Parametric Instability of Laminated Composite Doubly Curved Shell Panels Subjected to Hygrothermal Environment

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Abstract: The dynamic stability behavior of laminated composite shells subjected to hygrothermal loadings are studied in the present investigation. A simple laminated model is developed for the vibration and stability analysis of laminated composite shells subjected to hygrothermal conditions. A computer program based on FEM in MATLAB environment is developed to perform all necessary computations. An eight-node isoparametric element is employed in the present The analysis with five degrees of freedom per node. Element elastic stiffness matrices, mass matrices, geometric stiffness matrix due to mechanical and hygrothermal loads and load vectors are derived using the principle of minimum potential energy. Quantitative results are presented to show the effects of curvature, ply-orientation, degrees of orthotropy and static load factors of laminate on dynamic stability of composite shells for different temperatures and moisture concentrations.

Introduction

Composite materials are being increasingly used in aerospace, civil, naval and other high-performance engineering applications due to their light weight, high-specific strength and stiffness, excellent thermal characteristics, ease in fabrication and other significant attributes. Structures used in the above fields are more often exposed to high temperature as well as moisture. This wide range of practical applications demands a fundamental understanding of their vibrations, static and dynamic stability characteristics in different temperatures and moisture concentrations. Studies on vibration and buckling of composite plates and shells subjected to hygrothermal loads are available in literature. The vibration response of flat and curved panels subjected to thermal and mechanical loads are presented by Librescu and Lin [1]. Parhi *et al.*[2] investigated the effect of moisture and temperature on the dynamic behavior of composite laminated plates and shells with or without delaminations. The effect of hygrothermal conditions on the buckling and post buckling of shear deformable laminated cylindrical shells subjected to combined loading of axial compression and external pressure is investigated using micro-to-macro mechanical analytical model by Shen [3].

The parametric resonance characteristics of composite shells are studied by few investigators without considering the hygrothermal effects. The parametric resonance in cylindrical shells under periodic loads has attracted much attention due to its detrimental and de-stabilizing effects in many engineering applications. This phenomenon in elastic systems was first studied by Bolotin [4]. Bert and Birman [5] examined the parametric instability of thick orthotropic shells using higher-order theory. Argento and Scott [6] employed a perturbation technique to study the dynamic stability of layered anisotropic circular cylindrical shells under axial loading. The nonlinear parametric instability behavior of curved panels, the effects of curvature and aspect ratio on dynamic instability for a uniformly loaded laminated composite thick cylindrical panel is investigated by Ganapathi *et al.* [7] using finite element method (FEM). The dynamic stability of thin cross-ply laminated composite cylindrical shells under combined static and periodic axial force is examined by Ng *et al.* [8]. The dynamic stability behavior of laminated composite curved panels with cutout under ambient temperature and moisture but subjected to in-plane static and a periodic compressive load was studied by Sahu & Dutta [9]. The present study deals with the effect of temperature, moisture and curvature on parametric instability behaviour of composite shell panels using Bolotin's approach employing FEM.

Mathematical Formulation

The basic configuration of the problem considered here is a laminated composite doubly curved panel under hygrothermal conditions and subjected to in-plane periodic loads as shown in Fig. 1.

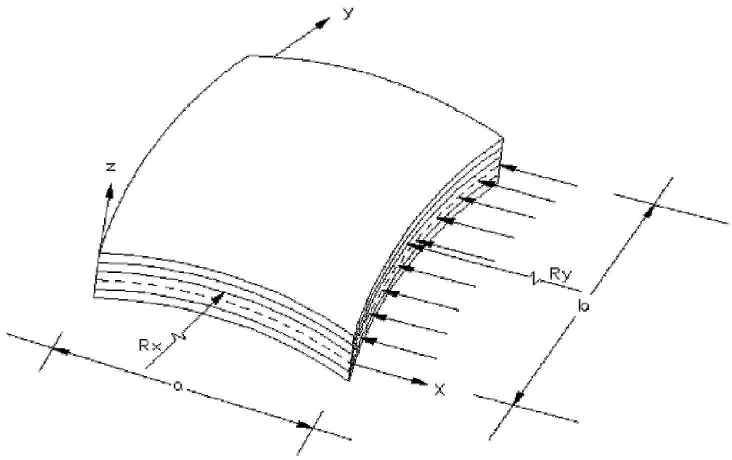

Fig.1 Laminated composite curved panel under in-plane loading and hygrothermal conditions

The governing equations for the dynamic stability of laminated composite doubly curved panels/shells subjected to hygrothermal loading are developed using first order shear deformation theory (FSDT) and omitted here for sake of brevity. The equation of motion represents a system of second order differential equation with periodic coefficients of the Mathieu-Hill type. The development of the regions of instability arises from Floquet's theory and the solution is obtaining using Bolotin's approach and FEM. The analysis is linear and the curved panels are with no initial imperfections. The loading considered is axial with a simple harmonic fluctuation with respect to time. All damping effects are neglected. An eight-node doubly curved isoparametric element is employed in the present analysis with five degrees of freedom u, v, w, θ_x and θ_y per node having all three radii of curvatures. The equation of motion for parametric resonance characteristics of a laminated composite panel in hygrothermal environment, subjected to in plane load $N_{(t)}$ can be expressed as[8]:

$$[[K_e]-\alpha P_{cr}[K_g]\pm\frac{1}{2}\beta P_{cr}[K_g]-\frac{\Omega^2}{4}[M]]\{q\}=0 \qquad (1)$$

Eq.(1) represents an eigenvalue problem for known values of α, β and P_{cr}. The two conditions under the plus and minus sign correspond to two boundaries (upper and lower) of the dynamic instability region. The above eigenvalues solution give Ω, which give the boundary frequencies of the instability regions for the given values of α and β. In this analysis, the computed static buckling load of the panel is considered as the reference load. Green-Lagrange's strain displacement relations are presented in general throughout the analysis. The linear part of the strain is used to derive the elastic stiffness matrix and the non-linear part of the strain is used to derive the elastic stiffness matrix and the non-linear part of the strain is used to derive the geometric stiffness matrix. A computer program is developed by using MATLAB environment to perform all the necessary computations. The element stiffness and mass matrices are derived using a standard procedure. Numerical integration technique by Gaussian quadrature is adopted for the element matrix. The overall matrices $[K_e]$, $[K_g]$, and [M] are obtained by assembling the corresponding element matrices, using skyline technique. The boundary conditions are imposed restraining the generalized displacements in different nodes of the discretized structure.

Results & Discussions

The convergence study is done for non-dimensional frequencies of free vibration of simply supported square 4 layer anti-symmetric angle ply laminated composite plates for elevated temperature and buckling analysis of shell under moisture conditions for different mesh division. The results are omitted here for sake of brevity. Based on convergence study, a mesh of 8×8 is employed throughout for free vibration, buckling and dynamic stability analysis of laminated composite plates in hygrothermal environment. The present formulation is validated for free vibration analysis of laminated composites panels under hygrothermal conditions with that of Sairam & Sinha [10] and Hung, Hen & Zheng [11]. As shown in Table 1, there exists excellent agreement between the present study with both of previous studies. .

Table 1: Comparison of non-dimensional free vibration frequencies for SSSS (0/90/90/0) panels at 325K temperature

a/b=1,a/t=100, At $T = 300K$ $E_1 = 130Gpa$, $E_2 = 9.5Gpa$, $G_{12} = 6Gpa$

$G_{13} = G_{12}$, $G_{23} = 0.5G_{12}$, $\nu_{12} = 0.3$, $\alpha_1 = -0.3\times10^{-6} /^0 K$, $\alpha_2 = 28.1\times10^{-6} /^0 K$

Non-dimensional frequency, $\lambda = \omega_n a^2 \sqrt{\rho / E_2 t^2}$

Methods	Non-dimensional frequencies at $325K$ Temperature			
	1	2	3	4
Sairam & Sinha [3] Present FEM	8.088 (8.0791)	19.196 (19.1002)	39.324 (39.3358)	45.431 (45.3505)
Hung, Hen & Zheng [27] Present FEM	8.043 (8.0791)	18.140 (19.1002)	38.364 (39.3358)	44.686 (45.3505)

The present formulation is also validated buckling analysis of composites plates for temperature and moisture and omitted here for sake of brevity. After validating the formulation, the parametric instability effects of laminated composite shells in hygrothermal environment is studied. Numerical results are presented on the dynamic stability of angle-ply laminated shell to study the effects of various parameters on instability regions. The geometrical properties of the shell are as follows:

a=500mm, b=500mm, t=5mm, E_1=130Gpa, E_2=9.5Gpa, G_{12}=6Gpa, G_{23}/G_{12}=0.5, υ12=0.3, $G_{13}=G_{12}$, $\alpha_1=-0.3\times10^{-6}$ /K, $\alpha_2=28.1\times10^{-6}$ /K.

The non-dimensional excitation frequency $\Omega = \Omega a^2 \sqrt{\rho / E_2 t^2}$ is used throughout the dynamic instability studies, where ³ is the excitation frequency in radian/sec. The principal instability regions of laminated composite curved panel subjected to in-plane periodic loads is plotted with non-dimensional frequency Ω/ω (ratio of excitation frequency to the free vibration frequency) versus the dynamic in-plane load β. The variation of excitation frequency with dynamic load factor of composite laminated simply-supported anti-symmetric angle-ply square shells subjected to uniform distribution of temperature from 300K, 325K, 350K, 375K & 400K is shown in Fig. 2. As shown, the onset of instability occurs earlier with wider DIR for anti-symmetric angle-ply laminated composite shells subjected to elevated temperature compared to composite shells with normal temperature. With increase in temperature from 300K to 350K, the excitation frequency is reducing by 65.2%.

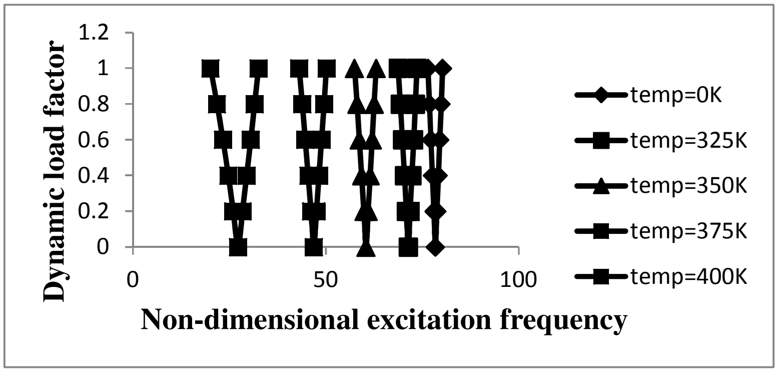

Fig 2: Variations of instability region with temperature of composite laminated antisymmetric angle-ply (45/-45/45/-45) curved panel

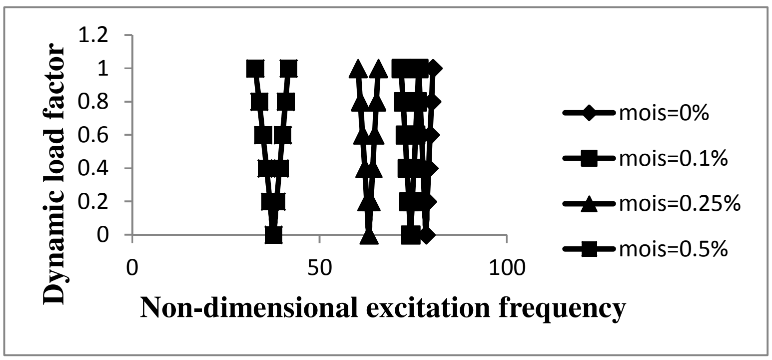

Fig 3: Variations of instability region with moisture of composite laminated anti-symmetric angle-ply (45/-45/45/-45) shell

The variation of excitation frequency with dynamic load factor of composite laminated simply-supported anti-symmetric angle-ply shell subjected to uniform distribution of moisture concentration from 0%, 0.1%, 0.25% & 0.5% is shown in fig.3. It is revealed that the onset of instability occurs earlier with wider DIR for anti-symmetric angle-ply laminated composite shells subjected to elevated moisture condition compared to composite shells with normal moisture concentration. When moisture concentration is increased from 0% to 0.25% then excitation frequency drop is happened for about 49.3%. Studies have also been made (Fig.4) for comparison of instability regions for different shell geometries. The effect of curvature on instability region of different curved panels for a/b=1, flat panel ($a/R_x = b/R_y = 0$), Cylindrical ($a/R_x = 0$, $b/R_y = 0.2$) & Spherical ($a/R_x = b/R_y = 0.2$) has investigated. It is observed that the excitation frequency increases with introduction of curvatures from plate to doubly curved panel in elevated temperature. The onset of dynamic instability region occurs earlier with wider dynamic instability region (DIR) coming from spherical laminated composite shell panel to laminated composite flat panel subjected to uniform distribution of temperature. The quantitative effects of other parameters on the instability of composite panels under hygrothermal conditions are studied but omitted here for sake of brevity.

Conclusion

In the present work, the conventional finite element formulation is modified to study the hygrothermal effects on the parametric resonance of laminated composite shells. The broad conclusions that can be made from the present study are summarized as follows:

- The excitation frequencies of laminated composite panels decrease with increase of temperature due to reduction of stiffness for all laminates.
- The excitation frequencies of laminated composite panels also decrease substantially with increase of moisture concentration for all laminates.
- Due to introduction of curvature, the onset of instability shifts to higher frequencies with narrow instability regions of the laminated composite panels under elevated temperature

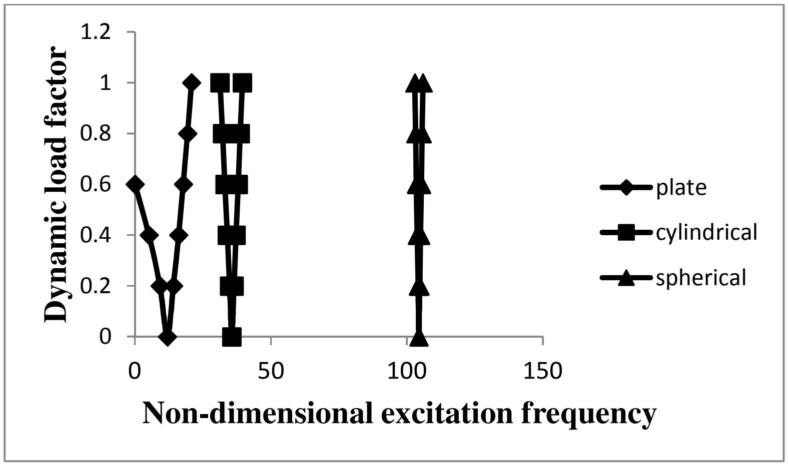

Fig 4: Variations of curvature of composite laminated symmetric cross-ply (0/90/90/0) curved panel with elevated temperature

From the present studies, it is concluded that the instability behaviour of laminate composite plates and shells is greatly influenced by the geometry, lamination parameter and hygrothermal condition. So the designer has to be cautious while dealing with structures subjected to hygrothermal loading. This can be utilized to the advantage of tailoring during design of laminated composite structures in hygrothermal environment.

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