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Multi User Chaos Based DS-CDMA System Using Orthogonal Chaotic Vector

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Abstract— **The quasi orthogonal characteristics of chaotic vectors used in conventional chaos based DS CDMA system leads to MAI. The effect of MAI deteriorates the performance of the multi-user communication system. Inorder to eliminate the effect of MAI we propose a scheme which uses orthogonal chaotic vector as spreading to modulate the message data. At the receiver a simple correlator type detector is used. Analytical expressions for BER is derived for the case of AWGN channel and compared with that of simulation results.**

Keywords -orthogonal chaotic vector, chaos, gram-schimdt orthonormalisation, AWGN.

I. INTRODUCTION

Application of chaotic waveforms to digital communication is of research interest for about two decades [1]-[5]. Properties of chaotic waveforms such as sensitive to initial conditions, wide spectrum, very low cross correlation and impulse like auto correlation function makes it very attractive to use as an alternate to conventional spreading sequences in spread spectrum communication.

Due to the quasi-orthogonal nature of the chaotic sequence results in cross correlation estimation problem [8], this causes co-channel interference in multi user communication systems. This co-channel interference due to non zero cross correlation of chaotic spreading sequences is termed as Multiple Access Interference (MAI). The presence of MAI deteriorates the performance of the system even with high signal to noise ratio.

In this paper we present an improved version of multi user chaos based DS – CDMA system using orthogonal chaotic vectors as spreading sequences. The use of orthogonal chaotic vectors as spreading sequence results in zero cross correlation between the users and thus eliminates the effect of MAI. The application of orthogonal chaotic vector for multi level chaotic communication has been previously discussed in [6] and [7].

This paper is organized as follows: Section II describes the system architecture of the proposed system. Performance of the proposed system over AWGN and fading channels are dealt in Section III and Section IV respectively. Finally, we present the simulation and conclusion in Section V and VI respectively.

II. SYSTEM ARCHITECTURE

The transmitter and receiver structure is as shown in Fig. 1. The transmitter contains *Nu* number of chaotic signal generators, where *Nu* is the number of users. The chaotic generators used to generate chaotic vectors can be from different chaotic maps or from the same map with different initial conditions. If $x(k)^{(i)}$ is the chaotic carrier for i^{th} user and defining the number of chaotic samples used to transmit single binary bit as the spreading factor, β. The mean value of chaotic carrier is made equal to zero, in order to avoid unwanted dc power transmission. To generate orthogonal chaotic vectors Gram-Schmidt ortho-normalization process is used and this process is depicted as linear transformation block in the block diagram. Gram-Schmidt ortho-normalization process [8] for *Nu* number of sequences is given by

$$\hat{x}(k)^{(p)} = \frac{x(k)^{(p)} - \sum_{q=1}^{p-1}\left[\sum_{k=1}^{\beta} x(k)^{(p)}\hat{x}(k)^{(q)}\right]\hat{x}(k)^{(q)}}{\sqrt{\sum_{k=1}^{\beta}\left[x(k)^{(p)} - \sum_{q=1}^{p-1}\left[\sum_{k=1}^{\beta} x(k)^{(p)}\hat{x}(k)^{(q)}\right]\hat{x}(k)^{(q)}\right]^2}} \quad (1)$$

Where p = 2, 3. . . *Nu*, For p = 1

$$\hat{x}(k)^{(1)} = \frac{x(k)^{(1)}}{\sqrt{\sum_{k=1}^{\beta}\left[x(k)^{(1)}\right]^2}} \quad (2)$$

Which is then modulated by data sequence $d_l \in \{-1\ +1\}$, assuming that transmitted symbols "+1"(for 1) and "-1"(for 0) are generated with equal probability. Then, modulated sequence is given by,

$$v(k)^{(i)} = \begin{cases} \hat{x}(k)^{(i)}\ if d_l^{(i)} = +1 \\ -\hat{x}(k)^{(i)}\ if d_l^{(i)} = -1 \end{cases} \quad (3)$$

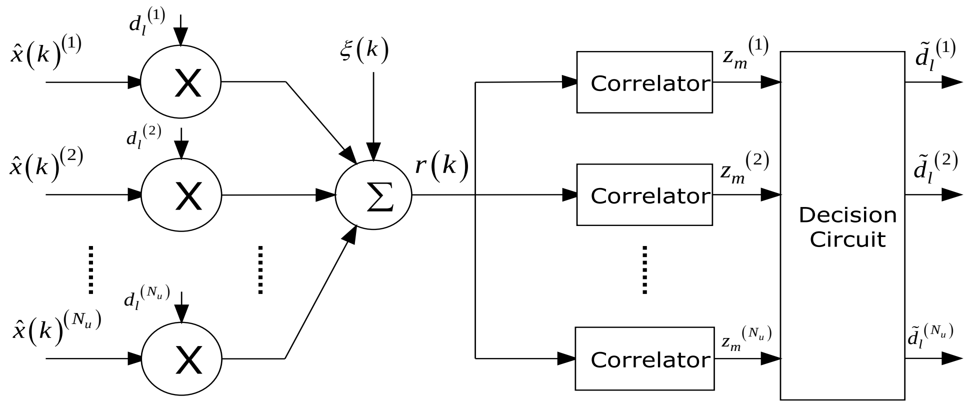

Fig. 1. Transmitter and Receiver structure

The transmitted signal $s(k)$ is the sum of modulated orthogonal chaotic vectors $\hat{x}(k)^{(i)}$ of each user. Given by,

$$s(k)=\sum_{i=1}^{N_u} v(k)^{(i)} \tag{4}$$

At receiver simple correlator type detection is used.

III. Performance Analysis Over AWGN Channel

Assuming that the signal is corrupted only due to AWGN, received signal *r(k)* can be represented as

$$r(k)=\sum_{i=1}^{N_u} v(k)^{(i)}+\xi(k) \tag{5}$$

Where $\xi(k)$ represents the additive white Gaussian noise with zero mean and variance *No/2*. At receiver it is assumed that exact replica of carrier is available and it is exactly synchronized with the transmitter then, the m^{th} decoded symbol for the j^{th} user, denoted by $\tilde{d}_m^{(j)}$, is determined according to the rule:

$$\tilde{d}_m^{(j)}=\begin{cases}+1 \; if \; \tilde{z}_m^{(j)}=\sum_{k=1}^{\beta} r(k)\hat{x}(k)^{(j)} > 0\\ -1 \; if \; \tilde{z}_m^{(j)}=\sum_{k=1}^{\beta} r(k)\hat{x}(k)^{(j)} \le 0\end{cases} \tag{6}$$

A. Derivation of BER:

Consider the j^{th} user. Without the loss of generality, we consider the probability of error for the first symbol. For brevity, the subscripts of the variables $\tilde{d}_m^{(j)}$ and $z_m^{(j)}$ are omitted. The decision parameter of the j^{th} user is given by

$$z^{(j)}=A+B+C \tag{7}$$

Where,

$$A=d^{(j)}\sum_{k=1}^{\beta}\left[\hat{x}(k)^{(j)}\right]^2$$

$$B=d^{(j)}\sum_{i=1,i\neq j}^{N_u}\sum_{k=1}^{\beta}\left(\hat{x}(k)^{(i)}\,\hat{x}(k)^{(j)}\right)$$

$$C=\sum_{k=1}^{\beta}\xi(k)\hat{x}(k)^{(j)}$$

Since, chaotic vectors used for each user is ortho-normal to each other, the second term in eq. (7) will be equal to zero

$$\left(\hat{x}(k)^{(i)}\,\hat{x}(k)^{(j)}\right)=0 \text{ for } i\neq j \tag{8}$$

Therefore,

$$z^{(j)}=A+C \tag{9}$$

Assuming, that $z^{(j)}$ has a Gaussian distribution, the BER for *j*th user can be written as

$$BER^{(j)}=\frac{1}{2}erfc\left(\frac{E\left(z^{(j)}\mid d^{(j)}=+1\right)}{\sqrt{\left(2\operatorname{var}\left(z^{(j)}\mid d^{(j)}=+1\right)\right)}}\right)$$
$$=\frac{1}{2}erfc\left(\frac{-E\left(z^{(j)}\mid d^{(j)}=-1\right)}{\sqrt{\left(2\operatorname{var}\left(z^{(j)}\mid d^{(j)}=-1\right)\right)}}\right) \tag{10}$$

Where, mean value of $\left(z^{(j)}\mid d^{(j)}=+1\right)$ is given by

$$E\left[z^{(j)}\mid d^{(j)}=+1\right]=\beta E\left[\left(x(k)\right)^2\right] \tag{11}$$

Where, E_b is energy per bit. And variance is given by

$$\operatorname{var}\left(z^{(j)}\mid d^{(j)}=+1\right)= \operatorname{var}\left[\sum_{k=1}^{\beta}\left[\hat{x}(k)^{(j)}\right]^2\right]+ \beta\frac{N_o}{2}E\left[\left(\hat{x}(k)^{(j)}\right)^2\right] \tag{12}$$

Since, the spreading sequences used for each user are ortho-normal to each other we can write,

$$E\left[\hat{x}(k)^{(j)}\right]=0 \tag{13}$$

$$E\left[\left(\hat{x}(k)^{(j)}\right)^2\right]=\frac{1}{\beta}=P_s \tag{14}$$

$$\operatorname{var}\left[\hat{x}(k)^{(j)}\right]=\frac{1}{\beta} \tag{15}$$

$$\operatorname{var}\left[\sum_{k=1}^{\beta}\left[\hat{x}(k)^{(j)}\right]^2\right]=0 \tag{16}$$

Where, P_s is average power transmitted by each carrier.
From equations 10 to 16 we can write,

$$BER^{(j)}=\frac{1}{2}erfc\left(\sqrt{\frac{E_b}{N_o}}\right) \tag{17}$$

From equation (17) it is clear that BER performance of the proposed system does not depends on number of users or spread factor.

From Fig. 2 we see that the BER performance of the system is better than that of the multi user DS-CDMA system which uses non orthogonal chaotic sequences for small values of spread factor. For larger values of spread factor both the system have same BER performance. From

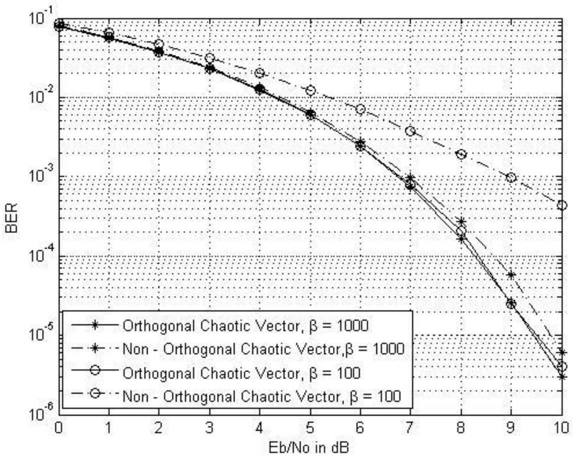

Fig. 2. Comparison of BERs v/s Eb/No plots for number of users = 5

Fig. 3 we observe that BER performance is not affected by the number of users when orthogonal chaotic vectors are used but for the system using non orthogonal chaotic vectors the BER performance deteriorates as number of user's increases. Fig. 4 gives the comparison of theoretical and simulation results and we see that the theoretical and simulated results are in close agreement thus confirming the procedure carried out for evaluation of theoretical BER.

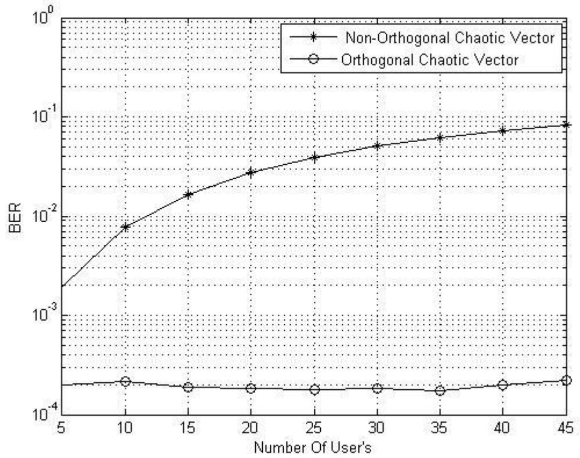

Fig. 3. Comparison of BER v/s Number of users plots for Eb/No = 8 dB

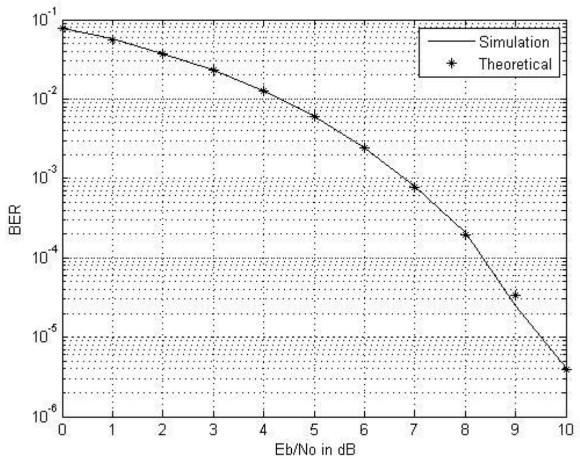

Fig. 4. Simulated and theoretical BERs versus Eb/No, for Number of users = 5 and β = 100

IV. SIMULATION

For generating chaotic sequence chebyshev map of order three is used, which is given by,

$$x(k+1)=4x^3(k)-3x(k) \tag{18}$$

To test the performance of the system we ran a Monte-Carlo simulation by transmitting 1,000,000 bits with equal probability for each user and the number of bits in error for each user is found and the average BER per user gives the BER of the complete system. The initial condition for chaotic sequence generator is chosen such that the map operates in chaotic regime.

V. CONCLUSION

In this paper, a new improved multi user chaos based DS-CDMA system using orthogonal chaotic vectors has been developed and analysed. We also compared the results of the proposed system with that of system using non orthogonal chaotic vectors and found that the use of orthogonal chaotic vector gives better performance for small values of spread factor, β. Theoretical expression for BER is derived for the case of AWGN channel. It is found that theoretical result matches with simulated results thus validating the analysis carried out.

Though the coherent chaotic communication system presented lacks practical implementation but, it provides the benchmark for evaluating the non coherent chaotic communication schemes which uses orthogonal chaotic vectors as spreading sequences.

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