

Image Compression Using Discrete Tchebichef Transform Algorithm

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Abstract— The Discrete Tchebichef Transform (DTT) based on orthogonal Tchebichef polynomials can be an alternative to Discrete Cosine Transform (DCT) for JPEG image compression standard. The properties of DTT are not only very similar to DCT; it has also higher energy compactness and lower computational advantage using a set of recurrence relation. Through extensive simulation, image reconstruction accuracy (Peak Signal to Noise Ratio) and the number of bits required to encode the coefficients for both DCT and DTT is verified. It has been demonstrated that, DTT requires lesser number of bits to encode the coefficients than DCT for a given compression ratio.

Keywords— DCT, DTT, Image compression, Peak signal to noise ratio (PSNR).

I. INTRODUCTION

Image Transform methods using orthogonal kernel functions are commonly used in image compression. One of the most widely known image transform method is Discrete Cosine Transform (DCT), used in JPEG compression standard [1]. The computing devices such as Personal digital assistants (PDAs), digital cameras and mobile phones require a lot of image transmission and processing. Therefore, it is essential to have efficient image compression techniques, which could be scalable and applicable to these smaller portable devices.

A new class of transform called Discrete Tchebichef Transform (DTT), which is derived from a discrete class of popular Tchebichef polynomials, is a novel orthonormal version of orthogonal transform. It has found applications on image analysis and compression [2-9]. The Tchebichef moment compression that is proposed in this paper is meant for smaller computing devices owing to its low computational complexity [3]. R.Mukundan [4] has proposed orthonormal version of Tchebichef moments and analysed some of their computational aspects. Mukundan has also shown details of various computational aspects of Tchebichef moments and their feature representation capability using methods of image reconstruction [5]. A block wise moment computation scheme which avoids numerical instabilities to yield a perfect reconstruction has been introduced in the literature [6]. Mukundan and Hunt [7] have shown that, for natural images, DTT and DCT exhibit similar energy compactness performance. It was very difficult to determine which of the two is better. Lang et al. [8] made a comparison between 4x4 Tchebichef moment transform and DCT. They have shown

that, there is a significant advantage for 4x4 Tchebichef moments in terms of error reconstruction and average length of Huffman codes. For computation of Tchebichef moments, a number of fast algorithms have been proposed [4], [9-10]. Ishwar et al. [9], have shown that DTT has lower complexity since it requires the evaluation of only algebraic (only add and shift operations, no multiplications) expressions whereas; implementation of DCT requires integer approximation or intermediate scaling, like Integer cosine transform (ICT) [11]. Some important characteristics of DTT are summarized as follows:

- A discrete domain of definition which matches exactly with image coordinates space.
- Absence of numerical approximation terms allows a more accurate representation of image features than others which is not possible using conventional transforms.

In this paper a 2-dimensional (2-D) 8x8 forward discrete Tchebichef transform algorithm is implemented in place of 8x8, 2-D DCT algorithm on a standard baseline JPEG encoder. The compression performance, in terms of number of encoded bits and peak signal to noise ratio (PSNR) of the proposed system is compared with that of DCT based system.

The organization of the paper is as follows: Section II presents the introduction about Discrete Image Transform. Section III describes the mathematical definition of Tchebichef Transform Algorithm. Similarities between DTT and DCT are compared in section IV. Implementation of Forward Discrete Tchebichef Transform (FDTT) algorithm on 8x8 blocks of image data is presented in section V. Section VI discusses the simulation result and finally in section VII conclusion and future work is given.

II. DISCRETE IMAGE TRANSFORM

From the point wise definition, the transform of an N X N pixel image is given by

$$\tau(u, v) = \sum_x \sum_y^{N-1} f(x, y) g(x, y, u, v) \quad (1)$$

$$f'(x, y) = \sum_u \sum_v^{N-1} \tau(u, v) h(x, y, u, v) \quad (2)$$

Equation (1) defines the image transform τ where u and v represents the frequencies in the transform domain, f is the image being transformed, f' is the inverse transform and $g(x, y, u, v)$ is the basis function used by the transform. Equation (2) defines the inverse transform of $h(x, y, u, v)$, where $h(x, y, u, v)$ represents the inverse of the basis function $g(x, y, u, v)$. The transform used in this study define their kernel (basis) function to be a product of two-dimensional function $g'(i, j)$ such that, $g(x, y, u, v) = g'(x, u)g'(y, v)$.

III. DISCRETE TCHEBICHEF TRANSFORM

The Discrete Tchebichef Transform (DTT) is relatively a new transform that uses the Tchebichef moments to provide a basis matrix. As with DCT, the DTT is derived from the orthonormal Tchebichef polynomials [5]. For image of size $N \times N$, the forward Discrete Tchebichef Transform of order $u+v$ is defined as:

$$T_{uv} = \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} t_u(x)t_v(y)f(x, y) \quad (3)$$

$u, v, x, y = 0, \dots, N-1.$

The inverse transformation of DTT is defined by:

$$f(x, y) = \sum_{u=0}^{N-1} \sum_{v=0}^{N-1} T_{uv}t_u(x)t_v(y) \quad (4)$$

$x, y, u, v = 0, 1, \dots, N-1.$

Equation (4) can also be expressed using a series representation involving matrices as follows:

$$f(i, j) = \sum_{u=0}^{N-1} \sum_{v=0}^{N-1} T_{uv}G_{uv}(i, j) \quad (5)$$

$u, v = 0, 1, \dots, N-1.$

where, G_{uv} is an 8×8 matrix (called basis images) and is defined as:

$$G_{uv} = \begin{bmatrix} t_v(0)t_v(0) & t_u(0)t_v(1) & \dots & \dots & t_u(0)t_v(7) \\ t_u(1)t_v(0) & t_u(1)t_v(1) & \dots & \dots & t_u(1)t_v(7) \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ t_u(7)t_v(0) & t_u(7)t_v(1) & \dots & \dots & t_u(7)t_v(7) \end{bmatrix}$$

The basis function of the DTT is defined as follows:

$$g'(x, u) = t_u(x). \quad (6)$$

where, $t_u(x)$ is the u^{th} order of the Tchebichef moments.

These can be defined using the following function over the discrete range $[0, N)$.

$$t_u(x) = u! \sum_{k=0}^u (-1)^{u-k} \binom{N-1-k}{u-k} \binom{u+k}{u} \binom{x}{k} \quad (7)$$

Due to the large dynamic range of the intermediate values generated by (7), it is not feasible to calculate the values of DTT on a point wise basis. Instead, we calculate the function using the following recurrence relation:

$$t_0(x) = \frac{1}{\sqrt{N}} \quad (8)$$

$$t_1(x) = (2x+1-N) \sqrt{\frac{3}{N(N^2-1)}} \quad (9)$$

$$t_u(x) = (A_1x + A_2)t_{u-1}(x) + A_3t_{u-2}(x) \quad (10)$$

where, $u \in [2..N)$, Coefficients A_1, A_2 and A_3 are as follows:

$$A_1 = \frac{2}{u} \sqrt{\frac{4u^2-1}{N^2-u^2}},$$

$$A_2 = \frac{1-N}{u} \sqrt{\frac{4u^2-1}{N^2-u^2}},$$

$$A_3 = \frac{u-1}{u} \sqrt{\frac{2u+1}{2u-3}} \sqrt{\frac{N^2-(u-1)^2}{N^2-u^2}}.$$

As noted by [4], the recurrence relation causes minor numerical errors to propagate through calculation. This error eventually manifests itself in the collapse of the basis function [7]. This problem is only apparent in this image as we are performing the transform over the entire image, rather than on a block-by-block basis.

IV. SIMILARITY IN PROPERTIES OF DTT AND DCT

A. Separability

The definition of DTT can be written in separable form as

$$T_{uv} = \sum_{x=0}^{N-1} t_u(x) \sum_{y=0}^{N-1} t_v(y) f(x, y). \quad (11)$$

Therefore, it can be evaluated using two dimensional transforms as follows:

$$g_v(x) = \sum_{y=0}^{N-1} t_v(y) f(x, y), \quad (12)$$

$$T_{uv} = \sum_{x=0}^{N-1} t_u(x)g_v(x). \quad (13)$$

The transform equation of DCT can be expressed as follows:

$$C(u,v) = \alpha(u)\alpha(v) \sum_{x=0}^{N-1} \cos\left[\frac{\pi(2x+1)u}{2N}\right] \sum_{y=0}^{N-1} \cos\left[\frac{\pi(2y+1)v}{2N}\right],$$

where

$$\alpha(u)\alpha(v) = \begin{cases} \sqrt{\frac{1}{N}} & \text{for } u, v = 0 \\ \sqrt{\frac{2}{N}} & \text{otherwise} \end{cases}. \quad (14)$$

From (13) and (14) it is clear that 2-D DTT and 2-D DCT are just one dimensional DTT and DCT applied twice by successive 1-D operations, once in x-direction, and the once in y-direction.

B. Even Symmetry

From [4], it can be shown that Tchebichef polynomials satisfy the property

$$t_p(N-1-x) = (-1)^p t_p(x), \quad p = 0, 1, \dots, N-1. \quad (15)$$

For DCT:

$$C_m(n) = (-1)^m C_m(N-n-1), \quad m = 0, 1, \dots, N-1. \quad (16)$$

The above two properties are commonly used in transform coding methods to get substantial reduction in the number of arithmetic operations.

C. Orthogonality

DTT and DCT basis functions are orthogonal. Therefore, this property renders some reduction in the pre-computation complexity. Fig. 1 (a) and (b) shows the 2-D basis images for the DTT and DCT.

In the basis images, it has been observed that the low frequencies reside in the upper left corner of the spectrum, while the high frequencies are in the lower right. The basis functions for rows are increasing frequencies in horizontal directions while the basis functions for columns are increasing frequencies in vertical directions.

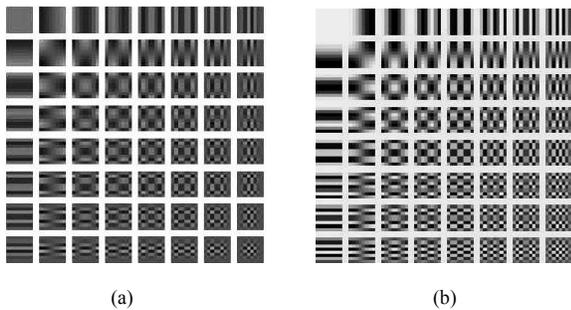


Fig. 1. Two dimensional Basis Image for (a) DTT, (b) DCT .

D. Energy Compaction

Efficiency of a transformation scheme can be gauged by its ability to pack input energy into as few coefficients as possible. Further, the quantizer discard coefficients with relatively small amplitudes without introducing visual distortion in the reconstructed image. DTT and DCT exhibit excellent energy compaction properties for highly correlated images. The energy of the image is packed into low frequency region (i.e. top left region).

V. JPEG BASELINE CODING VERSUS TCHEBICHEF TRANSFORM COMPRESSION

JPEG is an international compression standard which is designed to support a wide variety of applications for continuous-tone images. JPEG baseline which is based on DCT is a lossy technique commonly used today and is sufficient for a large number of applications [1].

Using the JPEG Compression platform, 8×8 forward DTT has been used in place of DCT. Fig. 2 shows how the FDTT is performed on the 8×8 blocks of image data in order to achieve good compression performance.

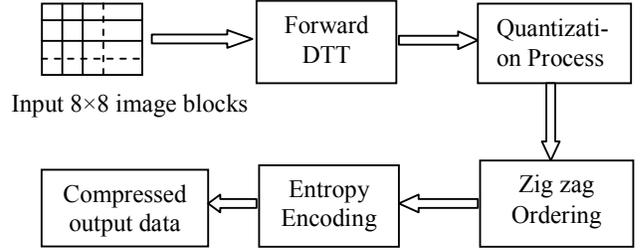


Fig. 2. Implementation of FDTT on the 8×8 blocks of image data.

To apply FDTT, the image is divided into 8×8 blocks of pixels. The 8×8 blocks are processed from left-to-right and from top-to-bottom. After transformation, two issues namely; quantization process and entropy coding are discussed below.

A. Quantization

Quantization is a process which removes the high frequencies present in the original image. This is due to the fact that the eye is much more sensitivity to lower frequencies than to higher frequencies. This is done by dividing values of high indexes in the vector (the amplitude of higher frequencies) with larger values. Values of low indexes are divided with amplitudes of lower frequencies. We have used the standard JPEG luminance quantization table in [1].

B. Entropy Huffman Coding

After the transformation and quantization over an 8×8 image sub-blocks, the new 8×8 sub-block shall be reordered in zigzag scan into a linear array. The first coefficient is the DC coefficient and the other 63 coefficients are AC coefficients. Because the DC coefficient contains a lot of energy, it has usually much larger value than AC coefficients. Since there is a

very close relation between the DC coefficients of adjacent blocks, the DC coefficients are differentially encoded. This process further reduces entropy. In our experiment we use basic entropy coding process.

The entropy coding process consists of Huffman coding tables as recommended in JPEG standard [1]. These tables are stored as header information during the compression process so that, it is possible to uniquely decode the coefficients during decompression process.

VI. SIMULATION RESULTS AND ANALYSIS

The superiority of the proposed technique is demonstrated through computer simulation running on Microsoft Window XP, Intel Core2 Duo CPU, 3 GHz Platform. The PSNR is a metric used for comparison. PSNR is expressed as:

$$PSNR = 10 \log_{10} \left(\frac{255^2}{MSE} \right), \quad (17)$$

where

$$MSE = \frac{1}{M \times N} \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} (X_{ij} - X'_{ij})^2. \quad (18)$$

Symbols X_{ij} and X'_{ij} are original and reconstructed pixel values at the location (i, j) respectively. $M \times N$ is the size of the image.

The experimental inputs for the proposed method are Lena and Slope images as shown in Fig. 3 (a) and (b) respectively. All the image dimensions are of 256×256 . Fig. 4(a) and (b) shows the comparison plot (PSNR Vs Compression Ratios) of Lena and Slope images respectively. Comparison is made between proposed 8×8 DTT algorithm and JPEG, which uses 8×8 DCT algorithm. From Fig. 4(a), it is clear that the PSNR of DTT and DCT differ by almost less than 0.2 dB, for any compression ratio. Fig. 4(b) shows that the PSNR of DTT reconstructed Slope image is better than DCT at any compression ratio.

In Table 1(a) and (b), a quantitative comparison is made between compression ratios (CR), Number of AC plus DC coefficients, PSNR and MSE between DTT and DCT reconstructed Lena and Slope images respectively. The ‘Scaling factor’ values indicated in the first column of Table 1, is the multiplication factor to scale the quantization matrix as described by [1]. For Lena image, it can be seen from Table 1 that, at a CR of 9.0 DCT needs around 300 less coefficients than DTT. But for higher CR DTT always needs lesser number coefficients than DCT. It can be also note that, from Table 1, DTT shows higher compression performance than DCT in most of the cases for Lena image. Furthermore, in case of Slope image, DTT shows always better compression performance than DCT, except at a CR of 35 to 37 as shown in Fig 4(b). This fact is also clearly evident form the second line of Table 1 in case of Slope image. For the same image case, the AC plus DC coefficients at a CR of 35 to 37, are around 200 bits more in DTT than DCT. But other CR DTT needs fewer coefficients than DCT.

In order to evaluate the compression performance between DTT and DCT, around 20 various kinds of images have been tested. We have observed that, for images having high intensity variations, DTT always performs better than DCT in terms of PSNR, MSE and number of AC and DC coefficients.

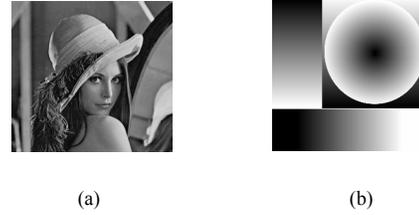


Fig. 3. Test images use for Experiment (a) Lena, (b) Slope.

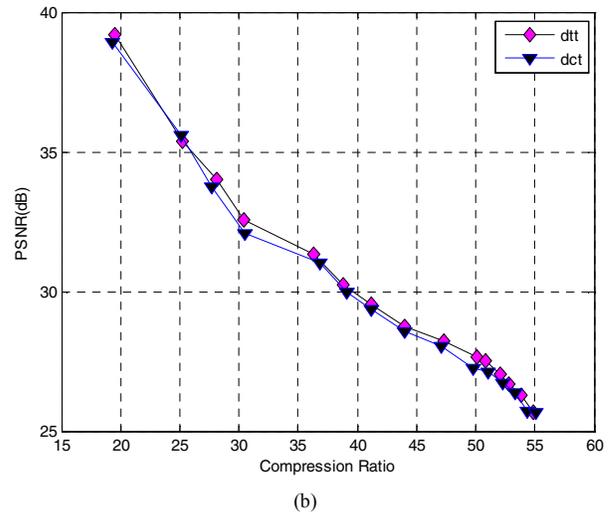
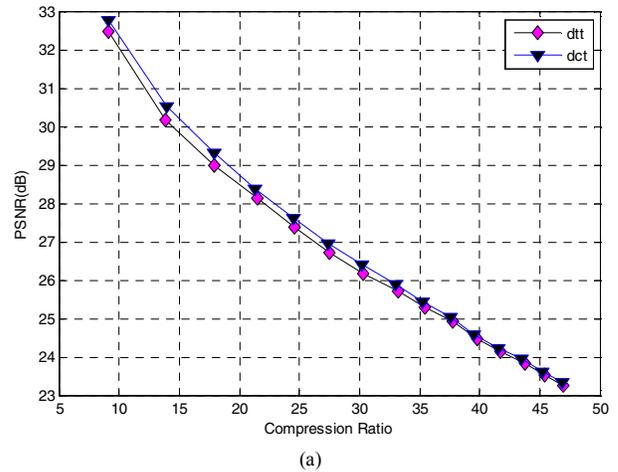


Fig. 4. PSNR-Compression Ratios curve between DTT and DCT of (a)Lena, (b) Slope image.

TABLE I
COMPARISON OF COMPRESSION RATIO (CR), NUMBER OF DC AND AC COEFFICIENTS, PSNR (dB) AND MSE BETWEEN DTT AND DCT

1. Lena Image:

Scaling factor	DTT(Proposed)						DCT(Used in JPEG)					
	CR	No. of bits to encode DC coefficients	No. of bits to encode AC coefficients	Total DC & AC Coefficients (in bits)	PSNR	MSE	CR	No. of bits to encode DC coefficients	No. of bits to encode AC coefficients	Total DC & AC Coefficients (in bits)	PSNR	MSE
1	9.0	5790	51854	57944	32	37	9.0	5789	51854	57643	33	34
5	24.63	4087	17196	21283	27	119	24.49	4088	17318	21406	28	112
10	37.72	3704	10194	13898	25	209	37.57	3705	10252	13957	25	203
15	46.92	3538	7635	11173	23	306	46.89	3540	7640	11180	23	300

2. Slope Image:

Scaling factor	DTT(Proposed)						DCT(Used in JPEG)					
	CR	No. of bits to encode DC coefficients	No. of bits to encode AC coefficients	Total DC & AC Coefficients (in bits)	PSNR	MSE	CR	No. of bits to encode DC coefficients	No. of bits to encode AC coefficients	Total DC & AC Coefficients (in bits)	PSNR	MSE
1	19.52	5568	21286	26854	39.2	7.83	19.32	5567	21571	27138	38.9	8.36
5	36.28	4342	10110	14452	31.36	47.60	36.86	4342	9879	14221	31	51.3
10	50.06	3754	6721	10475	27.64	112.1	49.82	3754	6771	10525	27.3	122
15	54.86	3550	6006	9556	25.69	175.7	55.04	3550	5976	9526	25.67	176

VII. CONCLUSIONS

In this paper, an 8×8 DTT algorithm is implemented in the JPEG in place of 8×8 DCT algorithm. By using various test images, it has been observed that DTT has energy compaction property competitive with that of DCT and thereby provides compression performance relatively close with DCT. Therefore, it can be a suitable candidate for applications such as PDAs, mobile phones and digital cameras where efficient image compression techniques are required. The proposed work is implementation of 8×8 DTT algorithm using Matlab. Future research work in this field is to develop a DTT fast algorithm for input block of size 8×8.

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