}{k},\quad B_2=\frac{h_2\delta}{k},$$

$$\mathrm{Pe}=\frac{\rho Cu\delta}{k},\quad Q=\frac{q\delta}{k(T_w-T_s)}\quad \text{and}\quad \theta_0=\frac{T_0-T_s}{T_w-T_s}.$$

It may be verified that for no heat flux condition with adiabatic dry side (by setting $Q=0$ $B_2=0$ and $Q/B_2=0$), the boundary conditions in Eq. (5) reduces to that of conventional two-region model (insulated dry wall and without a heating [2]). The main objective of the present analysis is to obtain the quench front temperature θ_0 in terms of wetside Biot number B_1, dryside Biot number B_2, Peclet number Pe and dimensionless heat flux Q. Although Eqs. (4) and (5) have been formulated for the case of bottom flooding, they are also valid for top flooding.

3 Analytical Solution

In order to employ the Wiener-Hopf technique, Eq. (4) is first transformed with a new variable φ, defined by $\theta(x,y)=1+(Q/B_2)+Q(1-x)-\varphi(x,y)e^{-sy}$, in which $s=\mathrm{Pe}/2$. The governing equation (Eq. (4)) then becomes

$$\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}-s^2\varphi=0\quad 0<x<1\quad -\infty<y<\infty \tag{6}$$

The boundary conditions can be written sequentially as

$$\begin{aligned}
&\frac{\partial\varphi}{\partial x}=0\quad \text{at } x=0\quad -\infty<y<\infty\\
&\frac{\partial\varphi}{\partial x}+B_1\varphi=B_1\left[1+\frac{Q}{B_2}-\frac{Q}{B_1}\right]e^{sy}\quad \text{at } x=1\quad y<0\\
&\frac{\partial\varphi}{\partial x}+B_2\varphi=0\quad \text{at } x=1\quad y>0\\
&\varphi=\left[1+\frac{Q}{B_2}-\frac{Q}{B_1}\right]e^{sy}\quad \text{at } y\to-\infty\\
&\varphi=0\quad \text{at } y\to+\infty
\end{aligned}\tag{7}$$

3.1 Fourier Transform. Fourier transformation of a partial differential equation and of its associated boundary conditions generally results in a less complicated problem in the plane of the transformed variable. If the solution of this subsidiary problem can easily be obtained and inverted, then the transform technique is straightforward and supposed to be efficient. In the next step of the analysis, Fourier transform is used to convert the partial differential equation (Eq. (6)) to an ordinary differential equation. The Fourier transform is defined by

$$\Phi(\alpha,x)=\Phi_+(\alpha,x)+\Phi_-(\alpha,x)=\int_{-\infty}^{\infty}\varphi(x,y)e^{i\alpha y}dy \tag{8}$$

with

$$\Phi_-(\alpha,x)=\int_{-\infty}^{0}\varphi(x,y)e^{i\alpha y}dy,$$

$$\Phi_+(\alpha,x)=\int_{0}^{\infty}\varphi(x,y)e^{i\alpha y}dy.$$

The parameter α used above is a complex quantity. The far-field boundary conditions in Eq. (7) indicate that $\varphi(x,y)$ is of the order $\exp(sy)$ at $y\to-\infty$, whereas $\varphi(x,y)$ is of the order $\exp(-sy)$ at $y\to+\infty$. Thus, the functions $\Phi_+(\alpha,x)$, $\Phi_-(\alpha,x)$ are analytic in the domains D_+ and D_- respectively ([8], p. 78). The domains D_+ and D_- are defined (Fig. 1(*b*)) in the entire complex domain as: $D_+:\mathrm{Im}(\alpha)>-s$, $D_-:\mathrm{Im}(\alpha)<+s$. Applying the Fourier transform, Eq. (6) assumes the form

$$\frac{d^2\Phi}{dx^2}-\gamma^2\Phi=0 \tag{9}$$

in which $\gamma=(\alpha^2+s^2)^{1/2}$. The transformed boundary conditions are

$$\begin{aligned}
&\Phi'(\alpha,0)=0\\
&\Phi'_-(\alpha,1)+B_1\Phi_-(\alpha,1)=-\frac{i}{\alpha-is}B_1\left[1+\frac{Q}{B_2}-\frac{Q}{B_1}\right]\\
&\Phi'_+(\alpha,1)+B_2\Phi_+(\alpha,1)=0
\end{aligned}\tag{10}$$

where prime denotes the transform of x-derivatives of $\varphi(x,y)$. The general solution of the second order ordinary differential equation (Eq. (9)) is

$$\Phi(\alpha,x)=C_1(\alpha)\cosh\gamma x+C_2(\alpha)\sinh\gamma x \tag{11}$$

Imposing the boundary conditions of Eq. (10) into Eq. (11) yields

$$\Phi_+(\alpha,1)+\left[\frac{1+(B_1\coth\gamma)/\gamma}{1+(B_2\coth\gamma)/\gamma}\right]\Phi_-(\alpha,1)=-\frac{i}{\alpha-is}\left(\frac{B_1\coth\gamma}{\gamma+B_2\coth\gamma}\right)\left(1+\frac{Q}{B_2}-\frac{Q}{B_1}\right) \tag{12}$$

3.2 Wiener-Hopf Technique. The technique of Wiener-Hopf, which has been fruitfully applied to the class of rewetting problems, uses the strategy of solving a functional equation (Eq. (9)) comprising of two unknown functions (Φ_+ and Φ_-) of complex variable. The crucial step in successful execution of the Wiener-Hopf technique depends on the factorization of a function, which is analytic in a strip, into the product of two functions that are analytic in the overlapping half-planes. In this context, let

$$K(\alpha)=K_+(\alpha)K_-(\alpha)=\frac{1+(B_1\coth\gamma)/\gamma}{1+(B_2\coth\gamma)/\gamma} \tag{13}$$

where the functions $K_+(\alpha)$, $K_-(\alpha)$ are analytic in the domains D_+ and D_- respectively. Now the kernel function $K(\alpha)$, in connection with Eq. (12), is to be decomposed to $K_+(\alpha)$ and $K_-(\alpha)$ in accordance with the Wiener-Hopf technique. This is accomplished by rearranging Eq. (12) to obtain

$$\begin{aligned}
&\frac{\Phi_+(\alpha,1)}{K_+(\alpha)}-\frac{i}{\alpha-is}\left(\frac{Q}{B_2}+\frac{1}{1-\lambda}\right)\left[\frac{1}{K_+(\alpha)}-\frac{1}{K_+(is)}\right]\\
&\quad=-\frac{i}{\alpha-is}\left(\frac{Q}{B_2}+\frac{1}{1-\lambda}\right)\left[K_-(\alpha)-\frac{1}{K_+(is)}\right]\\
&\qquad-\Phi_-(\alpha,1)K_-(\alpha)
\end{aligned}\tag{14}$$

where $\lambda=B_2/B_1$. In Eq. (14), each side characterizes the same "entire function", through its representation in the upper and lower halves of the α-plane. Since $\Phi_+(\alpha,1)$ and $\Phi_-(\alpha,1)$ tend to zero at infinity in their half planes of analyticity, while $K_+(\alpha)$ and $K_-(\alpha)$ remain bounded, it follows that the entire function vanishes according to Liouville's theorem ([8], p. 27). Therefore, equating both sides of the Eq. (14) to zero, $\Phi_+(\alpha,1)$ and $\Phi_-(\alpha,1)$ are determined as

$$\begin{aligned}
\Phi_+(\alpha,1)&=\frac{i}{\alpha-is}\left(\frac{Q}{B_2}+\frac{1}{1-\lambda}\right)\left[1-\frac{K_+(\alpha)}{K_+(is)}\right]\\
\Phi_-(\alpha,1)&=-\frac{i}{\alpha-is}\left(\frac{Q}{B_2}+\frac{1}{1-\lambda}\right)\left[1-\frac{1}{K_-(\alpha)K_+(is)}\right]
\end{aligned}\tag{15}$$

3.3 Quench Front Temperature. Using the above expressions of $\Phi_+(\alpha,1)$ and $\Phi_-(\alpha,1)$, quench front temperature may be obtained by inverting the Fourier transform (Eq. (8)). Such an attempt may become tedious because, in order to carry out the Fourier inversion, it would be necessary to evaluate the residues of the function $\Phi(\alpha,1)$ in the complex domain. Alternatively, in the present paper θ_0 has been calculated in a simplified approach [1] as follows.

$$\Phi_+(\alpha,1)=\int_0^\infty \varphi(1,y)e^{i\alpha y}dy$$

$$=\frac{i}{\alpha}\varphi(1,0)-\frac{1}{i\alpha}\int_0^\infty \frac{\partial\varphi(1,y)}{\partial y}e^{i\alpha y}dy \qquad (16)$$

In the limit $\alpha\to\infty$, the second integral appearing in Eq. (16) vanishes since the quantity $\partial\varphi/\partial y$ is bounded [1]. Then, by virtue of Eqs. (15)–(16) and invoking an assumption that $K_+(\alpha)$ approaches unity as $\alpha\to\infty$ (the assumption will be validated later), we obtain

$$\varphi(1,0)=\lim_{\alpha\to\infty}[-i\alpha\Phi_+(\alpha,1)]=\left(\frac{Q}{B_2}+\frac{1}{1-\lambda}\right)\left[1-\frac{1}{K_+(is)}\right] \qquad (17)$$

The quench front temperature then becomes

$$\theta_0=1+Q/B_2-\varphi(1,0)=\frac{1}{K_+(is)}\left[\frac{Q}{B_2}+\frac{1}{1-\lambda}\right]-\frac{\lambda}{1-\lambda} \qquad (18)$$

Now the function $K_+(is)$ may be expressed as an "infinite product series" or as a "contour integral." While the former leads to evaluation of the eigen values of a certain transcendental equation, the latter leads to an integral expression which is seemingly more convenient for numerical computation. On applying the Cauchy residue theorem within the strip, the function $\ln K(\alpha)$ can be represented by the following contour integral.

$$\ln K(\alpha)=\ln K_+(\alpha)+\ln K_-(\alpha)$$

$$=\frac{1}{2\pi i}\int_{C_+}\frac{\ln K(\xi)}{\xi-\alpha}d\xi-\frac{1}{2\pi i}\int_{C_-}\frac{\ln K(\xi)}{\xi-\alpha}d\xi \qquad (19)$$

where C_+/C_- is an infinite contour lying inside the strip and passing below/above the point α (Fig. 1(*b*)). It may be noted that, due to the asymptotic nature of $\ln K(\alpha)$ function (the order of $\ln K(\alpha)$ being $1/\alpha$), the contribution of vertical sides of the contour to the integral vanishes at $\mathrm{Re}(\alpha)\to\pm\infty$. Equation (19) can be succinctly written as

$$\ln K_\pm(\alpha)=\pm\frac{1}{2\pi i}\int_{C_\pm}\frac{\ln K(\xi)}{\xi-\alpha}d\xi \qquad (20)$$

from which it follows that $K_\pm(\alpha)=1$ as $\alpha\to\infty$, as assumed earlier. In order to evaluate the function $K_+(is)$, the contour C_+ may be shifted to the real axis to yield

$$\ln K_+(is)=\frac{1}{2\pi i}\int_{-\infty}^{+\infty}\frac{\ln K(\xi)}{\xi-is}d\xi \qquad (21)$$

Further, as the functions $\ln K(\xi)$, $\xi\ln K(\xi)$ exhibit even and odd properties respectively, Eq. (21) thereby reduces to

$$\ln K_+(is)=\frac{s}{\pi}\int_0^\infty\frac{\ln K(\xi)}{\xi^2+s^2}d\xi \qquad (22)$$

For computational purposes it is advantageous to transform ξ, by $\xi=s\tan\Omega$, to finally obtain the quench front temperature

$$\theta_0=\frac{1}{K_+(is)}\left[\frac{Q}{B_2}+\frac{B_1}{B_1-B_2}\right]-\frac{B_2}{B_1-B_2} \qquad (23)$$

in which,

$$K_+(is)=\exp\left[\frac{1}{\pi}\int_0^{\pi/2}\ln\left\{\frac{1+(B_1\coth s\sec\Omega)/s\sec\Omega}{1+(B_2\coth s\sec\Omega)/s\sec\Omega}\right\}d\Omega\right].$$

It is of interest to examine the limiting solution of the above equation for the case that has been investigated by Olek [2], namely, the rewetting of an infinite slab without any heating or precursory cooling. By assigning $Q=0$, $B_2=0$ and $Q/B_2=0$ in Eq. (23), the expression for θ_0 reduces to exactly the same as that of Olek [2] which, in turn, substantiates the present solution.

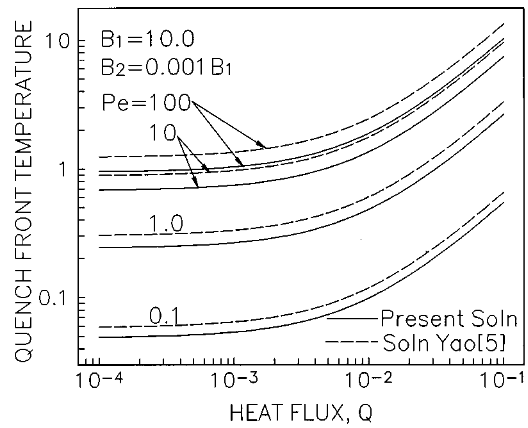

Fig. 2 Quench front temperature for various heat flux and Peclet number

3.4 Critical Heat Flux. The quench front temperature at the critical (dryout) heat flux has been deduced by specifying $s=0$ in Eq. (23). Thus, $K_+(is)$ simplifies to

$$K_+(is)=\exp\left[\frac{1}{\pi}\int_0^{\pi/2}\ln(B_1/B_2)d\Omega\right]=\sqrt{\frac{B_1}{B_2}} \qquad (24)$$

The quench front temperature at the critical heat flux is finally determined as

$$\theta_0=\frac{Q}{\sqrt{B_1B_2}}+\frac{\sqrt{B_2}}{\sqrt{B_1}+\sqrt{B_2}} \qquad (25)$$

The heat flux Q appearing in Eq. (25) may be regarded as the critical heat flux Q_{cri}, which characterizes the maximum allowable heat input to a slab to inhibit the dryout of the coolant.

4 Results and Discussion

Numerical values of the quench front temperature are obtained from the expressions in Eqs. (23) and (25), for a practical range of model parameters B_1, B_2, Pe and Q. For this purpose, the integral appearing in Eq. (23) has been numerically calculated by Simpson's 1/3 rule with 101 equally spaced base points. Experimental investigations on quenching [9] reveal the existence of four distinct heat transfer regimes along the wall, the regimes being demarcated by the characteristic hot surface temperature. These four zones are: forced convection of subcooled liquid, nucleate boiling, wet and dry transition boiling and film boiling. Quench front is observed to exist in the transition zone. The heat transfer coefficient in the transition zone is shown to be 10^5–10^6 W/m^2-K and the vapor cooling heat transfer coefficient in the film boiling zone is in the order of 10^2 W/m^2-K. In the present analysis the values of h_1 and h_2 are adopted from the experimental results of Barnea et al. [9]. Hence the values of h_2 are set equal to $10^{-3}h_1$ and, therefore, $B_2=10^{-3}B_1$.

The variation of quench front temperature with heat flux and Peclet number is shown in Fig. (2), for a fixed value of Biot number. Here θ_0 is found to increase with increase in Peclet number. With fixed material properties and dimensions, Peclet number and Biot number represent the quench front velocity and the heat transfer coefficient respectively. For prescribed values of heat flux and Biot number, θ_0 increases with increase in quench front velocity. This may be due to the fact that a higher relative velocity between the slab and the coolant allows less time for sufficient heat transfer to take place, resulting in a higher value of θ_0. The above trend also reflects the fact that, for the same rewetting rate,

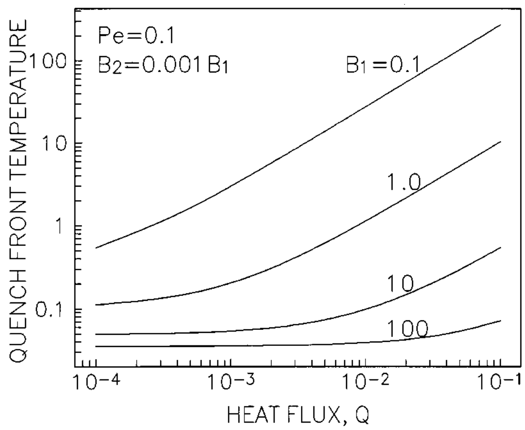

Fig. 3 Quench front temperature for various heat flux and Biot number

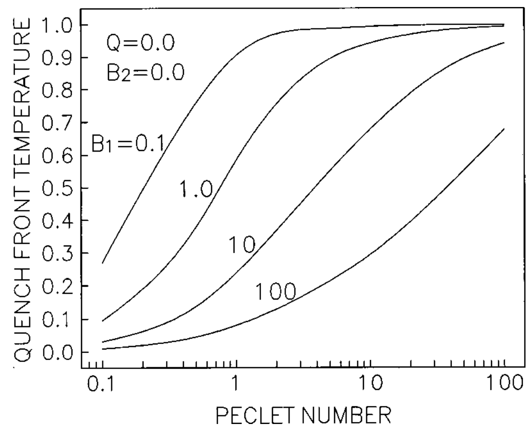

Fig. 5 Quench front temperature for various wetside Biot number and Peclet number without heating and precursory cooling

an increasing slab thermal diffusivity tends to reduce θ_0 whereas the increasing slab thickness has the opposite effect. Further, for a fixed Peclet number, θ_0 increases with increase in Q. Apparently, a higher heat flux causes more heat transfer to the slab and hence this would increase θ_0.

The dependence of quench front temperature on Biot number and dimensionless heat flux is shown in Fig. (3), where θ_0 decreases with increase in Biot number for a given Pe and Q. A higher Biot number results in a higher heat transfer coefficient. This enhanced heat transfer coefficient may cause to decrease θ_0. The above trends are in obvious accord with the predictions based on physical ground. In all cases, θ_0 decreases as Biot number increases, reflecting the fact that a quench front progresses more easily when the heat transfer to the coolant is increased. On a similar ground, conversely, one would conclude that an increasing Q has the opposite effect on the quench front velocity.

The dependence of quench front temperature on Biot number and dimensionless heat flux is shown in Fig. (4), with Pe=0. The physical meaning of Pe=0 is that the quench front ceases to move when Q approaches its critical value. This is the case that the surface can no longer be wetted. For $Q > Q_{\mathrm{cri}}$, the quench front will reverse its direction and the wetted surface will be dried. In this case, the slab will be heated by a heat flux that exceeds the maximum heat removal capacity by convection and boiling and, thus, dryout would occur. Further, the present solution has been compared with those of Yao [5] in Fig. (2) and in Fig. (4). The results are in good agreement with those in [5] for lower values of Biot and Peclet numbers while the deviation becomes more pronounced as Biot and Peclet numbers become large. In Yao's [5] analysis, the temperature distribution along the width of the slab was assumed to be quadratic and with this assumption, the solution was obtained for a two-dimensional conduction model. Apparently, the solution may deviate at higher Biot and Peclet numbers due to the above approximation. Finally, the model is reduced to the conventional model (by setting $Q=0$, $B_2=0$ and $Q/B_2=0$) and illustrated in Fig. (5). As expected, θ_0 increases with increase in Peclet number and with decrease in Biot number.

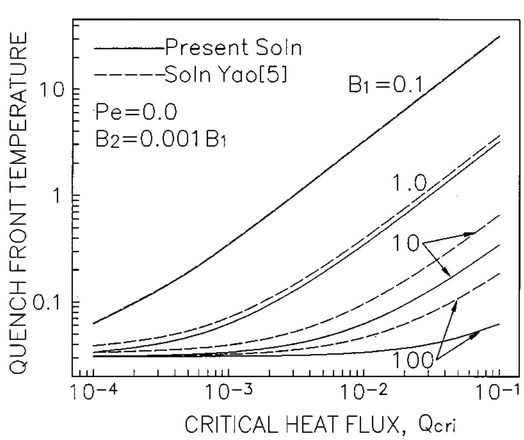

Fig. 4 Quench front temperature at the critical heat flux

The Wiener-Hopf technique yields a solution for the quench front temperature (Eq. (23)), which is more elegant and accurate than results obtained by other analytical methods. In particular, Wiener-Hopf solution is superior to the one by separation of variables, since it overcomes the accuracy problems due to slow convergence of the series expansions that stem from discontinuity of the surface heat flux at the quench front [2]. The technique makes use of decomposing a kernel function in the complex Fourier plane so as to resolve the singularity arising out of discontinuous boundary conditions at the quench front, as in the case of a rewetting problem. The explicit formula for the quench front temperature obtained in the present study is valid for all Biot and Peclet numbers. However, the present model is limited to small Peclet numbers with regard to heat pipes. This is due to the fact that, in the case of large Peclet numbers, a thermal boundary layer is formed near the cooling surface of a heat pipe and this has not been incorporated in the present model. Besides, large Biot numbers are usually associated with large Peclet numbers, unless the internal heating is large. Since the internal heating effect is also not considered in the model, the present analysis is limited to both small Biot and Peclet numbers in case of heat pipes.

5 Conclusion

An analytical solution for rewetting of an infinite slab with a uniform heating has been obtained, employing the Wiener-Hopf technique. In general, quench front temperature is found to increase with increase in Peclet number and dimensionless heat flux, and with decrease in Biot number. The boundary conditions in the present formulation require liquid/vapor temperatures and liquid/vapor heat transfer coefficients as input parameters, these limitations being inherent in a conduction-controlled rewetting model. The arbitrariness of the choice of their values may be eliminated if a conjugate heat transfer model is considered, where the energy equations of solid, liquid and vapor regions need to be solved simultaneously.

Nomenclature

B = Biot number
C = specific heat
h = heat transfer coefficient
k = thermal conductivity
L = length of the slab
Pe = Peclet number
q = heat flux
Q = dimensionless heat flux
s = half of the Peclet number
t = time
T = temperature
u = quench front velocity
$\underline{X}, \underline{Y}$ = physical coordinates
$\overline{x}, \overline{y}$ = coordinates in quasi-steady state
x, y = dimensionless coordinates in quasi-steady state

Greek Alphabets

δ = thickness of the slab
λ = ratio of dryside to wetside Biot numbers
θ = dimensionless temperature
ρ = density

Subscripts

0 = quench front
1 = wet region
2 = dry region
s = saturation
w = initial wall condition

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