

Particle Swarm Optimization Based Regularization for Image Restoration

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Abstract

Image restoration from a degraded observation has been a long standing problem in image processing. It requires a direct inversion of the degradation function in frequency domain which is ill posed in nature. So regularization has been used in the restoration process. The selection of regularization parameter still remains a difficult problem due to the amplification of noise in the inversion process. In this paper, we have proposed a PSO based regularization technique which adapts the regularization parameters depending on the noise and blurring conditions in the degraded image. Experimental results are presented to validate the efficiency of the proposed scheme.

1. Introduction

Because of imperfections in imaging systems and other non ideal conditions, the observed image often appears to be the degraded versions of the original image [1], [2]. The image degradation during acquisition makes it difficult for further analysis. So the need of image restoration is mandatory for various applications. The important issue in image restoration is to remove the blur in the presence of noise. In most of the applications, it is assumed that the image degradation model is linear and can be modeled as a two dimensional convolution between the original image ($f(x, y)$) with the point spread function ($h(x, y)$) as given below in equation 1. The image degradation and restoration model is shown in figure 1.

$$g(x, y) = f(x, y) * h(x, y) + n(x, y). \quad (1)$$

where $*$ denotes the two dimensional convolution and ($n(x, y)$) represents the additive noise. In matrix form the observed image can be written as

$$g = Hf + n \quad (2)$$

where g and f are lexicographically ordered vectors. The problem of deblurring has been addressed in many literatures [3], [4], [5]. It has been seen that image restoration is a ill posed problem and it can be effectively handled using regularization method [6]. The regularization theory assumes

that original image is smooth and therefore a more satisfactory result is obtained by adding a smoothness constraint to the original minimization function. Image restoration model using regularization is derived as

$$O(\hat{f}, \alpha) = \|g - H * \hat{f}\|^2 + \alpha \|Q(\hat{f})\|^2 \quad (3)$$

Where \hat{f} represents the restored image and Q is the regularization operator. The parameter α controls the tradeoff between the fidelity to the data and smoothness to the solution. The minimization of the equation 3 gives the restored image. This solution is called Tikhonov solution. The image stabilization is controlled by the function $Q(\hat{f})$. The value of Q is taken as Laplacian operator. The proposed method concentrates on the proper choice of regularization parameter α .

The degree of smoothness is controlled by the parameter α and is generally depends on the signal to noise ratio of the noisy blurred image. If the signal to noise ratio(SNR) is high then a small value of α gives the desired smoothness. Otherwise if SNR is low, a high value of α is required. The effects of α are shown in the figures 2. Number of methods have been proposed to find the optimum value of α . [7], [8], [9]. Reeves [10] proposed a spatial adaptive regularization technique which uses a multistage estimation procedure to estimate the optimal choice of local regularization weights. Reeves also have suggested a method of Generalized Cross Validation (GCV) [11] which gives a reliable estimate of the regularization parameter. However, the minimization of GCV function plays a important role in the correct estimation of the regularization parameter. It has been seen that traditional minimization algorithms fail to achieve the global minimum because GCV function have multiple minimums. Therefore even the GCV criterion is capable of optimizing α but fails to produce good results. In this paper, we have used the same GCV criterion to estimate the regularization parameter but we have minimized in a different manner. The minimization of the GCV function has been carried using particle swarm optimization which avoids local minimums and there is no chance of stuck in the local minimum.

This paper is organized as follows. Brief description of Generalized Cross Validation is provided in section 2. Section 3 describes the proposed scheme based on particle swarm optimization algorithm. Results of the restoration

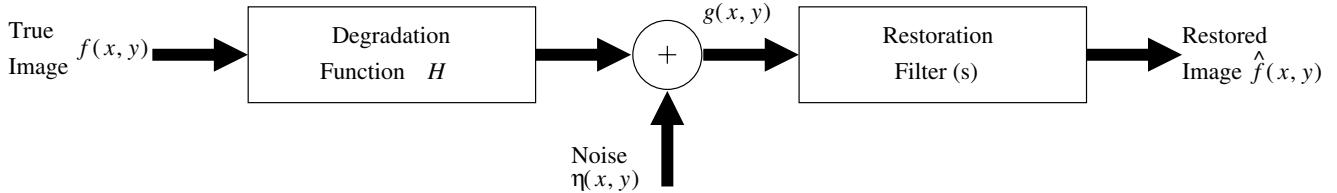


Figure 1. Model of image degradation/restoration

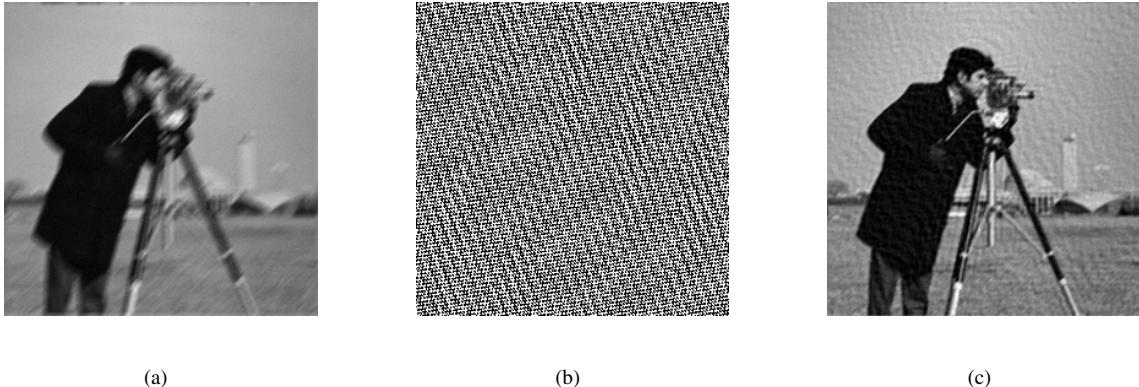


Figure 2. Effect of Regularized Restoration (a) Motion Blurred cameraman image, () Restored with inverse filter $\alpha = 0$, (c) Restored with regularization filter ($\alpha = 0.01$),

have been provided in section 4. Finally section 5 gives the concluding remarks and future works.

2. Generalized Cross Validation

Cross validation criterion is used to test the correctness of assumptions about a set of data by dividing the data set into two sets. One set is used for estimation and other set is used for validation. Instead of using the whole set of data in estimation, one portion of data is left out during estimation. The set that have been left out during estimation is used for validation. GCV criterion has been used efficiently for estimating the regularization parameter estimation [11]. The solution to the problem defined in equation 3 is given by

$$\hat{f}(\alpha) = (H^T H + \alpha Q^T Q) H^T g \quad (4)$$

For a optimum value of α the mean square error is defined as

$$e(\alpha) = \frac{1}{N} \|g - H\hat{f}(\alpha)\|^2 \quad (5)$$

To apply GCV, each pixel of the blurred image is considered as one set of data. For a fixed value of regularization parameter, the restored image is obtained using all the pixels leaving a single pixel. Then the restored image is blurred again to estimate the blurred image pixel that has been left out in the restoration process. Different estimates are observed for each pixel. Then the regularization parameter

which minimizes the mean square error over all the estimates gives the correct value of α . For implementation purpose mathematically it has been derived in compact form and is given by

$$GCV(\alpha) = \frac{\frac{1}{N} \|(I - A)g\|^2}{[\frac{1}{N} \text{tr}(I - A)]^2} \quad (6)$$

Where matrix A is defined as

$$A = (H^T H + \alpha Q^T Q) H^T \quad (7)$$

and I is the identity matrix. The minimization of the above GCV equation has been done using PSO technique.

3. Proposed Method

The main drawback of GCV error criterion is that has multiple number of local minima. So there is possibility that for any arbitrary image with any value of Gaussian noise, it may not give optimum value of regularization parameter. The important issue here is optimization of GCV function. We have proposed a PSO based optimization of GCV function.

3.1. Particle Swarm Optimization

PSO is a stochastic optimization technique developed in 1995 by Eberhart and Kennedy [12]. The algorithm is based

on population inspired by social behavior of bird flocking. The optimization starts by initializing with a population of random solutions and then searches for the optimum solution. Unlike Genetic algorithm, PSO has no crossover and mutation operators. In PSO the solutions are known as particles which fly through the problem space. The particles are also called swarms. In each iteration, the particles are updated using two values one is $pbest$ and other is $gbest$. $pbest$ is the best solution it has achieved in all iterations so far and $gbest$ is the best solution of all particles. The particle positions and velocity are updated after finding the $pbest$ and $gbest$. The updating equations for position and velocity are given in equations 8 and 9.

$$V = V + c1 * \text{rand}() * (pbest - presentx) \\ + c2 * \text{rand}() * (gbest - presentx) \quad (8)$$

$$presentx = presentx + V; \quad (9)$$

Where V is the particle velocity and $presentx$ is the current particle position and rand is the random number between 0 and 1. $c1$ and $c2$ are two weighting constants or accelerating constants. In our experiments, the value of $c1$ and $c2$ are kept equal and it is 1.4. The optimum value of the parameter is obtained when all the solutions become same. PSO systems have memory unlike GA and information sharing mechanism is different than other optimization algorithms. Compared to other optimization algorithms PSO is bit easier to implement. All particles tend to converge to the best solution much faster than GA. PSO has also been successfully applied in blind image deconvolution in 2008 [13]. In our method GCV function defined above have been taken as the objective function. The details formulation of our algorithm is described below.

3.2. GCV error minimization using PSO

The following steps gives the framework of the algorithm.

- 1) The blurred image is received and converted into lexicographic ordered vector.
- 2) The regularization parameter(α) values are initialized as random numbers between 0 and 1. These values denotes the solutions and are called particles of the PSO algorithm.
- 3) For each value of α , the GCV error or the objective function is evaluated by taking

$$Q = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

- 4) $pbest$ and $gbest$ are calculated in the following manner as discussed above.

$$pbest(i+1) = \begin{cases} pbest(i) & \text{if } GCV(\alpha_i) < GCV(\alpha_{i+1}) \\ pbest(i+1) & \text{otherwise} \end{cases} \quad (10)$$

and

$$gbest(i+1) = \arg \min_{pbest} GCV(pbest_n) \quad 1 < n \leq N \quad (11)$$

where i denotes the iteration number and N denotes the number of population

- 5) Then the velocity and position of each particle is updated as per the equations 8 and 9. The position of the particles are nothing but the regularization parameters.
- 6) If number of iterations is less than the maximum, repeat the steps from 3 to 5. Otherwise the algorithm is terminated.
- 7) The α value is obtained from the final $gbest$ after the maximum iteration are reached.

Finally the restored image is obtained using the optimum value of regularization parameter(α) with the help of equation 4.

4. Results and Discussion

The proposed algorithm was implemented for different standard images and two of the results are shown in this paper. Subjective as well as objective measurements are done. Peak signal to noise ratio is the performance metrics considered for comparison.

$$\text{PSNR} = 10 \log_{10} \left(\frac{255^2}{\text{MSE}} \right) \text{dB} \quad (12)$$

$$\text{MSE} = \frac{1}{MN} \sum_{x=1}^M \sum_{y=1}^N \left(f(x, y) - \hat{f}(x, y) \right)^2 \quad (13)$$

where, MN is the size of the image, and $f(x, y)$ and $\hat{f}(x, y)$ represent the pixel values at $(x, y)^{\text{th}}$ location of original and restored image respectively.

Cameraman image is degraded with motion blur ($L = 20$ and $\theta = 30^\circ$) and Gaussian is added to the image so that SNR is 30dB. The GCV function is minimized using PSO and traditional minimization method. The image is restored after obtaining the true value of value of α using PSO and traditional minimization. The corresponding restored images with α values and $PSNR$ are shown in figure 3.

Subsequently *Vase* image is blurred with ($L = 10$ and $\theta = 45^\circ$) and same amount of noise is added. The simulation is repeated for the noisy blurred *Vase* image. The restored image is shown in figure 4.

Similary experiments were carried out to restore the defocused pentagon image which is shown in figure 5.

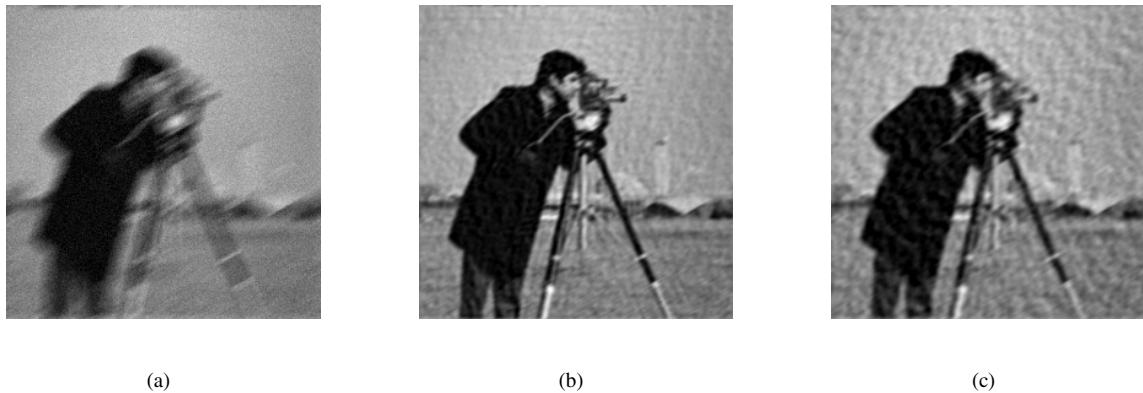


Figure 3. Restoration of Cameraman Image (a) Motion Blurred Cameraman image, (b) Restored after PSO Based GCV minimization $\alpha = 0.043$, and $PSNR = 21.33dB$ (c) Restored after minimization of GCV error with powel's method $\alpha = 0.052$ and $PSNR = 19.87dB$

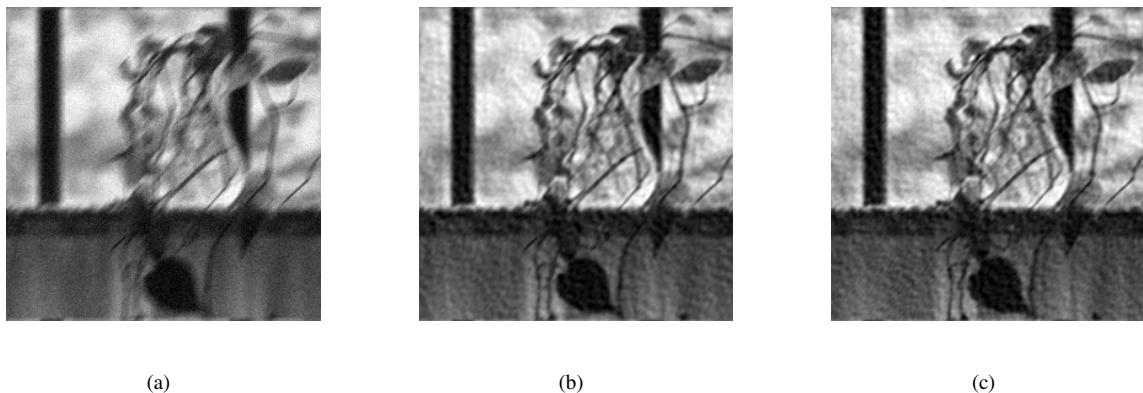


Figure 4. Restoration of Vase Image (a) Motion Blurred Vase image, (b) Restored after PSO Based GCV minimization $\alpha = 0.412$, and $PSNR = 25.53dB$ (c) Restored after minimization of GCV error with powel's method $\alpha = 0.03$ and $PSNR = 23.37dB$

The results reveal that proposed scheme produces a optimized value of α and the restored images are better than normal GCV based restoration with traditional minimization.

5. Conclusions and Future Work

We have proposed a evolutionary computation based GCV error minimization technique for regularized parameter estimation. The problem of determining the degree of smoothing for restoration of noisy blurred image is a old problem and has been addressed here nicely with PSO based GCV. The effectiveness of the proposed scheme have been verified for different noisy blurred images. The experimental results show that the proposed method gives a optimized value of regularization parameter for image restoration. There is a future scope that we can also determine a optimized value of regularization operator using PSO based GCV minimization instead of taking constant Laplacian operator for the whole

image. We believe that the proposed scheme would give better restored image if used to find both the regularization parameter and operator.

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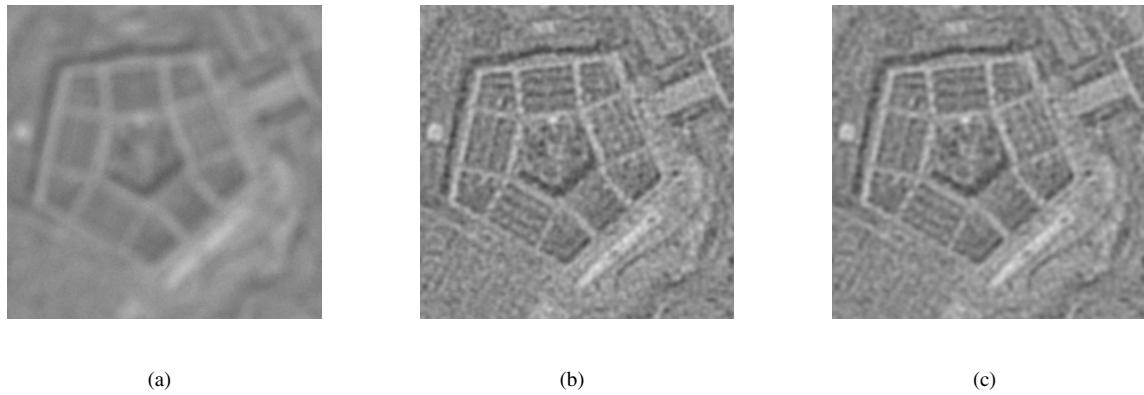


Figure 5. Restoration of Pentagon Image (a) Blurred Pentagon image, (b) Restored after PSO Based GCV minimization $\alpha = 0.0041$, and $PSNR = 32.13dB$ (c) Restored after minimization of GCV error with powel's method $\alpha = 0.0052$ and $PSNR = 29.51dB$

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