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An analysis of plastic flow through polygonal linearly converging dies: as applied to forward metal extrusion

S.K. Sahoo*

Department of Mechanical Engineering, Regional Engineering College, Rourkela 769 008, Orissa, India

Now National Institute of Technology (NIT)

Abstract

An approximate method based on the upper bound theory has been presented in order to investigate into non-axisymmetric metal extrusion process. In this approach the deformation region is divided into a finite number of rigid tetrahedral blocks that slides with respect to one another. The proposed method is successfully adapted to the extrusion of T-section bars from round billet through straight taper die. Computation for the upper bound pressure is carried out for various process variables such as area reduction, die angle and interface friction. The theoretical predictions are compared with that of known experimental results and found to be well within engineering accuracy.

Keywords: Upper bound analysis; Kinematically admissible velocity field; Extrusion; Converging dies

1. Introduction

Exact solutions are not available for many metal working processes and several attempts have been made to propose approximate methods which could be adopted for estimating the loads required to cause plastic deformation. A large number of approximate solutions are reported for the prediction of working stress in drawing and extrusion under plane-strain deformation and circular sections with axial symmetry [1]. The analyses under these conditions are relatively simple. However, in practice, non-circular sections, such as polygonal rods, channels, angles, etc. are also commonly drawn or extruded. In such cases, the analysis is more complicated. Furthermore, not many experimental studies of the flow through the polygonal die have been conducted. Therefore, the analysis of flow through a polygonal converging die has been rarely tried.

Among various methods of solution, the upper bound technique as an analytical method and finite element analysis as a numerical method have been widely used for the analysis of the extrusion process effectively. Even though the finite element method (FEM) gives detailed information, it takes considerable computation time for three-dimensional analysis, and this is not yet so practicable for optimising the extrusion process. The upper bound technique appears to be an useful tool for the analysis of three-dimensional metal forming problems when the objective of such an analysis is limited to prediction of the deformation load and/or to study metal flow during the process.

Despite the increasing demands for three-dimensional extrusion of arbitrarily shaped sections and advantage of straightly converging dies, a few theoretical approaches to the extrusion or drawing processes have been reported. Juneja and Prakash [2] analysed the plastic flow of metal through a converging die of any irregular cross-section, by using an equilibrium approach. The analysis is based on the assumption that the zones of plastic deformation is enclosed by two cylindrical surfaces of velocity discontinuity, at entry to and exit from the pyramidal portion of the die. Nagpal and Altan [3] introduced the stream function to express three-dimensional flow in the die and analysed the force of extrusion from round billet to elliptical bars. Basily and Sansome [4] made an upper bound analysis on drawing of square sections from round billets by using triangular elements at entry and exit of the die. Yang and Lee [5] formulated kinematically admissible velocity fields of billets having generalised cross-section for the extrusion, where the similarity in the profile of cross-section was assumed to be maintained throughout deformation. Prakash and Khan [6] made an upper bound analysis on extrusion and drawing through dies of polygonal cross-sections with straight stream lines, where the similarity in shape was presented. Boer et al. [7] made an upper bound approach to drawing of square rod from round bars, by employing a method of co-ordinate

Nomenclature

A_b	area of the billet cross-section
L	length of each side of the approximating polygon
m	friction factor on the die surface
M	number of sides of the approximating polygon
$\hat{n}$	unit vector normal to a surface
P_{av}	average extrusion pressure
R	billet radius
S_i	area of the ith face of the tetrahedral rigid region
S_{fj}	area of jth face having friction between die and work piece
ΔV	velocity discontinuity
V_b	billet velocity
W	upper bound energy consumption

Greek letters

ε_{ij}	components of strain rate tensor
θ	internal angle of a polygon
κ	yield stress in simple shear
σ_0	yield stress in uniaxial tension or compression
τ	shear stress between billet and die
ϕ	total metal flow through a face

transformation. Gunasekera and Hosino [8] obtained an upper bound solution for extrusion and drawing of square sections from round billets through converging dies formed by an envelope of straight lines. Yang et al. [9–11] derived kinematically admissible velocity fields assuming proper stream line functions. In addition to these, a number of techniques are reported [12–14] to construct kinematically admissible velocity field that leads to upper bound solution of three-dimensional metal forming problems in general and metal extrusion in particular. However, most of these procedures have been used to-date to analyse extrusion processes where billet and product sections are either similar or can be described by a continuous analytical function.

In the present study, the upper bound solutions for plane-strain problem are modified and applied to three-dimensional shape extrusion problems. Planer elementary rigid region (PERR) concept of Johnson and Kudo [15] is extended to solve non-axisymmetric extrusion problem from round billet. The deformation zone is discretised into rigid tetrahedral zone or so called, spatial elementary rigid region (SERR) blocks. However, this is applicable to the problems where the die walls are planer in nature. The objective of the present study is to apply the reformulated SERR technique to the extrusion of T-section from round billet through straightly converging die (Fig. 1) by approximating the circular cross-section of the billet by regular polygon, the number of sides of the polygon being successively increased until the extrusion pressure converges. The effect of various process variables such as area reduction, die angle and interface friction on the working stress is studied.

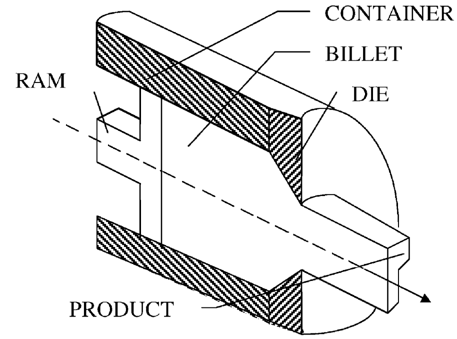

Fig. 1. Sectional view of extrusion of T-section from round billet.

2. SERR technique

In the SERR technique the volume of the solid is thought of as being made up of rigid polyhedra, with the plastic deformation localised on the faces of the same spatial figures, but allowed to glide over one another while maintaining contact. The polygon which divide the rigid adjacent polyhedra, are referred to as faces of discontinuity or shear planes. Such a scheme is an obvious generalisation of the model used extensively in the limit analysis of plane plastic deformation. The rigid motion inside each polyhedron due to plastic flow of material, according to SERR method, is described by a velocity vector consistent with the bounding conditions. These velocity vectors can be defined through their normal and tangential components with respect to plane of discontinuity. The continuity condition will be satisfied when the normal component of the velocities across each discontinuity boundary are constant.

If S_i is the area of the i_{th} face of the polyhedron of unit outer normal $\hat{n}_i$ and ϕ is the total metal flow through its faces, then

$$\phi = \sum_i S_i(V_i\hat{n}_i) = 0 \qquad (1)$$

The tangential component of the motion/velocity at the discontinuity planes suffers a sudden change which is responsible for the sliding of one rigid block on another.

Thus, if there are N rigid blocks, then the number of unknown internal velocity vectors is also N (thus, $3N$ spatial velocity components). The velocity at entry to the deformation zone (the billet velocity) is considered to be prescribed and the velocity at the exit has a single component since its direction is known from the physical description of the problem. Therefore, the total number of unknown velocity components in the global level becomes $3N + 1$. All these unknown velocity components can be uniquely determined if an equal number of equations are generated. This is done by applying the mass continuity condition (otherwise known as the volume constancy condition) to the bounding faces of all the tetrahedral rigid blocks taken together. It may be noted that the set of velocity equations so generated becomes consistent and determinate if and only if the SERR blocks are tetrahedral in shape, so that, the number of triangular bounding faces automatically becomes $3N + 1$.

The deformation zone in case of metal forming that occurs in a closed channel (like extrusion or drawing), can be subdivided into subzones that are prismatic, pyramidal or tetrahedral in shape or a combination of these shapes. Since the elementary blocks are to be tetrahedral in nature, the prismatic or pyramidal subzones are ultimately discretised into tetrahedrons. A pyramid can be discretised into two tetrahedrons by dividing the quadrilateral base into two triangles. Thus, there are two ways of discretising the pyramid into two tetrahedral blocks. In a similar manner, a prismatic subzone can be discretised into three tetrahedrons in six different ways.

3. Theoretical model

The model comprises the following assumptions:

(1) The material is rigid-plastic, and obeys the von Mises yield criterion.
(2) The shear stress between tools and workpiece is constant, $\tau = m\kappa = m\sigma_0/\sqrt{3}$.
(3) The material is incompressible.
(4) The die and punch are rigid and not deformed.
(5) The length of deformation zone is equal to that of the die.
(6) The centroid of the die aperture lies on the billet axis.
(7) No dead metal zones are formed on the sides of the die orifice.

As mentioned earlier, the SERR technique can be applied where there are plane boundaries. Hence, the curved surface is to be replaced by planer surfaces so as to accommodate the SERR analysis. For the present analysis the round billet is approximated by a 12-sided (as there is a negligible change of final computed value by further increasing the sides, Fig. 2) regular polygon. To approximate the circular cross-section of the billet into a regular polygon, the cross-sectional areas of the billet and the approximating polygon must be maintained equal. This condition is written as

$$\pi R^2 = \tfrac{1}{4} M L^2 \cot(\tfrac{1}{2}\theta) \qquad (2)$$

Since, the T-section has one-fold symmetry, half of the deformation zone (domain of interest) can be considered for the analysis. The subzones of deformation can be delineated in the domain of interest by taking suitably located floating points (since its location is not known a priori). Figs. 3–5 show one-half of deformation zone with one, two and three floating points, respectively. For single point formulation the floating point lies on the plane of symmetry, both floating points lie on the plane of symmetry in case of double point formulation and for triple point

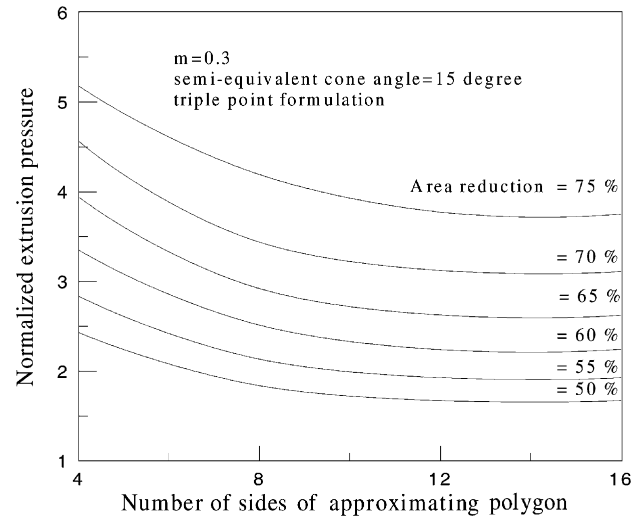

Fig. 2. Effect of number of sides of approximating polygon.

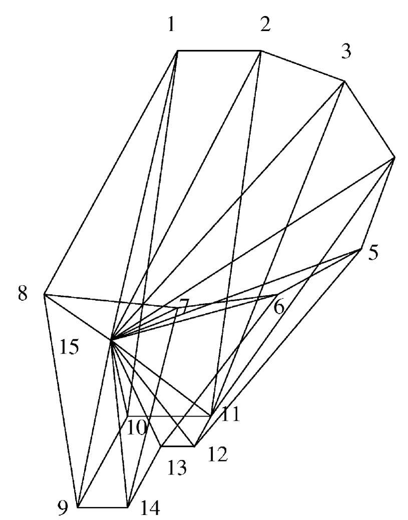

Fig. 3. Half of the deformation zone (single point formulation).

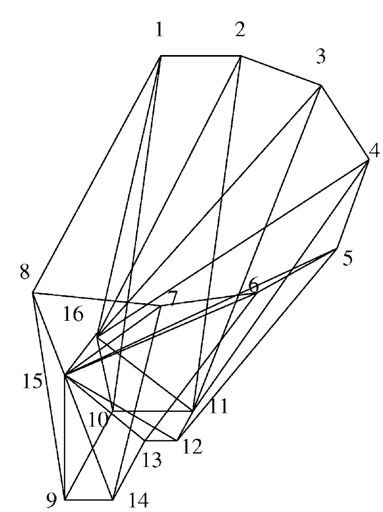

Fig. 4. Half of the deformation zone (double point formulation).

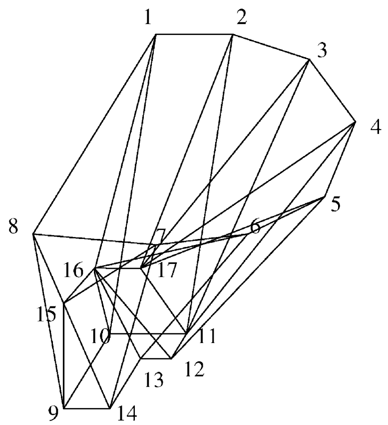

Fig. 5. Half of the deformation zone (triple point formulation).

Table 2
Details of subzones ($M = 12$)

Type of subzones	Formulation		
	Single point	Double point	Triple point
Prisms	–	4–11–16–5–12–15	1–16–10–2–17–11, 5–17–12–6–16–13, 6–16–13–7–15–14
Pyramids	15–1–2–11–10, 15–4–5–12–11, 15–12–13–6–5, 15–13–14–6–7, 15–7–8–9–14	16–1–2–11–10, 15–13–12–5–6, 15–13–14–7–6, 15–7–8–9–14	17–4–5–12–11, 15–7–14–9–8
Tetrahedrons	2–3–11–15, 3–4–11–15	2–3–11–16, 3–4–11–16	2–3–11–17, 3–4–11–17

formulation two on the plane of symmetry and third one at an arbitrary position in the deformation zone. All the corner points of the die orifice are joined to these floating points. The resulting pyramid, prism and tetrahedrons are the ultimate deformation subzone for the SERR formulation. For an illustration, as shown in Fig. 3, the single point formulation gives rise to five pyramids (15–1–2–11–10, 15–4–5–12–11, 15–12–13–6–5, 15–13–14–6–7, 15–7–8–9–14) and two tetrahedrons (2–3–11–15, 3–4–11–15). Hence it results in 12 tetrahedrons (SERR blocks) and the total number of alternative ways it can be obtained is 32 ($2 \times 2 \times 2 \times 2 \times 2$). All these subzones are interconnected and have common triangular faces. When continuity condition is applied to all the 37 bounding faces of these 10 SERR blocks equal number of velocity equations can be obtained for the respective formulation (details of discretisation and subzones are summarised in Tables 1 and 2, respectively). Though more number of subzones can be generated adding further floating points, three points is considered here as there is a marginal improvement of final computed value by further increasing the number of points.

4. Application of the upper bound theorem

If surfaces of velocity discontinuities are to be included and no body traction is taken into account, the actual velocity field minimises the expression [16]:

$$W = W_p + W_s + W_f \tag{3}$$

where

$$W_p = \frac{2\sigma_0}{\sqrt{3}} \int_v \sqrt{\frac{1}{2}\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij}}\, dv$$
$$= \text{work dissipated for internal deformation} \tag{4}$$

$$W_s = \tau \int_{S_i} |\Delta V_i|\, dS_i$$
$$= \text{work dissipated at surfaces of velocity discontinuity} \tag{5}$$

$$W_f = m\kappa \int_{S_{fj}} |\Delta V_j|\, dS_{fj}$$
$$= \text{work dissipated due to friction at die} - \text{work piece interface } (j\text{th face}) \tag{6}$$

In the present formulations with a discontinuous velocity field the strain rate components ε_{ij} are all zero inside the rigid blocks. This leads to

$$W_p = 0 \tag{7}$$

Since, velocity discontinuity $|\Delta V_i|$ and $|\Delta V_j|$ are constant over all the faces, it can be written as

$$W = \frac{\sigma_0}{\sqrt{3}} \sum [|\Delta V_i| S_i + m|\Delta V_j| S_{fj}] \tag{8}$$

Table 1
Summary of discretisation schemes ($M = 12$)

Description	Formulation		
	Single point	Double point	Triple point
Type of subzones	Five pyramids and two tetrahedrons	One prism, four pyramids and two tetrahedrons	Three prisms, two pyramids and two tetrahedrons
No. of rigid blocks	$5 \times 2 + 2 \times 1 = 12$	$1 \times 3 + 4 \times 2 + 2 \times 1 = 13$	$3 \times 3 + 2 \times 2 + 2 \times 1 = 15$
No. of discretisation schemes	$2 \times 2 \times 2 \times 2 \times 2 = 32$	$6 \times 2 \times 2 \times 2 \times 2 = 96$	$6 \times 6 \times 6 \times 2 \times 2 = 864$
No. of planes	37	40	46
No. of unknown velocity components	$12 \times 3 = 36$ for 12 SERR + 1 at exit, total = 37	$13 \times 3 = 39$ for 13 SERR + 1 at exit, total = 40	$15 \times 3 = 45$ for 15 SERR + 1 at exit, total = 46

The non-dimensional average extrusion pressure is can then be written as

$$\frac{P_{av}}{\sigma_0} = \frac{W}{A_0 V_b \sigma_0} \quad (9)$$

5. Computer program and optimisation parameters

The proposed model makes up the basis for a rapid, user-friendly program written for personal computers. A computer program is written in Fortran to make an upper bound analysis for the extrusion of T-section. The flow chart of the computer programme is shown in Fig. 6. Conjugate direction method for multivariable optimisation is used to minimise the total upper bound power with respect to the unknown co-ordinates of the floating points.

For the triple point formulation, two floating points lies on the plane of symmetry and third one at an arbitrary position in the deformation zone. Thus, in total there are seven undetermined co-ordinates, which serve as the optimisation parameters to minimise the extrusion pressure. Similarly, it is two and four for single point and double point formulation, respectively. Here, it is to be noted that the length of the die is taken as per the equivalent semi-cone angle. The equivalent semi-cone angle is defined as the semi-cone angle of a conical die where the reduction area is the same as that of polygonal sections.

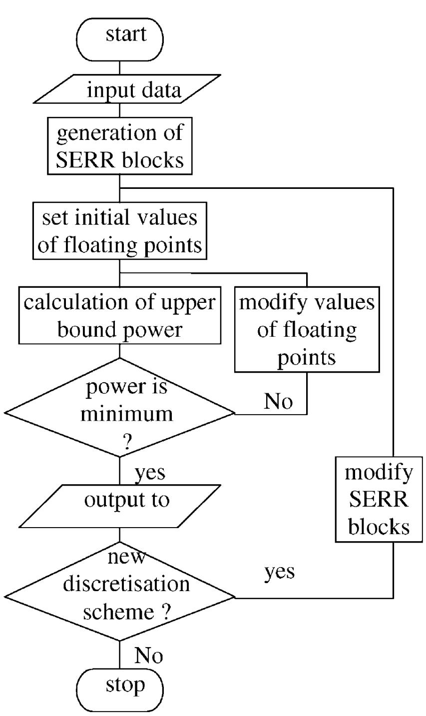

Fig. 6. Flow diagram of the computer program.

6. Results and discussion

All the 32, 96 and 864 discretisation schemes of single point, double point and triple point formulation, respectively, are checked to find out the optimum configuration. The discretised deformation zone corresponding to the least upper bound is named here as the optimum configuration. As the triple point formulation gives the lowest (Table 3) value all 864 schemes of discretisation of this formulation are tested and the scheme giving the least upper bound is identified. This optimum configuration is utilised for computation of normalised extrusion pressure variation with equivalent semi-cone angle in degrees and percentage of area reduction at different friction factor (Figs. 7 and 8). Fig. 8 exhibits that the optimal semi-cone angle which requires minimal extrusion pressure increases with the increase of friction. These results can be used to predict the forming stress and optimal die shape for designing the sectioned die, assessing the frictional condition either through an empirical way or a simulation test. Comparison of the present solution (at $m = 0.35$ and $m = 0.00$) is also made with the experimental results of Chitkara and Adeyemi [17] and Kar [18] at different area reductions (Fig. 9). It is to be noted that, in those previously reported experiments square billets of commercial lead are extruded through the rough square dies.

This comparison shows that the proposed upper bound method (UBM) reasonably can be used to model the non-axisymmetric extrusion through the straightly converging dies. This is so because the classical slip line field solution is not applicable to this class of problems and the FEM is constrained by computational difficulties to achieve accuracy in these cases. In this context, Kobayashi et al. [19] remarks that the economic constraint becomes too severe in three-dimensional metal flow analysis by FEM and suggest that special considerations must be given to achieve a balance between computational efficiency and solution accuracy. They further suggest that the use of simplified three-dimensional elements can be helpful in this regard, but warn that the scope of using such elements is limited. It is also interesting to note here that Lee et al. [13] compared the

Table 3
Comparison of results ($M = 12, m = 0.3$, semi-equivalent cone angle $= 15°$)

Area reduction (%)	Formulation		
	Single point	Double point	Triple point
40	2.283	1.531	1.288
45	2.747	1.912	1.461
50	3.213	2.133	1.672
55	3.751	2.514	1.928
60	4.133	2.739	2.240
65	4.325	3.113	2.626
70	4.914	3.624	3.123
75	5.587	4.361	3.773

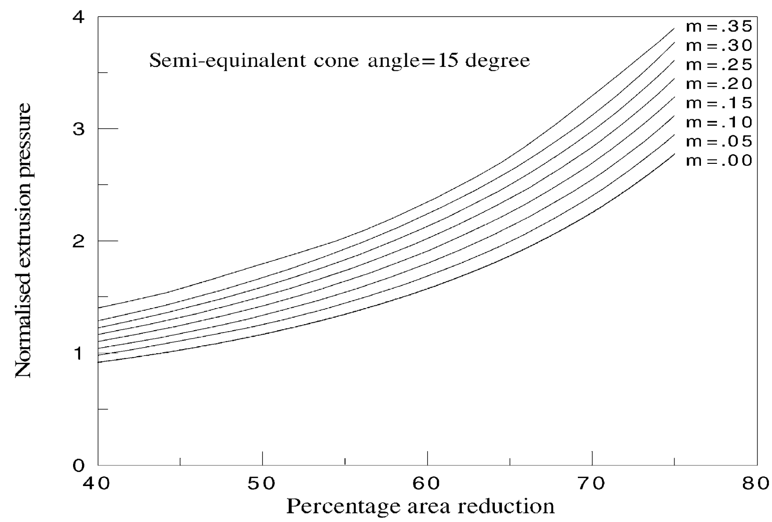

Fig. 7. Effect of friction on the extrusion pressure with reduction of area.

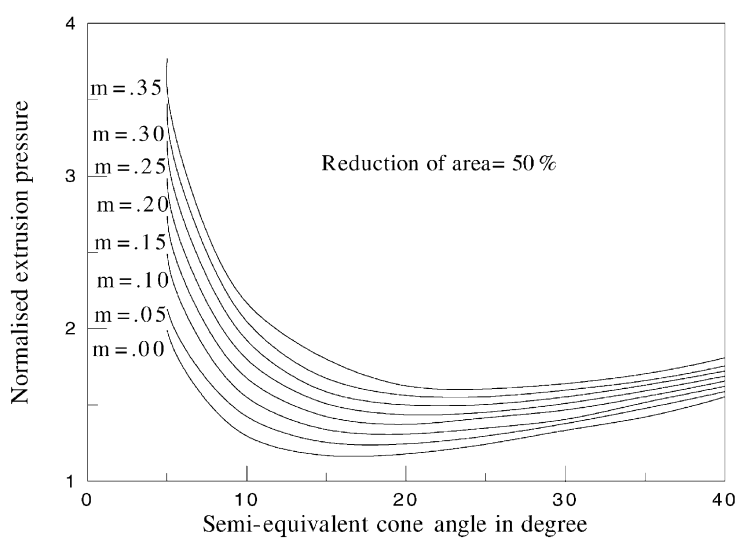

Fig. 8. Effect of friction on the extrusion pressure with die angle.

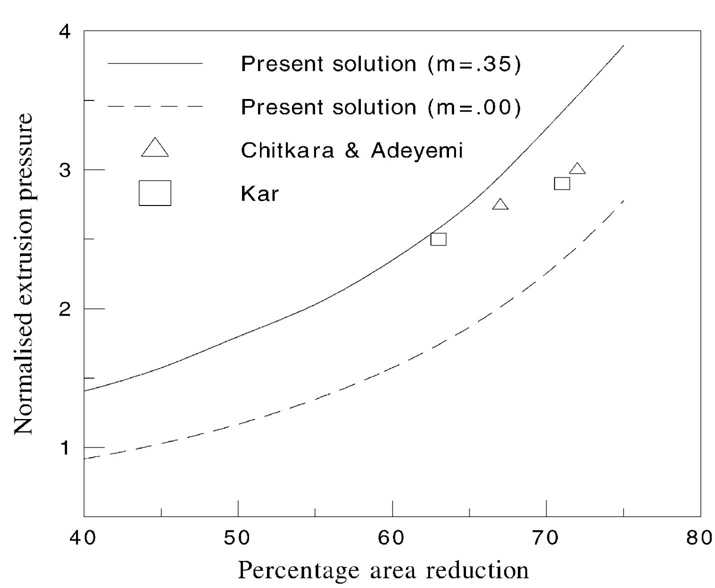

Fig. 9. Comparison of the present solution.

results obtained by the UBM, FEM and the weighted residual method (WRM) with experimental data for extrusion of various sectional shapes at an area reduction of 60%. They reported that the WRM, which is based on the velocity field of the UBM they adopted, gives the lowest result at relatively lower CPU time than the FEM. However, UBM is the simplest even though the predicted load, as reported by Lee et al. for the problems they solved, is about 9% higher than that of FEM. Nonetheless, either the FEM or the WRM has to be adopted when the stress field is needed from the solution.

7. Conclusions

In the present work, a simple analytical upper bound model is developed and it demonstrates that the proposed method is powerful and efficient for the analysis of the extrusion of section rods from round billet through straightly converging dies. The optimal die-geometry (equivalent semi-cone angle) which requires a minimal forming stress at different reduction of areas and friction conditions can be estimated reasonably using this technique. The results are in good agreement with the experiments, and provide adequate estimates of extrusion pressure needed for metal flow. It is hoped that the present method can be extended to obtain the solution of generalised problems of non-axisymmetric extrusion or drawing through converging dies.

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