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SOFT COMPUTING TECHNIQUES TO SOLVE THE RWA PROBLEM IN WDM OPTICAL NETWORKS WITH ILP FORMULATIONS

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ABSTRACT

This paper focuses on Integer Linear Programming formulations for the Routing and Wavelength Assignment problem in Wavelength Division Multiplexed optical networks where end-users communicate with each other by establishing all optical WDM channels which are referred to as lightpaths. The RWA problem is reducible to Graph Coloring problem in polynomial time and hence found to be NP-complete. So, Soft Computing techniques, Approximation schemes and Heuristic approaches can be applied to solve the RWA problem. In this work, we propose new ILP formulations by imposing additional constraints to the objective function, thus establishing lightpaths which are immune to signal distortion and crosstalk. After modeling the RWA problem as an optimization problem, we focus on applying Soft Computing techniques like Genetic Algorithms to find a sub-optimal solution for the RWA problem.

INTRODUCTION

Wavelength Division Multiplexing (WDM) technology in all optical networks has been gaining rapid acceptance as a mean to handle the ever-increasing bandwidth demands of Internet users [1]. WDM technique exploits the huge bandwidth of optical fibers by overcoming the optoelectronic bottleneck at intermediate nodes. WDM optical networks [2] use lightpaths to exchange information between source-destination node pairs. A lightpath is an all optical logical connection established between a node pair. Given a set of connection requests, the problem of setting lightpaths by routing and assigning wavelengths to each connection is called the Routing and Wavelength Assignment (RWA) problem.

Given a Demand matrix and the number of wavelengths supported by an optical fiber, the problem of maximizing the number of connection requests that can be established is known as MAX-RWA problem. Accordingly, the problem of establishing all the connection requests of a given Demand matrix using least number of wavelengths is termed as MIN-RWA problem.

In the absence of wavelength conversion, it is required that the lightpath occupy the same wavelength on all fiber links it uses. This requirement is referred to as the Wavelength Continuity Constraint. However, this may result in the inefficient bandwidth utilization of WDM channels. Alternatively, the routing nodes may have limited or full range wavelength conversion capability, whereby it is possible to convert an input wavelength to a subset of the available output wavelengths in the network. Since lightpaths are the basic building block of

WDM network architecture, their effective establishment is crucial. It is thus important to provide routes to the lightpath requests and to assign wavelengths on each of the links along the routes such that no two lightpaths that share a physical link use the same wavelength on that link. This requirement is known as Wavelength Distinct Constraint.

The traffic assumptions [3] generally fall into one of two categories: static or dynamic. In static RWA models, we assume that the demand is fixed and known, i.e., all the lightpaths that are to be set up in the network are known beforehand. The objective is typically to maximize in accommodating the demand while minimizing the number of wavelengths used on all links. By contrast, in a stochastic/dynamic setting, we assume that lightpath requests between source destination node pairs arrive one by one at random, and have random hold times. A typical objective in this case would be to minimize the call blocking probability, or the total number of blocked calls over a given period of time.

The Static Lightpath Establishment problem can be formulated as an Integer Linear Program which is found to be NP-Complete [4]. For large networks, randomized rounding heuristics [5] are used to convert the values of the variables of the ILP to either 0 or 1 thereby solving the routing sub-problem of the RWA problem. Once a route has been assigned to each lightpath, the number of lightpaths traversing any physical fiber link defines the congestion on that link. Assigning wavelength colors to the lightpaths, so as to minimize the number of used wavelengths under wavelength continuity constraint reduces to the following Graph Coloring problem in polynomial time.

1. Construct a graph G'(V, E) so that each lightpath in the system is represented by a node in the graph G'. There is an undirected edge between two nodes in the graph G' if the corresponding lightpaths share a common fiber link.

2. Color the nodes of the graph G' such that no two adjacent nodes have the same color and hence the minimum number of colors required for the node coloring problem of graph G' is known as the chromatic number of G'.

Let $\omega(G')$ be the size of the maximum clique and $\lambda(G')$ be the chromatic number of the graph G', then the following inequality holds:

$\omega(G') \le \lambda(G') \le \delta(G') + 1 \tag{6}$	(1))	
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Where $\delta(G') = \max(\delta(u) | u \in V(G'))$ (2)

Here, $\delta(u)$ defines the degree of node u. The degree $\delta(u)$ of node u is the number of edges connected to it.

The above graph coloring problem is found to be NP-complete and the minimum number of colors needed to color the graph is difficult to determine. So, approximation schemes are used to find feasible solutions that are close to the optimal one.

Let $A(\varepsilon)$ be the approximation scheme that produces a feasible solution for some $\varepsilon > 0$. Then, an approximation algorithm for the node coloring problem of graph G' is an absolute one such that $|FO(I) - FA(I)| \le 1$ where FO(I) produces the optimal solution for a problem instant I and FA(I) produces a feasible solution for the same problem instant I under the given approximation scheme. The complexity of the absolute approximation algorithm is found to be O(|V(G')| + |E(G')|).

The DLE problem is more difficult to solve. So, we use heuristic methods [2] to solve both the routing sub-problem and wavelength assignment sub-problem. To solve the routing subproblem the heuristic options used are: Fixed Routing, Fixed Alternate Routing and Dynamic Routing. Among these, the protocol overhead for Fixed Routing scheme is most simple while Dynamic Routing scheme provides the best performance in term of blocking probability. To solve the wavelength assignment sub-problem, various heuristics used are Random wavelength assignment, First Fit wavelength assignment, Least Used (LU) wavelength assignment and Most Used (MU) wavelength assignment scheme.

RELETED WORK

Various strategies have been proposed in current literature that addresses heuristic approaches to solve the RWA problem in all optical WDM networks. However, there are relatively few studies that investigate the performance of soft computing approaches to solve the RWA problem. A search of the IEEE Explorer database shows a published letter [6], where the Max-RWA model has been modified by introducing limited-range wavelength converters at the intermediate nodes. The optimization objective is to maximize the establishment of connection requests with least use of wavelength converters. The Max-RWA problem is formulated as an integer linear program and then solved using genetic algorithm.

In [7], Zhong Pan developed a new Fitness Function to solve the routing sub-problem of the RWA problem using genetic algorithm. The objective was to route each lightpath in such a way that would minimize the number of wavelengths needed to honor all the lightpaths in the Demand matrix. The secondary target was to minimize the total cost in setting all the lightpaths. The cost was calculated in term of route-length traversed by a lightpath from source to destination node.

In [8], D. Bisbal et al. proposed a novel genetic algorithm to perform dynamic routing and wavelength assignment in wavelength routed optical networks with no wavelength converters. By means of simulation experiments, they obtained a low average blocking probability and a very short computation time. Besides, by controlling the evolution parameters of the genetic algorithm, a high degree of fairness among the connection requests was achieved. They also developed an extension to the proposed algorithm with the aim at providing protection to the lightpaths in the optical layer.

In previous linear formulations [4] for the RWA problem the paths that the source–destination pair is allowed to take had to be specified beforehand. This is called as the path formulation ILP. As the number of paths between a node pair is exponential to the number of nodes of the graph; the path formulation will have to restrict itself to a few paths per node pair. When only a limited number of paths are considered, the path formulation ILP approach may yield a sub-optimal solution. In [9] Krishnaswami and Sivarajan proposed link based ILP formulations, i.e., the constraints are over the links (edges or arcs) of the network. The advantage of this formulation is that we do not specify the paths beforehand, but allow the Integer Linear Program solver to choose any possible path and any possible wavelength for a source–destination pair and also the number of constraints in this formulation grow polynomially in the number of nodes.

PROBLEM FORMULATION

The WDM optical network can be viewed as a graph G = (V, E) where V is the set of routing nodes and E is the set of edges in the network. Let W be the set of wavelengths supported by every fiber link in the optical network. Then a lightpath can be expressed as a path-wavelength vector x_p^w ; where p is a physical path between the source and destination node pair and w is the contiguous wavelength assigned to every fiber link of that physical path. Here, we are considering the connection requests individually, i.e., we are not grouping the connection requests according to their source-destination pairs. Let K be the set of

lightpaths to be established. Then the lower bound on the number of wavelengths supported by a fiber link can be formulated as:

$$|W| \ge \frac{\sum_{k \in K} |k|}{|E|} \tag{3}$$

The lower bound justifies the minimum number of wavelengths that should be supported by every fiber link of the network such that all lightpaths in the set K can be realized. This lower bound is known as aggregate network capacity bound. Here,

|k| = Length of a lightpath in term of all fiber links $e \in E$ traversed by it from source to destination edge node

|E| = Number of fiber links in the optical network

In our proposed work, we work on classical ILP formulations [4-6] [9] based on the literature survey and propose new ILP formulations by imposing additional constraints to the objective function thereby establishing lightpaths which are immune to signal distortion and crosstalk. A constraint on the number of intermediate hops traversed by a lightpath ensures less crosstalk accumulated by it while in the absence of wavelength continuity constraint, a restriction on the number of wavelength converters used by a lightpath ensures less signal distortion.

First ILP Formulation

Here, we group the connection requests according to their source-destination node pairs that is the set of lightpaths K can be viewed as $K = \sum_{i,j} K(i, j)$ where K(i, j) is the set of all

lightpaths between source node i and destination node j. Our basic objective is to maximize the number of established connection requests and hence to reduce the average blocking probability associated with a connection request. The variables of interest are defined as follows.

 $c_k(i, j)$ = This variable is set to 1 if k^{th} lightpath between the node pair (i, j) has established, otherwise it is reset to 0.

 $c_k^w(i, j)$ = This variable is set to 1 if the k^{th} lightpath between the node pair (i, j) has established with wavelength w, otherwise it is reset to 0.

 $c_k^{w,e}(i, j)$ = This variable is set to 1 if the k^{th} lightpath has established with wavelength w on link e.

Let, D is the Demand matrix where D_{ij} returns a positive integral value such that |K(i, j)| = D

$$|K(i, j)| = D_{ij}$$

The proposed ILP can be outlined as:

$$Maximize \sum_{(i,j):D_{ij}>0} \sum_{k \in K(i,j)} c_k(i,j) \text{ where } (i,j) \in V \times V$$

$$\tag{4}$$

Subject to:

1. Wavelength Continuity Constraint:

$$\sum_{w \in W} c_k^w(i,j) \le 1 \quad for \ all \quad k \in K(i,j) \quad and \quad (i,j) \in V \times V : D_{ij} > 0 \tag{5}$$

The continuity constraint justifies the use of at most one wavelength to honor a lightpath between a source-destination node pair.

2. Wavelength Distinct Constraint:

$$\sum_{(i,j)\in V\times V: D_{ij}>0} \sum_{k\in K(i,j)} c_k^{w,e}(i,j) \le 1 \quad for \ all \ w\in W, e\in E$$

$$\tag{6}$$

This constraint tells that a particular wavelength of a particular link can be at best allocated to a single lightpath.

3. Demand Constraint:

$$\sum_{k \in K(i,j)} c_k(i,j) \le D_{ij} \quad for \ all \ (i,j) \in V \times V : D_{ij} > 0 \tag{7}$$

This constraint signifies that the number of established lightpaths between a node pair can not be greater than its maximum demand.

4. Integer Constraint:

$$c_k(i, j), c_k^w(i, j), c_k^{w,e}(i, j) \in \{0, 1\} \text{ for all } w \in W, e \in E \text{ and } (i, j) \in V \times V : D_{ij} > 0$$

(8)

This constraint enforces the ILP solver not to keep fractional values in the variables concerned.

5. Wavelength Reservation Constraint:

$$\sum_{k \in K(i,j)} \sum_{e \in v^+} c_k^{w,e}(i,j) - \sum_{k \in K(i,j)} \sum_{e \in v^-} c_k^{w,e}(i,j) = 0 \qquad for \qquad all \qquad w \in W$$

and $(i,j) \in V \times V : D_{ij} > 0 \text{ where } v \in V$ (9)

This constraint tells that the number of lightpaths between a node pair entering and leaving an intermediate node v on a particular wavelength w must be reserved.

6. Consistency Constraint among variables:

$$c_k^{w,e}(i,j) \le c_k^w(i,j) \le c_k(i,j) \quad for \quad all \ w \in W, e \in E \ and \ (i,j) \in V \times V : D_{ij} > 0 \quad (10)$$

The consistency check among the variables of the ILP is straightforward. 7. Hop Count Constraint:

$$\sum_{e \in E} \sum_{w \in W} c_k^{w,e}(i,j) \le H \quad for \quad all \ k \in K(i,j) \ and \ (i,j) \in V \times V : D_{ij} > 0 \tag{11}$$

This constraint enforces us to keep a limit on the number of intermediate hops used by a lightpath so as to ensure less crosstalk accumulated by it. Here, H is the upper bound on the number of hops that can be used by a lightpath and can be calculated from the diameter of the graph G. The diameter d(G) of the graph G can be calculated as follows:

$$d(G) = \max(d(i, j) \mid i, j \in V(G))$$

$$(12)$$

Here, d(i, j) is the distance between the node pair (i, j) and calculated as the minimum number of edges in a path from node *i* to node *j*. The value that is to be chosen for *H* can not be less than d(G) and hence represented as:

$$H = d(G) + \alpha \tag{13}$$

The value for α is dependent on which heuristic we use to solve the routing sub-problem of the RWA problem.

Second ILP Formulation

Here, we modify the first ILP formulation by eliminating the variable $c_k(i, j)$ and thus making the second ILP more efficient one. For the sake of brevity, we are outlining the modifications over the first ILP only. The objective function can be modified as:

$$Maximize \sum_{(i,j):D_{ij}>0} \sum_{k \in K(i,j)} \sum_{w \in W} c_k^w(i,j) \text{ where } (i,j) \in V \times V$$

$$\tag{4a}$$

The Demand Constraint of the previous ILP can be replaced as:

$$\sum_{k \in K(i,j)} \sum_{w \in W} c_k^w(i,j) \le D_{ij} \text{ for all } (i,j) \in V \times V : D_{ij} > 0$$

$$\tag{7a}$$

Modifications over the Integer Constraint and Consistency Constraint are straightforward and depicted as:

$$c_k^w(i, j), c_k^{w,e}(i, j) \in \{0, 1\}$$
 for all $w \in W, e \in E$ and $(i, j) \in V \times V : D_{ii} > 0$ (8a)

$$c_k^{w,e}(i,j) \le c_k^w(i,j) \text{ for all } w \in W, e \in E \text{ and } (i,j) \in V \times V : D_{ii} > 0$$

$$(10a)$$

Third ILP Formulation

The above two proposed ILP formulations are applicable to networks where wavelength continuity constraint is maintained. Here, we modify the above two proposed linear formulations so that they can be applied to networks where wavelength continuity constraint is relaxed. In a network with sparse wavelength conversion capability, some of the nodes are equipped with wavelength converters and hence wavelength continuity constraint can be modified as:

$$\sum_{w \in W} c_k^w(i,j) \le \beta \text{ for all } k \in K(i,j) \text{ and } (i,j) \in V \times V : D_{ij} > 0$$
(14)

Where β denotes the upper bound on the number of wavelength converters the can be used by a lightpath and a low value on β ensures low signal distortion.

Atnodeswithoutwavelengthconverters:
$$c_k^{w,e1}(i,j) = c_k^{w,e2}(i,j)$$
 where $e1 \in v^-, e2 \in v^+, v \in V$ (15)Atnodesequippedwithwavelength $\sum wel = v^{-}$ $\sum wel = v^{-}$ $\sum wel = v^{-}$

$$\sum_{w \in W} c_k^{w.e1}(i,j) = \sum_{w \in W} c_k^{w,e2}(i,j) \text{ where } e1 \in v^-, e2 \in v^+, v \in V$$
(16)

If all the routing nodes of the optical network are equipped with limited range wavelength converters with degree of conversion Δ , then equation (16) will be replaced with the following equations.

$$c_{k}^{w,e1}(i,j) = \sum_{w' \in W} c_{k}^{w',e2}(i,j) \text{ where } e1 \in v^{-}, e2 \in v^{+}, v \in V \text{ and } w - \Delta \leq w' \leq w + \Delta \quad (16a)$$

$$\sum_{w' \in W} c_{k}^{w',e1}(i,j) = c_{k}^{w,e2}(i,j) \text{ where } e1 \in v^{-}, e2 \in v^{+}, v \in V \text{ and } w - \Delta \leq w' \leq w + \Delta \quad (16b)$$

SOFT COMPUTING APPROACHES FOR THE RWA PROBLEM

This section presents meta-heuristic approaches that allow us to solve large problem instances of the RWA problem. Heuristic approaches become important when the problem instant gets large due to increase in size of the physical network and traditional ILP solvers like Simplex method can not produce an exact solution in polynomial time due to the computational constraints. Results of these heuristic approaches compare favorably with the optimal results obtained by solving the exact problem formulation.

Genetic Algorithms Based Heuristic Approach to Solve the RWA Problem

Genetic Algorithms are a class of probabilistic searching algorithms based on the mechanism of biological evolution. A GA (Genetic Algorithm) begins with an initial population of individuals (also called chromosomes); each of which represents a feasible solution to the problem being tackled. Then the GA applies a set of genetic operations such as crossover or mutation to the current population to generate a better one. This process is repeated until a good solution is found or after predefined number of iterations. [10]

For the sake brevity, here, we only describe the proposed fitness function that can be used to discriminate the chromosomes in the current population such that fitter chromosomes will

have better chance to be selected and propagate their genetic materials to the successive generations. The proposed fitness function is the target function to be maximized can be outlined as:

$$fit _ fun = \frac{1}{0.8(u(W)) + 0.1h + 0.1\sqrt{l(p)}}$$
(17)

The above proposed fitness function is applicable to the network which is assumed to be static and circuit-switched. The fiber links are bidirectional. There is no limit on the number of wavelengths a fiber can carry. The symbols of the fitness function with their usual meanings are described as follows:

u(W) = The number of wavelengths used to honor all the static lightpaths and defines the congestion of the most congested link in the network

h = The maximum number of hops traversed by a lightpath in a chromosome

 $\sqrt{l(p)}$ = The maximum length of a lightpath in a chromosome and the square root is used to maintain normalization among too good chromosomes and too bad chromosomes

The working of the algorithm to solve the RWA problem is described as follows.

(i) The chromosome is a group of vectors where each vector p_i is a lightpath represented as $(n_{i0} \dots n_{ih(i)}), n_{i0}, \dots n_{ih(i)} \in V$ and h(i) represents the number of intermediate hops traversed by the lightpath and n_{i0} is the source node and $n_{ih(i)}$ is the destination node for the lightpath.

(ii) To create the initial population, we take the help of dynamic routing heuristic [3] that collects all possible routes from the source node to the destination node.

(iii) The chromosomes of the next generation are selected from the current population by a spinning roulette wheel method. [10]

(iv) According to a cross-over rate, we apply two-point crossover genetic operator technique for mating two selected chromosomes rendering two new chromosomes. Similarly, according to a certain mutation rate, a chromosome is mutated with randomness and the resulting chromosome is supposed to be a fitter one.

SIMULATION AND RESULTS

The standard network considered for simulation is ARPANET shown in Fig. 1; which has 20 nodes connected with 25 links. The genetic algorithm is implemented and simulated for this network with different set of connection requests and the result is obtained for the number of wavelengths required to establish these requests as shown in Fig. 2.

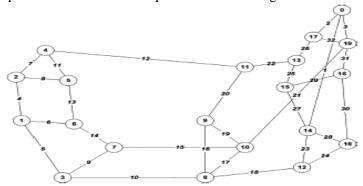


Fig. 1 ARPA Network

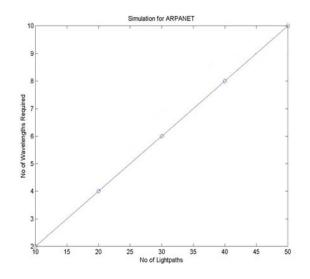


Fig. 2 No of Lightpath Vs. No of Wavelengths Required

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