

# Design of an Error-Based Robust Adaptive Controller

Ki-Young Song, and Madan M. Gupta\*

Division of Biomedical Engineering

University of Saskatchewan

Saskatoon, Canada

Debashisha Jena

Department of Electrical Engineering

National Institute of Technology

Rourkela, India

\*Corresponding author: [madan.gupta@usask.ca](mailto:madan.gupta@usask.ca)

**Abstract-** Design of an adaptive controller for complex dynamic systems is a big challenge faced by the researchers. In this paper, we propose a novel method for the design of an error-based robust adaptive controller to make the system response reasonably fast with no overshoot. Here the control action is designed by introducing the notion of ‘error-based adaptive controller’ (EB-AC). In the design of this feedback adaptive controller, parameters of the controller are designed as a function of the system error. For example, the position feedback parameter  $K_p(e, t)$ , which controls the bandwidth of the system as well as the dynamic response, is a function of the system error  $e(t)$ . In the design of the position feedback parameter  $K_p(e, t)$ , for large error  $K_p(e, t)$  is kept large, thus increasing the bandwidth of the system which yields a fast response, whereas for decreasing errors,  $K_p(e, t)$  is continuously decreased to a small value. Thus, during the dynamic response of the system, the bandwidth of the system is continuously controlled by the system error  $e(t)$ . Similarly, the velocity feedback parameter  $K_v(e, t)$  which controls the damping of the system is kept very small for large errors, and continuously increased to a large value for decreasing value of error. Hence, in the design of the proposed adaptive controller, the position feedback  $K_p(e, t)$  and the velocity feedback  $K_v(e, t)$  are formulated as a function of the system error, and this approach for formulating the adaptive controller yields a very fast response with no overshoot. In this paper, we present an error-based robust adaptive control design methodology for a linear system.

**Index Terms-** Adaptive controller, Dynamic Pole-Zero Locus (DPZL), position feedback, velocity feedback.

## I. INTRODUCTION

Recently there has been an increasing interest from the design of conventional controller to the design of intelligent based control approaches such as adaptive control for controlling a complex dynamic system containing nonlinearity like hysteresis. For decades various schemes of adaptive control have been proposed, and robust adaptive control for nonlinear systems with complex dynamics has received great attention. However, not many of these approaches are suitable for complex nonlinear systems [1-3]. The inverse optimal controller [4-6] using the Lyapunov function is one of the most effective way for designing controllers for nonlinear systems. Using this approach, the controller minimizes a cost function and guarantees the optimality and a stability margin.

In this paper, we present a novel design for a robust controller that yields a faster and stable response using the feedback parameters as a function of system error  $e(t)$  and its states  $\mathbf{x}(t)$ . In order to illustrate the robustness of this design procedure of the controller, some simulation studies are presented.

## II. DESIGN OF A ROBUST ADAPTIVE CONTROLLER

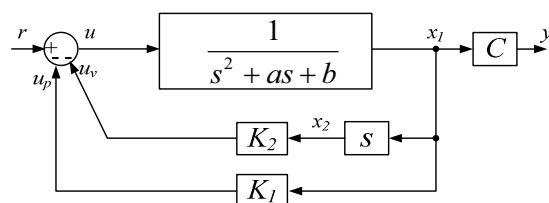
### A. Step response for a second-order system: some important observations

In our study, we consider a typical open-loop second order plant  $G_p(s)$  defined as

$$G_p(s) = \frac{1}{s^2 + as + b} \quad (1)$$

As shown in Fig. 1, With a feedback controller, the transfer function of the closed-loop system is given by

$$\frac{Y(s)}{R(s)} = \frac{b + K_1}{s^2 + (a + K_2)s + b + K_1} \quad (2)$$



$$\text{state vector: } \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{C} y \\ \dot{x}_1 \end{bmatrix}, \quad \text{output: } y = Cx_1 = (b + K_1)x_1$$

Fig. 1. A typical second-order model and its closed-loop system with two feedbacks  $K_1$  and  $K_2$

This transfer function can be compared with a general second-order system model as

$$\frac{b + K_1}{s^2 + (a + K_2)s + b + K_1} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (3)$$

Thus, we see that,

$$\begin{aligned}\omega_n^2 &= b + K_1 \triangleq K_p \\ 2\zeta\omega_n &= a + K_2 \triangleq K_v\end{aligned}\quad (4)$$

where the parameters  $K_p$  and  $K_v$  are defined as position feedback and velocity feedback respectively.

Generally the dynamic behavior of a second-order system can be described in terms of two parameters, the natural frequency ( $\omega_n$ ) and the damping ratio ( $\zeta$ ). The transient response of a typical control system often exhibits damped oscillations before reaching the steady-state. In specifying the transient response characteristics of a second-order control system to a unit-step input, following transient parameters in the design of a controller are usually considered:

$$\text{Rise time: } T_r = \frac{\pi - \theta}{\omega_d} = \frac{\pi - \theta}{\omega_n \sqrt{1 - \zeta^2}}, \theta = \cos^{-1}(\zeta) \quad (5)$$

$$\text{Settling time: } T_s = \frac{4}{\zeta\omega_n} \quad (2\% \text{ criterion}) \quad (6)$$

$$\text{Maximum overshoot: } M_p = e^{-\zeta\pi\sqrt{1-\zeta^2}} \times 100 \quad (\%). \quad (7)$$

It is important to note that in the step response of the second-order system, the transient values for  $T_r$ ,  $T_s$  and  $M_p$  are dependent on the natural frequency ( $\omega_n$ ) and the damping ratio ( $\zeta$ ). As shown in Fig. 2, the positions of the poles of the system are determined by the values of  $\omega_n$  and  $\zeta$ . In the transient response, it is to be noted that an underdamped system ( $\zeta < 1$ ) yields large overshoot ( $M_p$ ) with a larger settling time ( $T_s$ ), but faster rise time ( $T_r$ ). Whereas, an overdamped system ( $\zeta > 1$ ) yields no overshoot, i.e.  $M_p = 0$ , but it yields slower  $T_r$  and large  $T_s$ . Some typical transient responses for step inputs for both overdamped and underdamped systems are shown in Fig. 3.

### B. Development of a Error-Based Adaptive Controller Design Criterion

For the design of an appropriate feedback controller, let us consider the system error  $e(t)$  as an important parameter in our feedback design, and in our design methodology developed in this paper, we will make the parameters of the feedback controller as a function of the error. From the transient responses shown in Fig. 3(b), we can emphasize that for large responses a small  $\zeta$  and a large  $\omega_n$  i.e. an underdamped dynamics, will yield a very fast response with a very small rise time  $T_r$ . On the other hand, for small errors a large  $\zeta$  and a small  $\omega_n$ , i.e. an overdamped system dynamics, will inhibit any overshoot. Since  $\zeta$  and  $\omega_n$  are dependent upon the parameters of position feedback ( $K_p$ ) and velocity feedback ( $K_v$ ), if we define  $K_p(e, t)$  and  $K_v(e, t)$  as a function of the system error,  $e(t) = r(t) - y(t)$ , then we can achieve a very fast dynamic response with no overshoot, thus one can achieve a very fast response with a very small rise time and shorter settling time.

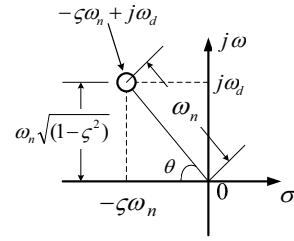


Fig. 2. Definition of the various parameters in the complex  $\sigma$ - $j\omega$  plane: natural frequency:  $\omega_n$ , damping ratio:  $\zeta$ , undamped natural frequency:  $\omega_d$ , and  $\theta = \cos^{-1} \zeta$ .

From these qualitative observations on the transient period of the step response, we derive the following design criteria for the feedback controller.

#### Design criteria for the adaptive controller:

- (i) When the system error is large, keep the damping ratio,  $\zeta$ , very small and natural frequency,  $\omega_n$ , very large.
- (ii) When the system error is small, damping ratio,  $\zeta$ , should be large and natural frequency,  $\omega_n$ , small.

#### Design of parameters for the adaptive controller:

- Position feedback  $K_p$  controls the natural frequency of the system  $\omega_n$ , i.e.  $K_p = \omega_n^2$ , and the bandwidth of the system is determined by the system natural frequency  $\omega_n$ .
- Velocity feedback  $K_v$  controls the damping ratio  $\zeta$ , i.e.  $K_v = 2\zeta\omega_n$

Thus, we can design the adaptive controller parameters for position feedback  $K_p(e, t)$  and velocity feedback  $K_v(e, t)$  as a function of the error:

*'As error changes from a large value to a small value,  $K_p(e, t)$  is varied from a very large value to a small value, and simultaneously,  $K_v(e, t)$  is varied from a small value to a large value'.*

Hence, as discussed above, a desired transient response from the systems can be obtained by using sliding values of  $\omega_n$  and  $\zeta$  during the operation of the system. As given in Eqn. (4),  $\omega_n$  and  $\zeta$  are dependent upon the position feedback  $K_p$  and velocity feedback  $K_v$ , respectively. The response curves for a second-order closed-loop system with varying  $K_p$  and  $K_v$  are shown in Fig. 4.

### C. Design of adaptive controller parameters $K_p(e, t)$ and $K_v(e, t)$

Using the design criterion for the adaptive controller stated above, one can develop many functions for  $K_p(e, t)$  and  $K_v(e, t)$  which satisfy the design criteria with respect to the system error and time. Here, we give one such function for  $K_p(e, t)$  and  $K_v(e, t)$ . Defining the system error as

$$e(t) = r(t) - y(t) \quad (8)$$

where the system output  $y(t)$  is given by

$$y(t) = K_p(e, t)x_1(t), \quad (9)$$

Using the design criterion for the adaptive controller as given above, we define the position feedback  $K_p(e, t)$  and the velocity feedback  $K_v(e, t)$  gains as a function of  $e(t)$  as,

$$K_p(e, t) = K_{pf}(1 + \alpha e^2(t)) \quad (10)$$

$$K_v(e, t) = K_{vf} \exp[-\beta e^2(t)] \quad (11)$$

where  $\alpha$  and  $\beta$  are some gain constants which decide the slope of the functions and affect the system response as illustrated in Fig. 5,  $K_{pf}$  and  $K_{vf}$  are the final steady-state values of  $K_p(e, t)$  and  $K_v(e, t)$ , and  $\exp(\bullet)$  is the exponential function. The other possible functions for  $K_p(e, t)$  and  $K_v(e, t)$  are given in Table 1.

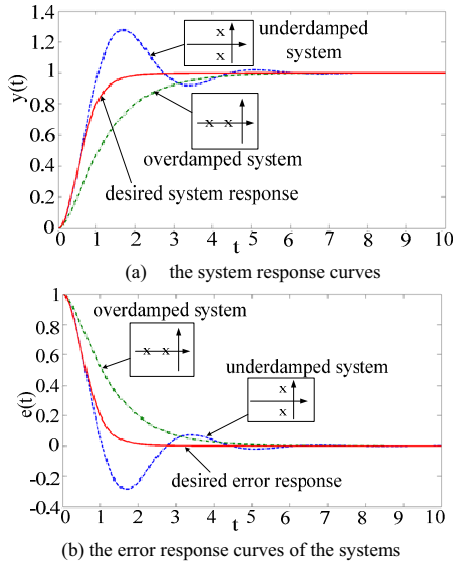


Fig. 3. System responses to a unit-step input with two different locations of poles (i) underdamped case ( $\zeta < 1$ ) and (ii) overdamped case ( $\zeta > 1$ ). The optimal system response initially follows the underdamped curve for large errors, and then settles down to a steady-state value for decreasing errors.

#### D. Design of Robust Adaptive Controller:

As shown in Fig. 6, the control signal  $u$  is derived as a function of the error  $e(t)$  and time  $t$  as

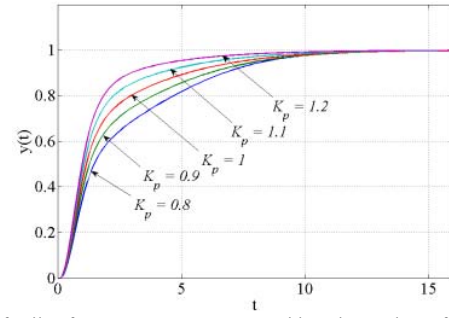
$$\text{position feedback: } u_p(e, t) = K_p(e, t)x_1(t) \quad (12)$$

$$\text{velocity feedback: } u_v(e, t) = K_v(e, t)x_2(t). \quad (13)$$

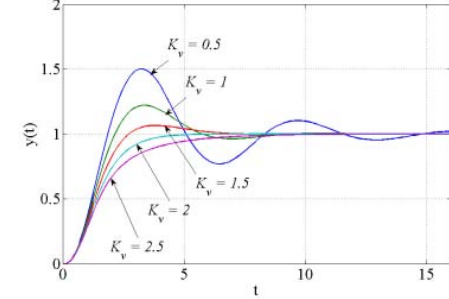
And, thus the feedback control signal is

$$u(e, t) = r(t) - \{u_p(e, t) + u_v(e, t)\} \quad (14)$$

where  $K_p(e, t)$  and  $K_v(e, t)$  are defined in Eqns.(10) and (11) respectively. This proposed novel error-based adaptive controller is illustrated in Fig. 6.

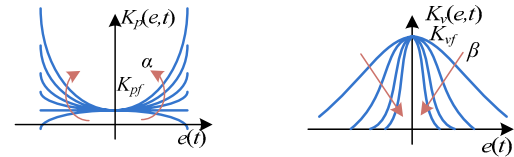


(a) A family of system response curves with various values of  $K_p$  and a constant  $K_v = 2$



(b) A family of system response curves with various values of  $K_v$  and a constant  $K_p = 1$

Fig. 4. System response curves varying  $K_p$  and  $K_v$ .



(a) Changes in slope for  $K_p(e, t) = K_{pf}(1 + \alpha e^2)$  for various values of  $\alpha$ : the direction of arrows indicates the increasing value of  $\alpha$  from negative to positive values  
(b) Changes in slope for  $K_v(e, t) = K_{vf} \exp[-\beta e^2]$  for various values of  $\beta$ : the direction of arrows indicates the increasing value of  $\beta$ .

Fig. 5. The change of the slopes by varying values of  $\alpha$  and  $\beta$

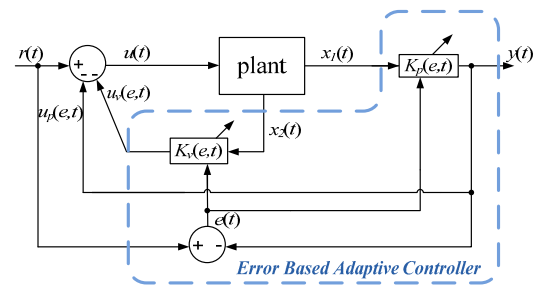


Fig. 6. The proposed robust adaptive controller:  $y(t) = K_p(e, t)x_1(t)$  and  $x_2(t) = \dot{x}_1(t)$ .  $K_p(e, t)$  and  $K_v(e, t)$  are defined in Eqns.(10) and (11).

TABLE 1. VARIOUS POSSIBLE FUNCTIONS AND THEIR GRAPHIC EXPRESSIONS FOR FEEDBACK GAINS  $K_p$  AND  $K_v$

$K_p(e, t)$		$K_v(e, t)$	
$K_{pf}(1 + \alpha e )$		$K_{vf} \frac{1}{1 + \beta e }$	
$K_{pf}(1 + \alpha e^2)$		$K_{vf} \frac{1}{1 + \beta e^2}$	
		$K_{vf} \exp(-\beta e^2)$	

Note: Many other functions can be derived for  $K_p(e, t)$  and  $K_v(e, t)$  for example taking the hyperbolic tangent and cosine functions.

### III. DESIGN OF A ROBUST ADAPTIVE CONTROLLER: A CASE STUDY

In this section, we present the design and simulation studies of a robust adaptive controller for a third-order dynamic system with two complex poles and one real pole such as

$$G_p(s) = \frac{10}{s^3 + 11s^2 + 11s + 10} \quad (15)$$

where the poles are located at  $-10$  and  $-0.5 \pm j0.87$  on real-imaginary plane. The open-loop system response to the unit-step input is shown in Fig. 7.

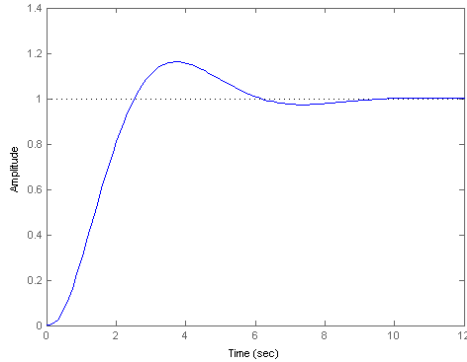


Fig. 7. The transient response of the open-loop system to unit-step input with  $T_r = 2.24$  (sec) (90%) and  $M_p = 16\%$ .

#### A. Design of Robust Adaptive Controller for the Systems

Now we design a robust adaptive controller for the system. For the proposed robust adaptive control, the control input signal  $u$  is derived using Eqn.(14) as

$$u(e, t) = r(t) - \{u_p(e, t) + u_v(e, t)\} \quad (16a)$$

where,

$$(16b)$$

$$u_p(e, t) = \left\{ \left[ K_{pf} + \alpha K_{pf} (r(t) - K_p(e, t)x_1(t)) \right]^2 x_1(t), \right.$$

$$\left. u_v(e, t) = \exp \left[ -\beta (r(t) - K_p(e, t)x_1(t))^2 \right] x_2(t) \right\} \quad (16c)$$

and,  $x_1(t) = x$  and  $x_2(t) = \dot{x}$  are the states of the system,  $K_{pf}$  and  $K_{vf}$  are the steady-state values of feedbacks  $K_p(e, t)$  and  $K_v(e, t)$  respectively, and  $\alpha$  and  $\beta$  are some gain constants,  $r(t)$  is the reference input of the system and  $e(t) = [y(t) - r(t)]$  is the system error. The control objective is to design the function for  $u(t)$  to make the system stable, so that the system output  $y(t)$  follows the reference input signal  $r(t)$  as  $t \rightarrow \infty$  achieving fast rise time  $T_r$  and fast settling time  $T_s$  with minimal overshoot  $M_p$ .

Referring to the error-based design criteria of the robust adaptive controller, using Eqns.(10) and (11), we change the closed-loop system dynamics continuously: initially for large errors we make large  $\omega_n$  and very small  $\zeta$ , which are continuously forced to change to a small  $\omega_n$  and a large  $\zeta$  for zero errors. Thus, initially when the system error is large, the system is started as an underdamped system with large  $\omega_n$  and very small  $\zeta$ , and continuously, with decreasing error, the system is changed to an overdamped system with a small  $\omega_n$  and a large  $\zeta$ . Considering the design criteria, we decide the values  $\alpha = 1$  and  $\beta = 1.5$  for desired transient response to the unit-step input determining final steady-state feedback values  $K_{pf} = 10$  and  $K_{vf} = 5$ .

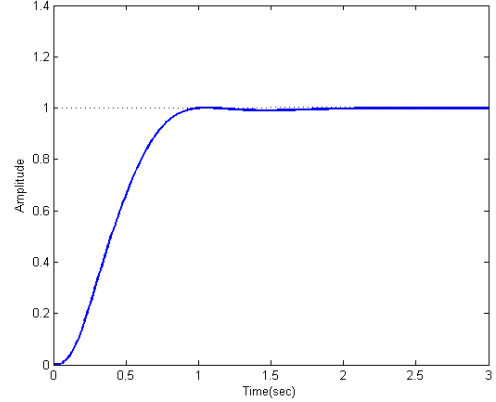
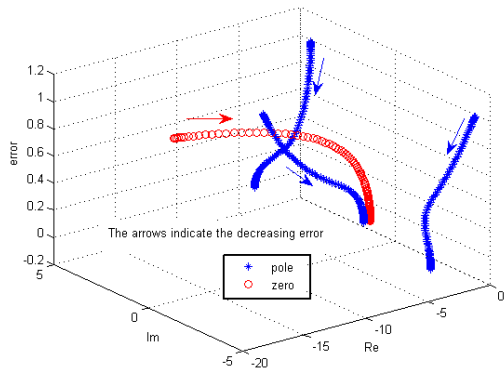


Fig. 8. The transient response of the closed-loop system to unit-step input with  $T_r = 0.7$  (sec) (90%) and  $M_p = 0\%$ .

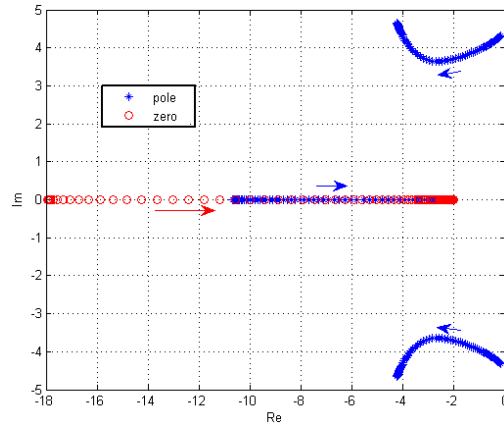
The transient response curve of the closed-loop system for a unit-step input is shown in Fig. 8. This response shows a very fast response ( $T_r = 0.7$ ) with no overshoot improving 30% of the rise time. In Fig. 12, we present the movement of the closed-loop poles and zeros as a function of the system error. The trajectory of the poles and zeros is named Dynamic Pole-Zero Locus (DPZL).

REFERENCES

- [1] Y. C. Chang, "An adaptive H-infinity tracking control for a class of nonlinear multiple-input-multiple-output (MIMO) systems," *Ieee Transactions on Automatic Control*, vol. 46, pp. 1432-1437, Sep 2001.
- [2] D. G. Taylor, P. V. Kokotovic, R. Marino, and I. Kanellakopoulos, "Adaptive Regulation of Nonlinear-Systems with Unmodeled Dynamics," *Ieee Transactions on Automatic Control*, vol. 34, pp. 405-412, Apr 1989.
- [3] Y. S. Liu and X. Y. Li, "Robust adaptive control of nonlinear systems represented by input-output models," *Ieee Transactions on Automatic Control*, vol. 48, pp. 1041-1045, Jun 2003.
- [4] M. Jankovic, R. Sepulchre, and P. V. Kokotovic, "Global adaptive stabilization of cascade nonlinear systems," *Automatica*, vol. 33, pp. 263-268, Feb 1997.
- [5] R. Sepulchre, M. Jankovic, and P. V. Kokotovic, "Integrator forwarding: A new recursive nonlinear robust design," *Automatica*, vol. 33, pp. 979-984, May 1997.
- [6] E. D. Sontag, "A Universal Construction of Artstein Theorem on Nonlinear Stabilization," *Systems & Control Letters*, vol. 13, pp. 117-123, Aug 1989.
- [7] F. Al-Bender, W. Symens, J. Swevers, and H. Van Brussel, "Theoretical analysis of the dynamic behavior of hysteresis elements in mechanical systems," *International Journal of Non-Linear Mechanics*, vol. 39, pp. 1721-1735, Dec 2004.
- [8] Y. Feng, Y. M. Hu, C. A. Rabbath, and C. Y. Su, "Robust adaptive control for a class of perturbed strict-feedback non-linear systems with unknown Prandtl-Ishlinskii hysteresis," *International Journal of Control*, vol. 81, pp. 1699-1708, 2008.
- [9] I. D. Mayergoyz, "Mathematical-Models of Hysteresis," *Ieee Transactions on Magnetics*, vol. 22, pp. 603-608, Sep 1986.
- [10] H. J. Shieh, F. J. Lin, P. K. Huang, and L. T. Teng, "Adaptive displacement control with hysteresis modeling for piezoactuated positioning mechanism," *Ieee Transactions on Industrial Electronics*, vol. 53, pp. 905-914, Jun 2006.
- [11] S. N. Huang, K. K. Tan, and T. H. Lee, "Adaptive motion control using neural network approximations," *Automatica*, vol. 38, pp. 227-233, Feb 2002.
- [12] P. Ge and M. Jouaneh, "Tracking control of a piezoceramic actuator," *Ieee Transactions on Control Systems Technology*, vol. 4, pp. 209-216, May 1996.



(a) DPZL: As the error decreases, the poles move from initial underdamped position to the final overdamped positions.



(b) Projection of (a): The poles and zeros of the closed-loop system.

Fig. 9. DPZL with respect to the system error to a unit-step input. The red and blue arrows indicates the decreasing error and the direction of movement of poles and zeros.

IV. CONCLUSIONS

In this paper, we have proposed the design of an error-based robust adaptive controller for controlling the dynamic response of a system. The proposed error-based adaptive controller is the controller with continuously changing feedback parameters as a function of the system error: initially for large errors an underdamped system which is forced to approach to become an overdamped system for small errors. In the beginning, for large errors, the system is underdamped, thus, it makes the system faster with a wider bandwidth. As the error changes, the value of the feedback gains  $K_p$  and  $K_i$  change. The design of this robust adaptive controller is conceptually error-based and can be used to handle the complexity of systems. From the simulation studies, it is shown that the transient response of the closed-loop system is very fast yielding a very small rise time with no overshoot. A proper design of the controller guarantees that the changing pole position is always are in the left-hand plane, thus guaranteeing the stability of the controlled system. Further work is under way to extend this error-based robust adaptive controller design philosophy for higher-order complex dynamic systems.