Comparison of Mixing Index for Binary and Ternary Mixtures of Irregular Particles in a Gas-Solid Fluidized Bed

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Abstract

Ternary mixtures of equi-density particles have been fluidized in a gas-solid fluidized bed to study the mixing characteristics. A correlation for the mixing index has been developed for these mixtures. The mixing index for the ternary mixtures has also been calculated by the already available method. Finally a comparison has been made for the values of the mixing index calculated by both the methods against the experimental ones. The values of the mixing index for the ternary mixtures thus obtained have also been compared with those of the binary mixtures thereby indicating its application for real life multi-component mixing over a wide range of parameters.

Key words: Ternary mixture, Mixing index, Take-over velocity, Sauter-mean diameter

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1. Introduction:

Ideal particulate system consisting of mono-sized particles of equal density seldom occurs in practical fluidized bed applications where simultaneous treatment of dissimilar materials is normally encountered in a fairly large number of industrial applications of gas–solid fluidization technology as observed by Nienow and Chiba (1985). Both in the case of chemical conversions and of physical operations, the presence of two or more solids, differing in one or more of their constitutive properties, is often demanded by some specific characteristic of the process; in other situations, having to deal with a multi-component solid charge may quite be the real reason for resorting to fluidization as the most efficient mode for fluid–particle contact. The fluidized bed reactor is extensively used in the process industry for its characteristics of efficient mixing, high rates of mass and heat transfer and capability of continuous operation.

Mixing index concept is an important factor in quantifying the quality of the fluidization phenomenon. Many studies have been made in the past in order to understand the underlying mechanism and predict the mixing behaviors’ including the investigation of factors affecting the mixing and the development of mechanistic models. The ability to predict or regulate the extent of mixing or stratification of a given mixture is an important concern for industrial operations. A remarkable research effort to achieve a clear picture of the fluidization behavior has been made by many investigators in the last two decades. The apparent complexity of the problem is attributable to the large number of factors that are reported to affect the mixing pattern of the system. Further, it is not clear as to how and at what extent the theory of fluidization developed for mono-disperse beds can be adapted to a multi-component system. Indeed, the latter aspect of
the problem has too often been neglected, and this may perhaps explain the lack of generality of many results and relationships so far produced.

A lot of efforts have been made by various groups of researchers in last two decades to explain mixing/segregation in gas-solid fluidized bed for binary mixtures. But in most of the industries the mixtures handled are of multi-component types. The ability to predict or regulate the extent of mixing/segregation stratification of a given mixture is, therefore an important concern for industrial operations. Many researchers have provided the ways to proceed further with various variables, in order to deal with the ternary or multi-component mixtures. Nienow et al. (1978) have reported that behavior of the ternary system could be predicted from the binary system relationships. In practice, the operation of a fluidized bed depends on the properties of the bed materials for which different sizes, densities and shapes may be used.

2. Literature:

Generally, the gas fluidized beds have excellent and rapid mixing characteristics for non-segregating particle systems. Much effort, both experimental and theoretical has been spent in explaining this feature [(Kunii and Levenspiel, 1991) and (Sahoo and Roy, 2005)]. Nienow and Cheesman (1980), Nienow et al. (1978) and Nienow and Chiba (1978) introduced the terms ‘flotsam’ and ‘jetsam’ to describe the solids which occupy the top and the bottom of the bed respectively. Even the denser or the larger particles carried up in the bubble wake from the bottom segregated layer will be shed from the wake and descend rapidly (Naimer et al., 1982).

For binary systems of different densities and sizes as well as those containing particles having a continuous size distribution and of equal density [(Chen and Keairns, 1975), (Nienow et al., 1978a) and (Hoffman and Romp, 1991)], a substantial amount of research investigations
involving mixing phenomenon have been reported on bubble behavior with operation time, superficial gas velocity, particle species and sizes, particle density and mass-fraction of jetsam as well as bed height as the parameters.

Complete mixing of two particle types occurs only when both the components have the identical terminal velocities, i.e. perfect mixing (zero segregation) occurs around. Above and below the line of perfect mixing the degree of mixedness of two species varies from 100% to nearly 0% in the upper region of the bed. The degree of mixing has been quantified by the term the mixing index by several investigators. Nienow et. al. (1978 c) developed the correlation for an index of uniform mixing in terms of depth of penetration of large flotsam particles circulating in the bed.

Knowing the percentage of jetsam at different heights of the bed, the experimental values for the mixing index (M) at those heights have been calculated as per the following expression (Naimer et al., 1982) as mixing index at any section of the bed has been defined as the ratio of the concentration of jetsam particles in that section to the ratio of the jetsam concentration in the whole bed.

\[ M = \frac{X^*}{X_{bed}} \]  

(1)

Where, \( X^* \) is percentage of jetsam particles in any layer and \( X_{bed} \) is percentage of jetsam particles in the bed.

Nienow et al. (1978b) investigated binary, ternary and quaternary systems of different size particles of equal density in a fluidized bed with porous, perforated plate and standpipe distributors. They observed same segregation patterns for binary systems of irregular particles with equal and unequal densities. They have also proposed a correlation for the equilibrium
mixing index for an equal-size, density-variant binary mixture in a three dimensional fluidized bed as follows.

\[ M = \left(1 + e^{-z}\right)^{-1} \]  \hspace{1cm} (2)

where, \( Z = \left(\frac{U - U_{To}}{U - U_F}\right) \times e^{U/UTo} \)

Rice and Brainnovich (1986) modified the equation of Nienow et al. [1978b] as per the following

\[ \frac{U_{To}}{U_M} = 1.0 \times \left(\frac{U_L}{U_M}\right)^{0.46} + 0.47 \times \left(\frac{D_B}{d_{sv}}\right)^{-0.26} \]  \hspace{1cm} (3)

\[ M = 0.5 + 0.5 \text{erf} (Z) \]  \hspace{1cm} (4)

Where \( \text{erf} \) is the error function defined by

\[ \text{erf} (Z) = \left(\frac{2}{\sqrt{\pi}}\right) \times \int_0^z \exp(-y^2) \, dy \]  \hspace{1cm} (5)

With \( y \) being a dummy variable of integration.

\( U_{TO} \) is the take-off or take-over velocity which is the superficial velocity at which the mixing-index is equal to 0.5 (\( M = 0.5 \)) for jetsam-rich conditions.

Although Eq. (4) was an advance over available correlations for a particular type of system, it failed to predict correctly for the systems used by Wu and Baeyens (1998). Therefore they revised the equations as proposed below:

\[ \text{Model-1} : \quad M = \frac{1}{1 + e^{-z^*}} \]  \hspace{1cm} (6)
**Model- 2:** \[ M = a_1 + a_2 Z^* \]  

Where,

\[ Z^* = \frac{U - U_{TO}}{U - U_S} \exp\left( \frac{U}{U_{TO}} \right) \sqrt{f_s} \]  

And \( a_1, a_2 \) are constants.

Where, \( f_s = \frac{d_R}{3} \) for \( U < U_{TO} \) and \( f_s = \frac{3}{d_R} \) for \( U > U_{TO} \)

\[ U_{TO} = (U_S U_L)^{0.5} + (2H/D)^{-0.2} \]

Where, \( U_L \) and \( U_S \) are values of the minimum fluidizing velocity of the larger and the smaller components of the mixture respectively, \( d_R \) is the particle diameter ratio of the larger relative to the smaller particles at the take-over velocity.

The experimental mixing index for equal density particles can also be evaluated by the equation proposed by Chiba et al. (1980) as

\[
M = \frac{\frac{h}{H}}{\int f(d_{SV}) \, dh} \left( f\left(\frac{d_{SV}}{H - h}\right) - f\left(\frac{d_{SV}}{H - h}\right) \right)
\]

Where,

\[ f(d_{SV}) = \frac{(d_{SV} - d_S)}{(d_B - d_S)} \]

In examining the mixing and segregation of equal density systems, Nienow et al. (1987) have proposed a relationship to predict \( U_{TO} \) based on the values of the minimum fluidization velocity of the mixtures and the jetsam particles as follows
\[ U_{TO} = U_M \left( \frac{U_L}{U_M} \right)^{0.5} \] (11)

Wormsbecker et al. (2002) studied the axial profile of the mixing index for the pharmaceutical granules. It was observed that the segregation patterns are because the local velocities in the dilute central core region of the conical bed are less than the terminal velocities of the largest particles in the particle size distribution.

Sahoo and Roy (2008) have proposed a correlation for mixing index for binary mixtures of homogeneous irregular materials (dolomite) as follows

\[ M = 0.8995 \times \left[ \left( \frac{d_L}{d_S} \times \frac{d_M}{d_S} \right)^{0.046} \left( \frac{H_S}{D_C} \right)^{0.059} \left( \frac{h_B}{D_C} \right)^{-0.207} \left( \frac{U}{U - U_M} \right)^{-0.036} \right] \] (12)

3. Experimentation:

A mixture of dolomite particles of irregular sizes, having density of 2860 kg/m\(^3\) and average particle sizes of 1.7mm, 1.09mm, 0.725mm was fluidized in a 100cm height and 14cm inside diameter perspex column. Different sized particles of dolomite were taken in different compositions and filled in the column to a particular height at a time. The mixture was fluidized at a particular fluid mass velocity till it attained steady state. Then the bed was brought to static condition by closing the air supply suddenly. The static bed was then divided into different layers each of two (2.0) cm height. The layers were drawn with the help of vacuum pump and analyzed for the amount of jetsam particles present. Such a system was referred as the static bed condition.
This procedure was repeated with variation in different system parameters. The different system parameters studied and the set-up used for experimentation is shown in Table-1(A) and Fig.-1 respectively.

4. Results and Discussion:

The correlation for the mixing index for ternary mixture was developed using different system parameters on the basis of dimensional analysis as follows.

\[
M = 0.8924 \times \left( \frac{d_p}{d_s} \right)^{0.122} \times \left( \frac{H_s}{D_c} \right)^{-0.062} \times \left( \frac{h_g}{D_c} \right)^{0.015} \times \left( \frac{U - U_M}{U_M} \right)^{0.067}
\] (13)

The correlation plot for the above is shown in Fig-2. The values of minimum fluidization velocity for different mixtures tabulated in Table-1(B) were calculated as per the following.

\[
U_M = \frac{\mu_g}{d_p \rho_g} \times \left[ \left(33.7\right)^2 + 0.0408 \times \frac{d_p^3 \rho_g (\rho_s - \rho_g) g}{\mu_g^2} \right]^{1/2} - 33.7
\] (14)

The effects of individual parameters on mixing were studied [Fig-3 A, B, C and D].

It is observed from Fig-3(A) that M.I decreases with increases of Hs for a particular particle size, dp. It is further observed that M.I decreases with increase in particle sizes initially then increases. As seen from fig -3(A), M.I. for particle sizes of 1.109 and 1.225mm are almost same and less than that for the case of particle size of 1.048mm, where as it is found to be greater for the particle size of 1.237mm.

For a particular size of the ternary mixture four different static bed heights, Hs were cosidered. From this (Fig-3(B)) it is observed that for a particular Hs, M.I. increases in axial direction i.e. with increase in h_B height up to 12cm. Above h_B height of 12cm the effect is observed to be reverse.
Again it is observed M.I. decreases with increase of Hs as seen from Fig-3(B). For static bed heights of 10.5 and 13.5 cm, effect is clear except for the first point whereas M.I. values for Hs of 17.5 and 21.5 cm are almost same. If it is analysed carefully, it is seen that M.I. value is almost constant around 0.9 for all the axial position indicating uniform mixing irrespective of the different static bed heights and particle size of the systems.

It is observed from the Fig-3(C) that M.I. increases with take over velocity, $U_{TO}$ initially then decreases and again increases. But the overall effect is an increasing one. This may be due to the fact that initially particles move upward as the wake materials along with the bubbles, then move downward with breakage of bubbles due to its higher terminal velocity compared to the gas velocity. Again at higher velocity which exceeds terminal velocity jetsam particles move upward causing uniform mixing.

It is observed from the Fig-3(D) that M.I. decreases initially with increase of particle size which may be due to the fact that jetsam particles sinks to the bottom at low flow rates of air. It is also observed that M.I. for larger particle size (fourth point in Fig-3D) is maximum irrespective of different static bed heights.

Although there is variation in the measured values of M.I., it is found that all the values are around 0.9 implying that the effect of Hs, dp, $U_{TO}$ and $h_B$ are not so prominent as seen in correlation, Eq.-13.

The values of the takeover velocity of the equal density ternary mixtures were calculated by eq. (11) and are tabulated in Table-1(B).

Using statistical analysis (factorial design) the following correlation was developed for the mixing index of the ternary mixtures.

\[
M = 0.9386 - 0.0183A + 0.0279B - 0.0042AB - 0.0016C - 0.0002AC - 0.0039BC - 0.0108ABC + 0.0035D \\
- 0.0036AD - 0.0153BD + 0.0039ABD - 0.005CD + 0.0031ACD + 0.0148BCD - 0.0054ABCD
\]  

(15)

Where, $a_0, a_1, a_2, \ldots, a_{23}, a_{123}, a_{234}, a_{1234}$ etc. are the mean effects for different factors and A, B, C, D, AB, AC, ABC, ….ABCD are the factors for factorial design.
Further the values of mixing index were also calculated by eq-(10) (proposed by Chiba et al. 1980) on the basis of area under the plot of $f(d_{sv})$ versus $h_0$. Sixteen sets of data were obtained during the experimentation for which sixteen plots were obtained. Sample plot of $f(d_{sv})$ versus $h_0$ for the different mixture compositions has been presented in Fig.-4. Accordingly the mixing index values were calculated for sixteen runs. Experimental values for mixing index were obtained with the help of eq-(1). Further the mixing index values were calculated as per the above developed correlation, [eq-(13)] for these sixteen data sets. The effects of individual parameters on mixing index were also analysed. The calculated values of mixing index obtained by both the methods [eq-10 and eq-13] have been compared along with the experimental values [obtained with the help of eq-1] in Table-2.

The values of the mixing index for ternary mixtures have also been calculated with the help of the expression developed for the binary mixtures of equal density materials (eq-12) and compared against the values obtained from eq-(13) in Table-2. The experimentally observed data of the binary system (Sahoo & Roy, 2008) has also been used for the verification of the correlation developed by statistical analysis (Eq-15). It is observed that the developed expression (Eq-15) by factorial design can be applied for binary as well as for multi-component systems of the homogeneous mixture of particles. The standard deviation and mean deviation of both the systems i.e. the binary and the ternary mixtures by both the approaches (viz. dimensionless analysis and statistical analysis) have been compared in Table-3. It has been found that the deviation is within +16 to -8% by dimensionless analysis and 9% by statistical analysis implying that the correlation for binary mixtures can be used for ternary mixtures of the similar systems without any modification. The above deviation may be due to the fact that in case of
binary mixtures different sets (ratios) of jetsam and flotsam particles were considered in addition
to different compositions of the mixture whereas in the present study only one set of jetsam and
flotsam particles with different proportions of mixing has been considered for the ternary
mixtures. Further takeover velocity has been considered as the velocity parameter for the ternary
mixtures in eqs-(13 and 15) while minimum fluidization velocity has been considered for the
binary mixtures in eq-(12).

5. Conclusion:

The developed experimental models (eqs-13 and 15) based on dimensional analysis and
statistical analysis approaches can be used for analyzing the mixing characteristics of equal
density ternary mixtures of particles over a wide range of the operating parameters. As in some
reactions better mixing is required, the calculation of mixing index for particular solid mixtures
in which particles vary in size or density is very much essential to know the reactivity of the
components from the beginning.

In the present case, the developed correlation establishes that the concentration of jetsam
particles (and hence the mixing index) decreases with the increase in static bed height and size of
the particles up to 1.225mm. Beyond particle size of 1.225mm the mixing index again increases.
Similarly M.I. increases with increase in axial height of the particle layers up to 12cm measured
from the distributor. Such a result gives the rough idea for selecting the particle size, bed height
and the superficial velocity of the fluid for any process which can be modified accordingly for
industrial applications depending upon the requirements (Wormsbecker et al., 2005).

Although there are not much differences in the values of the mixing index, calculated from the
developed correlations, eqs-(13 and 15) and from eq-(10) when compared with the experimental
ones, but improvements are always possible. More work need to be incorporated to improve the result obtained by developed correlations. Expression of mixing index for the binary mixture can be combined to the ternary one to come out with better result in the form of a single equation. The developed correlation can further be generalized for other kinds of homogeneous/heterogeneous mixtures through further experimentation.

**Nomenclature:**

- \( a_1 \) : a constant, intercept of the plot of \( M \) versus \( Z^* \)
- \( a_2 \) : a constant, slope of the plot of \( M \) versus \( Z^* \)
- \( d_p \) : Particle diameter, mm
- \( \bar{d}_p \) : Average particle diameter of the mixture, m
- \( D_C \) : Diameter of the column, m
- \( D_B \) : Bubble diameter, mm
- \( d_L \) : Larger particle diameter, mm.
- \( d_i \) : Particle diameter, mm
- \( d_M \) : Mean particle diameter, mm
- \( d_R \) : Particle diameter ratio, (larger particle/smaller one, Dimensionless
- \( d_S \) : Smaller particle diameter, mm
- \( d_{SV} \) : Particle sauter mean diameter in the uniform section, mm \( (= 1/\sum (x_i/d_i)) \)
- \( f(d_{SV}) \) : \((d_{SV}-d_S)/(d_B-d_S)\)
- \( f_s \) : particle size ratio correction factor defined by Eq. (8), Dimensionless
- \( g \) : Gravitational constant, m/sec^2
- \( H \) : Bed height, m
- \( h \) : Height from bottom of bed, m
- \( h_B \) : Height of particle layer in the bed measured from the distributor (m)
- \( H_S \) : Initial static bed height (m)
- \( M \) : Mixing index, Dimensionless
- \( U \) : Superficial velocity of the fluidizing medium (m/s)
- \( U_L \) : Minimum fluidization velocity of larger particles in binary mixtures (m/s)
US : Minimum fluidization velocity of smaller particles in binary mixtures (m/s)
UM : Minimum fluidization velocity of binary mixtures (m/s)
UTO : Take-off velocity (critical velocity above which mixing predominates), (m/s)
X* : Percentage of jetsam particle in any layer
X bed : Percentage of jetsam particle in the bed
y : A dummy variable of integration
Z : Reduced gas velocity, Dimensionless
Z* : Dimensionless parameter defined by Eq. (8)
µg : Viscosity of gas, cp
ρs : Density of solid, kg/m³
ρg : Density of gas, kg/m³

Abbreviations:

erf : Error function defined by Eq-5

MI-Exp : Experimental values of Mixing Index
MI-Cal : Calculated values of Mixing Index
MI-LIT : Calculated values of mixing index as per the literature
MI-DA : Calculated values of mixing index as per the dimensional analysis approach
MI-ternary: Calculated values of Mixing Index for ternary mixture.
MI-binary: Calculated values of Mixing Index for binary mixture.

References:


Figure Caption:

Fig. 1(A) : Experimental set-up

Fig. 1(B) : Vacuum system used in the experiment to draw the samples layer wise.

Fig.-2 : Correlation plot for the mixing index of ternary mixtures against the system parameters

Fig.-3 : Effect of different system parameters on mixing index values of the ternary mixtures.

(A) effect of static bed heights

(B) effect of layer height

(C) effect of take over velocity

(D) effect of particle size

Fig.-4 : Sample plots of $f(dsv)$ versus $h_B$ for getting the M.I. as per available literature [13] for the following compositions.

(i) For bed mixture of 20:35:45 composition

(ii) For bed mixture of 40:30:30 composition

(iii) For bed mixture of 45:20:35 composition

(iv) For bed mixture of 30:25:45 composition
### TABLE-1-A (Scope of the Experiment)

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Particle size, $d_p \times 10^3$, m</th>
<th>Composition of the bed mixture</th>
<th>Average particle size, $\overline{d_p} \times 10^3$, m</th>
<th>Initial static bed height, $H_s \times 10^2$, m</th>
<th>Superficial velocity of gas, $U$, m/s</th>
<th>Heights of any layer of particles, $h_B$, $\times 10^2$, m</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.7, 1.09, and 0.725</td>
<td>20:35:45</td>
<td>1.048</td>
<td>21.5</td>
<td>1.35</td>
<td>2,4,6,8,10,12,14,16,18,20</td>
</tr>
<tr>
<td>2</td>
<td>1.7, 1.09, and 0.725</td>
<td>20:35:45</td>
<td>1.048</td>
<td>17.5</td>
<td>1.35</td>
<td>2,4,6,8,10,12,14,16</td>
</tr>
<tr>
<td>3</td>
<td>1.7, 1.09, and 0.725</td>
<td>20:35:45</td>
<td>1.048</td>
<td>10.4</td>
<td>1.35</td>
<td>2,4,6,8,10</td>
</tr>
<tr>
<td>4</td>
<td>1.7, 1.09, and 0.725</td>
<td>20:35:45</td>
<td>1.048</td>
<td>13.5</td>
<td>1.35</td>
<td>2,4,6,8,10,12</td>
</tr>
<tr>
<td>5</td>
<td>1.7, 1.09, and 0.725</td>
<td>40:30:30</td>
<td>1.225</td>
<td>21.5</td>
<td>1.35</td>
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</tr>
<tr>
<td>6</td>
<td>1.7, 1.09, and 0.725</td>
<td>40:30:30</td>
<td>1.225</td>
<td>17.5</td>
<td>1.35</td>
<td>2,4,6,8,10,12,14,16</td>
</tr>
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<td>7</td>
<td>1.7, 1.09, and 0.725</td>
<td>40:30:30</td>
<td>1.225</td>
<td>10.4</td>
<td>1.35</td>
<td>2,4,6,8,10</td>
</tr>
<tr>
<td>8</td>
<td>1.7, 1.09, and 0.725</td>
<td>40:30:30</td>
<td>1.225</td>
<td>13.5</td>
<td>1.35</td>
<td>2,4,6,8,10,12</td>
</tr>
<tr>
<td>9</td>
<td>1.7, 1.09, and 0.725</td>
<td>45:20:35</td>
<td>1.237</td>
<td>21.5</td>
<td>1.35</td>
<td>2,4,6,8,10,12,14,16,18,20</td>
</tr>
<tr>
<td>10</td>
<td>1.7, 1.09, and 0.725</td>
<td>45:20:35</td>
<td>1.237</td>
<td>17.5</td>
<td>1.35</td>
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</tr>
<tr>
<td>11</td>
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<td>1.237</td>
<td>10.4</td>
<td>1.35</td>
<td>2,4,6,8,10</td>
</tr>
<tr>
<td>12</td>
<td>1.7, 1.09, and 0.725</td>
<td>45:20:35</td>
<td>1.237</td>
<td>13.5</td>
<td>1.35</td>
<td>2,4,6,8,10,12</td>
</tr>
<tr>
<td>13</td>
<td>1.7, 1.09, and 0.725</td>
<td>30:25:45</td>
<td>1.109</td>
<td>21.5</td>
<td>1.35</td>
<td>2,4,6,8,10,12,14,16,18,20</td>
</tr>
<tr>
<td>14</td>
<td>1.7, 1.09, and 0.725</td>
<td>30:25:45</td>
<td>1.109</td>
<td>17.5</td>
<td>1.35</td>
<td>2,4,6,8,10,12,14,16</td>
</tr>
<tr>
<td>15</td>
<td>1.7, 1.09, and 0.725</td>
<td>30:25:45</td>
<td>1.109</td>
<td>10.4</td>
<td>1.35</td>
<td>2,4,6,8,10</td>
</tr>
<tr>
<td>16</td>
<td>1.7, 1.09, and 0.725</td>
<td>30:25:45</td>
<td>1.109</td>
<td>13.5</td>
<td>1.35</td>
<td>2,4,6,8,10,12</td>
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</table>

### Table-1-B: (Particle properties)

<table>
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<tr>
<th>Composition of the mixture</th>
<th>Particle size, $d_p \times 10^3$, m</th>
<th>Minimum fluidization velocity, m/s</th>
<th>$U_{TO}$, m/s</th>
<th>$(U-U_{TO})/U_{TO}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>20:35:45</td>
<td>1.048</td>
<td>$U_M = 0.56$</td>
<td>0.75</td>
<td>0.81</td>
</tr>
<tr>
<td>30:25:45</td>
<td>1.109</td>
<td>$U_M = 0.52$</td>
<td>0.73</td>
<td>0.87</td>
</tr>
<tr>
<td>40:30:30</td>
<td>1.225</td>
<td>$U_M = 0.68$</td>
<td>0.83</td>
<td>0.64</td>
</tr>
<tr>
<td>45:20:35</td>
<td>1.237</td>
<td>$U_M = 0.69$</td>
<td>0.84</td>
<td>0.63</td>
</tr>
<tr>
<td>Jetsam particle</td>
<td>1.7</td>
<td>$U_L = 1.01$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flotsam particle</td>
<td>0.725</td>
<td>$U_S = 0.39$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Fig. 1 (A): Experimental set-up

Fig. 1(B): Vacuum system used in the experiment to draw the samples layer wise
Fig.-2 : Correlation plot for the mixing index of ternary mixtures against the system parameters

Fig.-3(A)
Fig.-3(B)

Fig.-3(C)
Fig.-3(D)

Fig.-3: Effect of different system parameters on mixing index values of the ternary mixtures

Fig.-4(i) For bed mixture of 20:35:45 composition
Fig.-4(ii) For bed mixture of 40:30:30 composition

\[ y = 0.713x - 0.248 \]
\[ R^2 = 0.193 \]

Fig.-4(iii) For bed mixture of 45:20:35 composition

\[ y = -0.643x + 0.394 \]
\[ R^2 = 0.051 \]
Fig.-4(iv) For bed mixture of 30:25:45 composition

Fig.-4 Sample plots of f (dsv) versus Hb for getting the M.I. as per available literature [13]

Table-2: Comparison of calculated values of mixing index against the experimental values

<table>
<thead>
<tr>
<th>RUN No.</th>
<th>Avg. M.I.-DA</th>
<th>M.I.-LIT</th>
<th>M.I.-Exp</th>
<th>%dev_DA</th>
<th>%dev-LIT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.917989</td>
<td>0.925585</td>
<td>0.954105</td>
<td>3.79</td>
<td>2.99</td>
</tr>
<tr>
<td>2</td>
<td>0.929774</td>
<td>0.958622</td>
<td>0.957252</td>
<td>2.87</td>
<td>-0.14</td>
</tr>
<tr>
<td>3</td>
<td>0.944849</td>
<td>0.935095</td>
<td>0.940793</td>
<td>-0.43</td>
<td>0.61</td>
</tr>
<tr>
<td>4</td>
<td>0.959647</td>
<td>0.947333</td>
<td>0.963671</td>
<td>0.42</td>
<td>1.70</td>
</tr>
<tr>
<td>5</td>
<td>0.927785</td>
<td>0.921556</td>
<td>0.940111</td>
<td>1.31</td>
<td>1.97</td>
</tr>
<tr>
<td>6</td>
<td>0.923701</td>
<td>0.95445</td>
<td>0.938791</td>
<td>1.61</td>
<td>-1.67</td>
</tr>
<tr>
<td>7</td>
<td>0.908345</td>
<td>0.931026</td>
<td>0.95285</td>
<td>4.67</td>
<td>2.29</td>
</tr>
<tr>
<td>8</td>
<td>0.916892</td>
<td>0.94321</td>
<td>0.949432</td>
<td>3.43</td>
<td>0.66</td>
</tr>
<tr>
<td>9</td>
<td>0.905726</td>
<td>0.921258</td>
<td>0.914321</td>
<td>0.94</td>
<td>-0.76</td>
</tr>
<tr>
<td>10</td>
<td>0.899443</td>
<td>0.954141</td>
<td>0.944556</td>
<td>4.78</td>
<td>1.01</td>
</tr>
<tr>
<td>11</td>
<td>0.954226</td>
<td>0.930724</td>
<td>0.939038</td>
<td>-1.62</td>
<td>0.89</td>
</tr>
<tr>
<td>12</td>
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<td>0.942904</td>
<td>0.939637</td>
<td>-2.32</td>
<td>-0.35</td>
</tr>
<tr>
<td>13</td>
<td>0.932211</td>
<td>0.939174</td>
<td>0.932948</td>
<td>0.079</td>
<td>0.67</td>
</tr>
<tr>
<td>14</td>
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<td>0.972697</td>
<td>0.950172</td>
<td>2.02</td>
<td>-2.37</td>
</tr>
<tr>
<td>15</td>
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<td>0.948825</td>
<td>0.944896</td>
<td>0.949</td>
<td>-0.42</td>
</tr>
<tr>
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<td>0.959851</td>
<td>0.960445</td>
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<td>0.06</td>
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</tbody>
</table>
Table-3:
Averaged error values for each of the approaches presented (with reference to Eqs. 12, 13 and 15) in comparison with the experimental values for different types of bed mixtures.

<table>
<thead>
<tr>
<th>Items</th>
<th>Dimensionless Analysis</th>
<th></th>
<th>Statistical Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ternary Mixture</td>
<td>Binary Mixture</td>
<td>Ternary Mixture</td>
</tr>
<tr>
<td>Standard Deviation, %</td>
<td>3.51</td>
<td>15.53</td>
<td>2.59</td>
</tr>
<tr>
<td>Mean Deviation, %</td>
<td>1.43</td>
<td>-7.61</td>
<td>1.32</td>
</tr>
</tbody>
</table>