A Study on Multi-Objective Evolutionary Algorithms and its Applications to Economics and Finance

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Abstract:- *The use of evolutionary algorithms in diversified application domains has gained ever increasing popularity in the last few years producing a wide range of interesting applications ranging from engineering and computer science to ecology, sociology and medicine. From these diversified application areas of evolutionary algorithms, economics and finance constitutes a very promising field. The use of evolutionary algorithms for solving multi-objective optimization problem emerges as a potential field of research in recent years. This paper presents the use of multi-objective evolutionary algorithms (MOEAs) for solving problems in economics and finance. Different applications of MOEA are explained briefly and a specific simulation work has been done for one particular application i.e. investment portfolio optimization.*

Index Terms— Multi-objective optimization, Pareto-optimal solutions, global optimization, Crowding distance, Pareto front.

I. INTRODUCTION

This paper presents a survey on the use of multi-objective evolutionary algorithms (MOEAs) for solving problems in economics and finance. The use of MOEAs in this research field is relatively rare as compared to single-objective evolutionary algorithms [2].One of the main motive of this survey is to attract the attention of EMOO researchers towards this field. Excellent survey has been done by Ma. Guadalupe Castillo Tapia and Carlos A. Coello Coello [2] in this regard. The paper is organized in a hierarchical order in which section II presents basic concepts of multi-objective optimization. Similarly section III includes many areas of applications and their brief description. Section IV outlines two well know multi-objective algorithms i.e. NSGA II and Micro-GA and the investigation of these two MOEAs in Section V. Conclusions and further directions in which research work to be carried out are discussed in the Section VI.

II. BASIC CONCEPTS

In a single-objective optimization problem, an optimal solution is the one which optimizes the objective with certain associated constraints. It is not possible to find a single solution for a multiobjective problem and due to the contradictory objectives a set of solutions is obtained. The solution to a multi-objective optimization problem is a set of vectors are not dominated by any other vector, and Pareto-equivalent to each other. This set is known as the Pareto-optimal set. Grouping these Pareto optimal set generates a plot, often discontinuous known as the Pareto front or Pareto border. Its name refers to Vilfredo Pareto [1], who generalized these concepts in 1896.

III. MULTIOBJECTIVE OPTIMIZATION (MOO)

Most real world optimization problems require to make decisions involving more than one goal. A Multiobjective Optimization Problem (MOOP) is defined as the problem of finding a vector of decision variables that satisfies some restrictions and optimize a vector function whose elements represent the values of the functions. A MOOP may be formulated as:

Maximize / minimize: $f_m(x)$ Where $m = 1,2,\ldots M$

Subjected to: $g_j(x) \geq 0$, $j = 1,2,\ldots J$ and

$h_k(x) = 0$, $k = 1,2,\ldots K$, $x_i^L \leq x_i \leq x_i^U$, $i = 1,2\ldots n$ .Where x is the vector of decision variables $x = (x_1, x_2, \ldots, x_n)^T$ and $f_m(x)$ are the m objective functions. The values x_i^L and x_i^U represent respectively the minimum and maximum acceptable values for the variable. These values define the boundary of the search space. The J inequalities g_j and the K equalities h_k are known as constrain functions.

IV. AREA OF APPLICATIONS

The taxonomy of applications of MOEAs in economics and finance are:

A. Investment portfolio optimization
B. Financial time series
C. Stock ranking

D. Risk-Return analysis
E. Economic modeling
F. Model discovery
G. Data mining
H. Forecasting stock prices
I. Risk management

A.Investment Portfolio Optimization

One of the most promising fields of application is investment portfolio optimization. It can vary from simple portfolios held by individuals to huge portfolios managed by professional investors. The portfolio contains stocks, bank investments, real estate holdings, bonds, treasury bills etc. The moto of it is to find an optimal set to invest on, as well as the optimal investment for each asset. This optimal selection and weighting is a multi-objective problem where total profit of investment has to be maximized and total risk is to be minimized. There are also different constraints, depending on the type of problem to be solved. For example, the weights normally have lower bounds, upper bounds and many other constraints. This is the so-called optimal investment portfolio that one wishes to obtain by using optimization techniques. This problem is traditionally studied using the Markowitz portfolio selection model [3].

B.Finantial Time Series

In this application, the idea is to find patterns in financial time series, such that predictions can be made regarding the behavior of a certain stock. Different MOEAs have also been reported in this application domain. Ruspini and Zwir [4] used the Niched-Pareto Genetic Algorithm (NPGA) [5] for this purpose. The authors apply their methodology to the identification of significant technical-analysis patterns in financial time series.

C.Stock Ranking

The aim of this problem is to classify stocks as strong or weak performers based on technical indicators and then use this information to select stocks for investment and for making recommendations to customers. Many MOEAs has been reported in this application area. Mullei and Beling [6] use a GA with a linear combination of weights to select rules for a classifier system adopted to rank stocks based on profitability.

D.Risk-Return analysis

It is slight different from risk-return trade up made in investment portfolio. Credit portfolios handled by banks operate under different rules and therefore they are not modeled using the original Markowitz approach. Schlottmann and Seese [7] use an approach similar to the NSGA-II [8] for solving portfolio selection problems relevant to real-world banking. In the problem studied by the authors, a bank has a fixed supervisory capital budget.

E.Economic Modelling

Mardle et al. [9] use a GA with a weighted goal programming approach to optimize a fishery bioeconomic model. Bioeconomic models have been developed for a number of fisheries as a means of estimating the optimal level of exploitation of the resource and for assessing the effectiveness of the different management plans available.

F.Model discovery

This is an interesting area in econometrics in which non-parametric models are assumed and one tries to use an evolutionary algorithm to derive a model for a certain type of problem (e.g., forecasting nonlinear time series). Normally, artificial neural networks (ANNs) have been used for the model itself, but several researchers have used evolutionary algorithms to find the most appropriate ANN that models the problem of interest.

G.Data mining

The use of data mining techniques for learning complex patterns is a very promising research area in economics and finance. For example, the mining of financial time-series for finding patterns that can provide trading decision models is a very promising topic[10].

H.Forecasting stock prices

Although long-term forecasting is not possible for the stock market, it is normally possible to perform short-term forecasting with heuristics. The use of genetic programming (GP) in this area has become increasingly popular, since GP can be used for symbolic regression, emulating the tasks traditionally performed by ANNs.

I.Risk management

The study of risk and the reaction of an agent to it, is a very interesting research area. Some researchers have studied, for example, the formation process of risk preferences in financial problems [10].

V. MULTIOBJECTIVE EVOLUTIONARY ALGORITHMS

This section deals with the basics of two standards multi-objective algorithms which are used in this paper. The steps involved in two different multi-objective algorithms are outlined in sequel.

A.Nondominated Sorting Genetic Algorithm II

Dev and Pratab [8] have proposed NSGA II where selection criteria is based on the crowding comparison operator. Here the pool of individuals is split into different fronts and each front has assigned a specific rank. All individuals from a front are ordered according to a crowding measure which is equal to the sum of distance to the two closest individuals along each objective. The environmental selection is processed based on these ranks. The archive will be formed by the non dominated individuals from each front and it begins with the best ranking front.

NSGA II Algorithm:

1. Initialize population
2. Generate random parent population p_0 of size N
3. Evaluate objective Values
4. Assign fitness (or rank) equal to its non dominated level
5. Generate offspring Population Q_0 of size N with binary tournament selection, recombination and mutation.
6. For $t=1$ to Number of Generations
6.1 Combine Parent and Offspring Populations
6.2 Assign Rank (level) based on Pareto Dominance.
6.3 Generate sets of non-dominated fronts
6.4 until the parent population is filled do
6.4.1 Determine Crowding distance between points on each front F_i
6.4.2 Include the ith non dominated front in the next parent population (P_{t+1})
6.4.3 check the next front for inclusion
6.5 Sort the front in descending order using Crowded comparison operator
6.6 Choose the first N - card (P_{t+1}) elements from front and include them in the next parent population (P_{t+1})
6.7Using binary tournament selection, recombination and mutation create next generation
7. Return to 6

B. Micro-GA Algorithm

The micro genetic algorithm employs a small population and involves a reinitialization process. Initially random population is generated which is feeds to the population memory. It is divided in two parts (i) a replaceable and (ii) a nonreplaceable portion. The nonreplaceable portion of the population memory remain unchanged during the entire run and provides the required diversity. But the other portion undergoes changes after each cycle. The microGA uses three forms of elitism[8]: (i) it retains nondominated solutions found within the internal cycle.(ii) it uses a replaceable memory whose contents is partially refreshed at certain intervals and (iii) it replaces the population by the best solutions found after a full internal cycle of the microGA.

The steps of the algorithm are as follows.

1. Generate initial population P of size N
2. Store its contents in the population memory M
3. Divide the population memory M in to replaceable and nonreplaceable parts.
4. For $t=1$ to number of generations
4.1. Obtain the initial population of micro-GA (P_i) from M
4.2. Apply the binary tournament selection based on nondominance.
4.3. Apply two point crossover and uniform mutation to the selected individual
4.4. Apply elitism and create next generation
4.5. Until nominal convergence is reached copy two nondominated vectors from P_i to the external memory E.
4.6. Use adaptive grid when E is full .
4.7. Copy two nondominated vectors from P_i to M
5. Return to step 4

VI. MOEAs APPLICATION TO INVESTMENT PORTFOLIO OPTIMIZATION

Portfolio p consisting of N assets. The basic mean-variance portfolio selection problem can be formalized as:

$$\text{Min } V(w) = W^T Q W \quad (1)$$

$$\text{Max } W^T \mu = E \quad (2)$$

$$W^T e = 1 \quad (3)$$

$$0 \le w_i \le 1 \text{ and } i = 1,2...,N \quad (4)$$

Where N is the number of assets available Q denotes the covariance matrix of all investment alternatives, μ_i is the expected return of asset i and e is the unit vector. The decision variables w_i determines what share of the budget should be distributed in asset i. Here $W = \{w_1 w_2 w_3 ... w_N\}$ and equation 1 and 2 give the two competing objectives which are to be optimized. Equations 3 and 4 show the constraints for a feasible portfolio

In this paper we consider a multi-objective portfolio assets selection and optimal weighting of assets where the total profit is maximized while total risk is minimized simultaneously. The present study employs NSGA II and micro-GA for modeling the Pareto front and for optimizing the portfolio performance. The results obtained with these two algorithms are finally compared by performing different numerical experiments.

A. Simulation studies

In this section we present the simulation results obtained when searching the general efficient frontier that resolves the problem formulated in equation 1 and 2. The efficient frontier is computed using NSGA II and Micro-GA. All the computational experiments have been computed with a set of benchmark data available online and obtained from OR-Library being maintained by Prof. Beasley. Five data sets port1 to port5 represent the portfolio problem. Each data set corresponds to a different stock market of the world. The test data comprises of weekly prices from March 1992 to September 1997 from the following indices: Hang Seng in Hong Kong, DAX 100 in Germany, FTSE 100 in UK, S&P 100 in USA and Nikkei in Japan. For each set of test data, the numbers of different assets are 31,85,89,98 and 225. In the paper we have used the first data set which corresponds to Hang Seng stock having 31 assets. The data can be found at http://people.brunel.ac.uk/~mastjjb/jeb/orlib /portinfo.html. The NSGA II has population size of 100, number of generations 100, crossover rate 0.8 and mutation rate 0.05. The number of real-coded variables is equal to number of assets and the selection strategy used is tournament selection. . In microGA an external memory of 100 individuals, a number of iterations to achieve nominal convergence, a population memory of 50 individuals, five percentage of non replaceable memory, a population size of four individuals and 25 subdivisions of the adaptive grid are used. The crossover rate of 0.9 and mutation rate of $1/L$ (L = length of the chromosome string) are chosen.

B. Performance Measures for Comparison

1. S metric

The S metric proposed in [11] indicates the extent of objective space dominated by a given nondominated set A. If the S metric of a non dominated front f_1 is less than another front f_2 then f_1 is better than f_2. It has been proposed by Zitzler [11].

2. Δ metric

This metric called as spacing metric (Δ) measures how evenly the points in the approximation set are distributed in the objective space. This formulation introduced by K. Deb [8] is given by

$$\Delta = \frac{d_f + d_l + \sum_{i=1}^{N-1} \left| d_i - \bar{d} \right|}{d_f + d_l + (N-1)\bar{d}} \quad (5)$$

Where d_i be the Euclidean distance between consecutive solutions in the obtained nondominated set of solutions. $\bar{d}$ is the average of these distances. d_f and d_l are the Euclidean distance between the extreme solutions and the boundary solutions of the obtained non dominated set and N is the number of solutions from nondominated set. The low value for Δ indicate a better diversity and hence better is the algorithm.

3. C metric

Two sets of non dominated solutions are compared using C metric. The definition of C metric given in [11] for convergence of two sets A and B is given by:

$$C(A,B) = \frac{\left|\{b \in B \mid \exists a \in A : a \succ b\}\right|}{|B|} \quad (6)$$

C. Simulation results and Pareto fronts of the two MOEA algorithms

TABLE I
THE RESULTS OBTAINED FOR S AND Δ METRICS

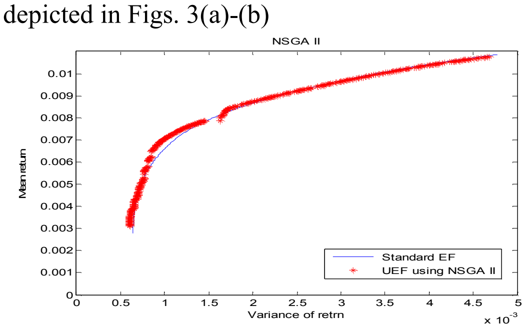
Algorithm	NSGA II	Micro-GA
Metric S	0.0000057624	0.0000067874
Metric Δ	0.5967844252	0.8227976192

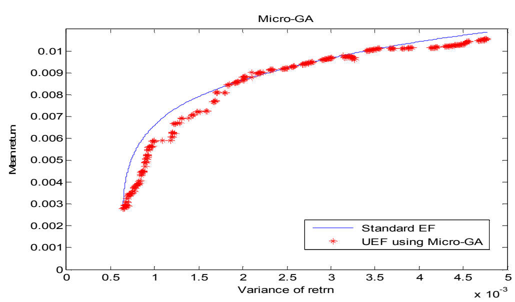
Table I shows the S and Δ metrics obtained using two algorithms. It may be observed from the Table I that NSGA II performs better as its S and Δ metric values are less than those obtained by Micro-GA algorithms.

TABLE II
THE RESULTS OBTAINED FOR C METRIC

	NSGA II	Micro-GA
NSGA II	—	0.9656
Micro-GA	0.4543	—

Table II demonstrates the results of C metric. A magnitude of 0.9656 on the first line, first column signifies that almost all of the solutions from final populations obtained by NSGA II dominate the solutions obtained by Micri-GA. The standard efficient frontier corresponding to Hang Seng benchmark problem and pareto fronts (between risk and return) obtained by two algorithms are depicted in Figs. 3(a)-(b)





(a)NSGA II (b) Micro-GA
Fig.Plots of Pareto fronts achieved by two methods

VII.CONCLUSION

This paper has presented a state-of-the-art study on the use of MOEAs for solving problems in economics and finance. We have identified a taxonomy of applications that consists of ten large groups. From these ten groups, the first (investment portfolio optimization) is most popular and here we have shown two multiobjective evolutionary algorithms NSGA II and Micri-GA for solving the bi-objective portfolio optimization problem. The paper makes a comparative study between these two algorithms. Experimental results reveal that the NSGA II algorithm outperforms Micro-GA algorithms in different experiments conducted. Future work include introduction of different operators for local search in the existing models which allow better exploration and exploitation of the search space when applied to portfolio optimization problem. We expect that this paper can motivate researchers interested in economics and finance to learn more about MOEAs, and to apply them in more problems within these areas.

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