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LOCAL NUSSELT NUMBERS FOR FLOW THROUGH ASYMMETRICALLY HEATED PARALLEL PLATE CHANNELS BY SUPERPOSITION

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ABSTRACT

Explicit relations to calculate local Nusselt number from the values corresponding to the boundary condition of first kind have been developed for forced convective flow through asymmetrically heated parallel plate channels. The validity of the superposition relations has been established from the numerical results obtained for the simple model of hydrodynamically developed and thermally developing flow. However, the superposition relations can be expected to be valid, as long as geometric and flow symmetry are preserved.

NOMENCLATURE

- A parameter to characterize asymmetry, $(T_{w2}-T_i)/(T_{w1}-T_i)$
- C_p specific heat of the fluid, J/ (kg-K)
- D_h hydraulic diameter, (= $2L$), m
- h_{1x} local heat transfer coefficient based on T_{w1} and T_b at the wall at $y = -L/2$, W/(m^2-K)
- h_{2x} local heat transfer coefficient based on T_{w2} and T_b at the wall at $y = + L/2$, W/(m^2-K)
- h_{11} local heat transfer coefficient corresponding to $A = 0$ at the wall at $y = -L/2$ based on T_{w1} and the bulk mean temperature
- h_{12} local heat transfer coefficient corresponding to $A = 0$ at the wall at $y = L/2$ based on T_i and the bulk mean temperature
- h_{21} local heat transfer coefficient corresponding to $A \to \infty$ at the wall at $y = -L/2$ based on T_i and the bulk mean temperature
- h_{22} local heat transfer coefficient corresponding to $A \to \infty$ at the wall at $y = L/2$ based on T_{w2} and the bulk mean temperature
- k thermal conductivity of the fluid, W/(m-K)
- L spacing between the two plates, m
- Nu_x local Nusselt number based on hydraulic diameter when $A = 1$
- Nu_{1x} local Nusselt number at the wall at $Y = -1/4$ based on T_{w1}, T_b and hydraulic diameter, $A \neq 1$
- Nu_{2x} local Nusselt number at the wall at $Y = 1/4$ based on T_{w2}, T_b and hydraulic diameter, $A \neq 1$
- Nu_{11} local Nusselt number at the wall at $Y = -1/4$ corresponding to $A = 0$
- Nu_{12} local Nusselt number at the wall at $Y = 1/4$ corresponding to $A = 0$
- Nu_{21} local Nusselt number at the wall at $Y = -1/4$ corresponding to $A \to \infty$
- Nu_{22} local Nusselt number at the wall at $Y = 1/4$ corresponding to $A \to \infty$
- Pe Peclet number, $(Re.Pr = U_{avg}\, D_h / \alpha)$
- Pr Prandtl number, $(Re = (\mu\, C_p)/k)$
- Re Reynolds number, $(=U_{avg}\, D_h / \nu)$
- T dimensional fluid temperature, K
- T_b dimensional bulk mean temperature, K
- T_{b1} dimensional bulk mean temperature for the case of $A = 0$
- T_{b2} dimensional bulk mean temperature for the case of $A \to \infty$
- T_i dimensional inlet fluid temperature, K
- T_w dimensional temperature at the walls at $y = \pm L/2$, K
- T_1 dimensional fluid temperature for the case of $A = 0$
- T_2 dimensional fluid temperature for the case of $A \to \infty$
- T_{w1} temperature at the wall at $y = -L/2$, K
- T_{w2} temperature at the wall at $y = L/2$, K
- $\overline{T}_w$ average wall temperature, $(T_{w1} + T_{w2})/2$, K
- U non-dimensional velocity in X direction = u/ U_{avg}
- u dimensional velocity in x direction, m/s
- U_{avg} average velocity through the channel employed as the reference velocity, m/s
- X non-dimensional coordinate in the axial direction = x/D_h
- X^* X/Pe
- x coordinate in the axial direction
- Y non-dimensional coordinate normal to the axis of the channel
- y coordinate normal to the axis of the channel

Greek Symbols

- α thermal diffusivity, m^2/s
- θ non-dimensional temperature, $\{= (T - \overline{T}_w)/ (T_i-\overline{T}_w)\}$
- θ_i non-dimensional temperature at the inlet
- θ_b non-dimensional temperature based on bulk mean temperature, $\{= (T-\overline{T}_w)/(T_b-\overline{T}_w)\}$
- θ_1 non-dimensional temperature for the case of $A = 0$, $\{(T_1 - T_{w1})/(T_i - T_{w1})\}$

θ_2 non-dimensional temperature for the case of $A \to \infty$, $\{(T_2 - T_{w2})/(T_i - T_{w2})\}$

θ_{w1} non-dimensional temperature at the wall at $Y = -1/4$, $\{-(1 - A)/(1 + A)\}$

θ_{w2} non-dimensional temperature at the wall at $Y = +1/4$, $\{(1 - A)/(1 + A)\}$

θ^* non-dimensional bulk mean temperature of the fluid in general, $\{= (T_b - \overline{T}_w)/(T_i - \overline{T}_w)\}$

θ_1^* non-dimensional bulk mean temperature for the case of $A = 0$, $(T_{b1}(x) - T_{w1})/(T_i - T_{w1})$

θ_2^* non-dimensional bulk mean temperature for the case of $A \to \infty$, $(T_{b2}(x) - T_{w2})/(T_i - T_{w2})$

μ dynamic viscosity, kg/m-s

ρ density of the fluid, kg / m^3

INTRODUCTION

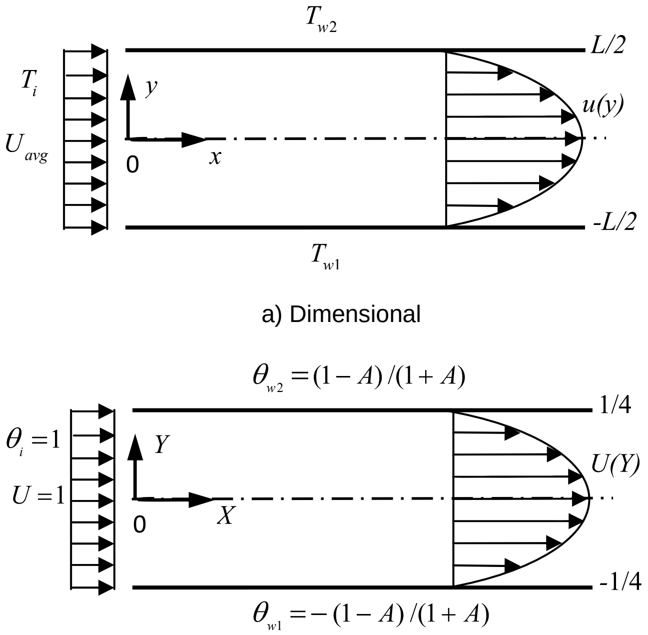
Increasing attention is being paid to study heat transfer in asymmetrically heated ducts in general owing to newer applications, such as cooling of electronic equipment, using materials involving hyper porous media or micro channels, and fuel cells. Early pioneering studies by Graetz dealing with forced convective heat transfer in pipes assumed hydrodynamically and thermally fully developed conditions. An excellent review on the studies involving laminar heat transfer in ducts prior to 1978 is available in Shah and London [1]. Heat transfer in parallel plate channel subjected to constant but unequal temperatures has been first studied by Hatton and Turton [2]. Hatton and Turton assumed fully developed flow and developing thermal field but neglected axial conduction and obtained series solution. Recently, Mitrovic et al. [3] referred this problem as asymmetric Graetz problem and obtained numerical solutions within the same framework of Hatton and Turton. Mitrovic and Maletic [4, 5] also investigated laminar forced convection heat transfer in asymmetrically heated annuli and channels filled with porous material. In connection with mixed convection studies in vertical channels, Barletta [6] described asymmetry in a binary form; unity when the temperatures are unequal and zero when the temperatures are equal. Ramjee Repaka [7] characterized the asymmetry by a parameter, A, defined as the ratio of wall temperatures in excess of the fluid inlet temperature.

It has been mentioned about the applicability of superposition in the literature in obtaining the solutions of energy equation and derived quantities (such as Nusselt number) for different boundary conditions. Superposition relations for Nusselt number in terms of influence coefficients are available, say in, Kays and Crawford [8], for annuli and channels when the walls are subjected to uniform but unequal heat flux. Such explicit relations could not be found by the authors even after considerable efforts for the case of constant but unequal temperatures.

In this article superposition relations to obtain the local Nusselt number values Nu_{1x} and Nu_{2x} at the two walls (kept at constant, but unequal temperatures, T_{w1} and T_{w2}) of the channel in terms of the Nusselt number values corresponding to the boundary condition of the first kind [Shah and London [1], p. 33] have been developed. The relations developed have been validated with the numerically obtained solutions employing the Successive Accelerated Replacement Scheme (Satyamurty [9], Marpu and Satyamurty [10]), the details of which are also available in Ramjee Repaka and Satyamurty [11].

PHYSICAL MODEL AND ASYMMETRY DESCRIPTION

The physical model in general and the coordinate system are shown in Fig. 1. The parallel plates are L distance apart. x coordinate is measured along the axis of the channel and the normal coordinate y is measured from the axis of the channel. The fluid enters at a uniform temperature of T_i. The plates at $y = -L/2$ and $L/2$ are kept at temperatures T_{w1} and T_{w2} respectively. The flow and temperature fields may be developing or developed.



a) Dimensional

(b) Non-Dimensional

Fig.1 PHYSICAL MODEL AND THE COORDINATE SYSTEM

X and Y, the non-dimensional axial distance and normal coordinates are defined as,

$$X = x/D_h,\ Y = y/D_h \tag{1}$$

In Eq. (1), $D_h = 2L$ is the hydraulic diameter.

A, the asymmetry parameter, to characterize the unequal wall temperatures, is defined by,

$$A = (T_{w2} - T_i)/(T_{w1} - T_i) \tag{2}$$

$A = 0$ and $A = \infty$ correspond to boundary condition of the first kind described in, p. 33 of Shah and London [1]. As per the present definition of A given by Eq. (2), $A = 0$, corresponds to $T_{w2} = T_i$, and $A = \infty$, corresponds to, $T_{w1} = T_i$.

Further, θ , the non-dimensional temperature and θ^* the non-dimensional bulk mean temperature are defined by,

$$\left.\begin{array}{l} \theta = (T - \overline{T}_w)/(T_i - \overline{T}_w) \text{ and} \\ \theta^* = (T_b - \overline{T}_w)/(T_i - \overline{T}_w) = \int_{-1/4}^{1/4} U\theta dY \Big/ \int_{-1/4}^{1/4} UdY \end{array}\right\} \tag{3}$$

$\overline{T}_w$ $\{=(T_{w1} + T_{w2})/2\}$ is the average wall temperature. U (= u/U_{avg}) is the non-dimensional velocity in the axial direction. In Eq. (3), T_b, the dimensional bulk mean temperature is defined by,

$$T_b = \int_{-L/2}^{L/2} uTdy \Big/ \int_{-L/2}^{L/2} udy \tag{4}$$

In Eq. (4), $u(y)$ is the velocity in the axial direction.

The local heat transfer coefficient at the wall at $y = -L/2$, h_{1x}, at x, is defined by,

$$-k\left(\partial T/\partial y\right)\big|_{y=-L/2} = h_{1x}\,(T_{w1} - T_b) \tag{5}$$

From Eq. (5), it follows that Nu_{1x}, the local Nusselt number at $Y = -1/4$, based on the hydraulic diameter is given by,

$$Nu_{1x} = \left(h_{1x}\, D_h\right)/k = \{1/(\theta^* - \theta\big|_{Y=-1/4})\}\left(\partial\theta/\partial Y\right)\big|_{Y=-1/4} \tag{6}$$

Similarly, the local Nusselt number at the second wall, $Y = 1/4$, is given by,

$$Nu_{2x} = \left(h_{2x}\, D_h\right)/k = -\{1/(\theta^* - \theta\big|_{Y=1/4})\}\left(\partial\theta/\partial Y\right)\big|_{Y=1/4} \tag{7}$$

For the sake of completeness, we state, $A = 1$ corresponds to walls kept at equal temperature, and, $\theta\big|_{Y=-1/4} = \theta\big|_{Y=1/4} = 0$. When $A = 1$, Eqs. (6) and (7) yield,

$$Nu_{1x} = Nu_{2x} = Nu_x = (1/\theta^*)\left(\partial\theta/\partial Y\right)\big|_{Y=-1/4} = -(1/\theta^*)\left(\partial\theta/\partial Y\right)\big|_{Y=1/4} \tag{8}$$

It is easy to see that, it is sufficient to obtain solutions in the finite interval $-1 \le A \le 1$, since it can be envisaged that $\theta(Y;\, A) = \theta(-Y;\, 1/A)$. Thus, the Nusselt numbers at the two walls for a given A and $1/A$ get interchanged.

SUPERPOSITION RELATIONS

Non-Dimensional Temperature

As per the present definition of A, the configurations corresponding to $A = 0$ and $A \to \infty$ are shown in Figs. 2 and 3 respectively. The non-dimensional temperature profiles are shown in Fig. 2 (b) and Fig. 3 (b). For the purpose of developing the superposition relations the non-dimensional temperatures $\theta_1(X, Y)$ and $\theta_2(X, Y)$ for the two cases, depicted in Fig. 2 and Fig. 3 respectively, are defined as,

$$\theta_1(X, Y) = (T_1(x, y) - T_{w1})/(T_i - T_{w1}) \tag{9}$$

$$\theta_2(X, Y) = (T_2(x, y) - T_{w2})/(T_i - T_{w2}) \tag{10}$$

Slight difference in defining $\theta_1(X, Y)$ and $\theta_2(X, Y)$ compared to $\theta(X, Y)$, {Eq. (3)} is to be noted. Reference temperatures are T_{w1} and T_{w2} in defining θ_1 and θ_2, whereas it is $\overline{T}_w = (T_{w1} + T_{w2})/2$ in defining θ as per Eq. (3). The cases depicted in Figs. 2 and 3 continue to be described by the asymmetry parameter $A = 0$ and ∞.

It is easy to see that, $\theta_1(X, Y)$ varies from 0 to 1 and $\theta_2(X, Y)$ varies from 1 to 0 as Y varies from $-1/4$ to $1/4$ and it follows,

$$\theta_1(X, Y) = \theta_2(X, -Y) \tag{11}$$

$$\theta_2(X, Y) = \theta_1(X, -Y) \tag{12}$$

The symmetry of the fluid flow is implicit in writing Eqs. (11) and (12). The general depiction when the walls are kept at T_{w1} and T_{w2} is shown in Fig. 4.

From the definition of $\theta(X, Y)$ in Eq. (3), it follows,

$$T(x, y) = \theta(X, Y)\,(T_i - \overline{T}_w) + \overline{T}_w \tag{13}$$

By superposition, $T(x, y)$ can be expressed as,

$$T(x, y) - T_i = T_1(x, y) - T_i + T_2(x, y) - T_i \tag{14}$$

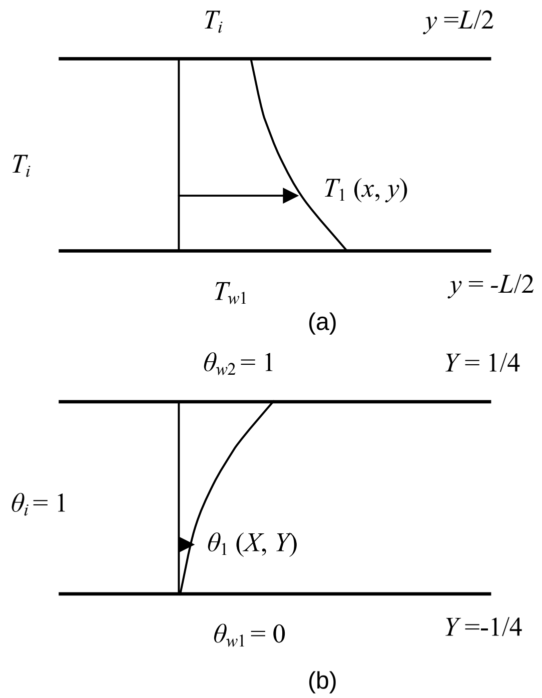


Fig. 2 REPRESENTATION for A = 0; (a) DIMENSIONAL (b) NON-DIMENSIONAL

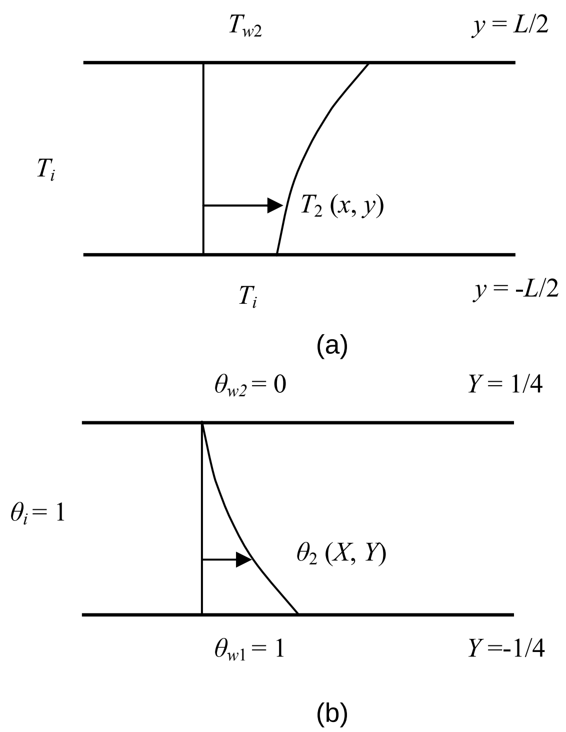


Fig. 3 REPRESENTATION for $A = \infty$; (a) DIMENSIONAL (b) NON-DIMENSIONAL

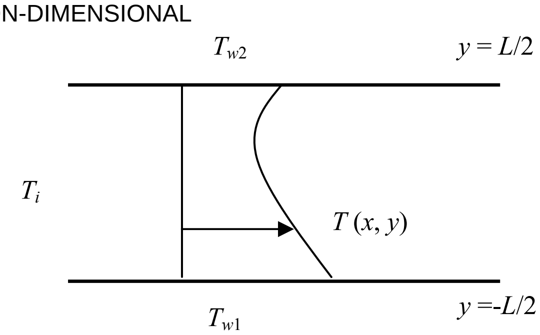


Fig. 4 REPRESENTATION FOR A GENERAL A

It may be recalled that $T_1(x, y)$ and $T_2(x, y)$ correspond to $A = 0$ and ∞ as depicted in Fig. 2 and Fig. 3 respectively.

Using Eqs. (9) and (10) in Eq. (14),

$$T(x, y) - T_i = \theta_1(X, Y)\,(T_i - T_{w1}) + T_{w1} + \theta_2(X, Y)\,(T_i - T_{w2}) + T_{w2} - 2\,T_i \qquad (15)$$

Employing Eq. (3) to construct $\theta(X, Y)$, it follows from Eq. (15) that,

$$\theta(X,Y) = 1 + \left\{1 - \theta_1(X,Y)\right\}(\theta_{w1} - 1) + \left\{1 - \theta_2(X,Y)\right\}(\theta_{w2} - 1) \qquad (16)$$

Equation (16) can be rewritten by using Eq. (12) as,

$$\theta(X,Y) = 1 + \left\{1 - \theta_1(X,Y)\right\}(\theta_{w1} - 1) + \left\{1 - \theta_1(X,-Y)\right\}(\theta_{w2} - 1) \qquad (17)$$

It may be noted that the dimensional temperature $T(x, y)$ obtained from $\theta(X, Y)$ for $A = 0$, is the same as $T_1(x, y)$ obtained from $\theta_1(X, Y)$ for a given T_{w1} and T_i. Further, $\overline{T}_w$ in $\theta(X, Y)$ for $A = 0$ is given by, $(T_{w1} + T_i)/2$. Thus,

$$T(x,y) = \{\theta(X, Y; A = 0)\{T_i - (T_{w1} + T_i)/2\}\} + (T_{w1} + T_i)/2\}$$
$$= T_1(x,y) = \theta_1(X, Y)\,(T_i - T_{w1}) + T_{w1} \qquad (18)$$

Considering the equality $\{\theta(X, Y; A = 0)\{T_i - (T_{w1} + T_i)/2)\} + (T_{w1} + T_i)/2\} = \theta_1(X, Y)\,(T_i - T_{w1}) + T_{w1}$ of Eq. (18), simplification yields,

$$\theta(X, Y; A = 0) = 2\,\theta_1(X, Y) - 1 \qquad (19)$$

Making use of Eq. (19) in Eq. (17), the non-dimensional temperature $\theta(X, Y)$ for any desired A can be obtained from $\theta(X, Y)$ for $A = 0$ as,

$$\theta(X,Y;A) = 1 + \frac{\{1 - \theta(X,Y;A=0)\}}{2}(\theta_{w1} - 1) + \frac{\{1 - \theta(X,-Y;A=0)\}}{2}(\theta_{w2} - 1) \qquad (20)$$

Non-Dimensional Bulk Mean Temperature

Equation (14) can be applied to relate the bulk mean temperatures, resulting in,

$$T_b(x) - T_i = \{T_{b1}(x) - T_i\} + \{T_{b2}(x) - T_i\} \qquad (21)$$

Where $T_{b1}(x)$ and $T_{b2}(x)$ are the bulk mean temperatures calculated adapting Eq. (4) suitably for the two cases depicted by Figs. 2 and 3. The corresponding non-dimensional bulk mean temperatures θ_1^* and θ_2^*, are defined by,

$$\theta_1^* = \left(T_{b1}(x) - T_{w1}\right)/\left(T_i - T_{w1}\right) \qquad (22)$$

$$\theta_2^* = \left(T_{b2}(x) - T_{w2}\right)/\left(T_i - T_{w2}\right) \qquad (23)$$

Introducing the non-dimensional bulk mean temperatures θ_1^* and θ_2^* in Eq. (21),

$$T_b(x) - T_i = \theta_1^*\,(T_i - T_{w1}) + \theta_2^*\,(T_i - T_{w2}) + (T_{w1} - T_i) + (T_{w2} - T_i) \qquad (24)$$

Equation (24) can be rearranged as,

$$T_b(x) - T_i = (T_{w1} - T_i)\,(1 - \theta_1^*) + (T_{w2} - T_i)\,(1 - \theta_2^*) \qquad (25)$$

Dividing Eq. (25) throughout by $(T_i - \overline{T}_w)$,

$$(\theta^* - 1) = (1 - \theta_1^*)\{(T_{w1} - T_i)/(T_i - \overline{T}_w) + (1 - \theta_2^*)\{(T_{w2} - T_i)/(T_i - \overline{T}_w)\} \qquad (26)$$

$$(\theta^* - 1) = (1 - \theta_1^*)(\theta_{w1} - 1) + (1 - \theta_2^*)\,(\theta_{w2} - 1) \qquad (27)$$

By virtue of Eqs. (11) and (12), it may be noted that,

$$\theta_1^* = \theta_2^* \qquad (28)$$

Making use of Eq. (28) and noting that $\theta_{w1} = -\theta_{w2}$ {see, the boundary conditions given in Fig. 1 (b)}, Eq. (27) reduces to

$$\theta^* = 1 - 2\left(1 - \theta_1^*\right) \qquad (29)$$

Equation (29) implies that θ^* is independent of A since θ_1^* is for a specific value of A whereas θ^* may be for any A. It is easy to envisage that the bulk mean temperature is independent of the asymmetry for a given average of wall temperatures.

Nusselt Number

Nu_{11} and Nu_{12}, the Nusselt numbers at the walls $y = -L/2$ with temperature T_{w1} and at $y = L/2$ with temperature T_i, for the case $A = 0$ depicted in Fig. 2 are given by,

$$Nu_{11} = \frac{h_{11}\,D_h}{k} = \frac{1}{\theta_1^*}\frac{\partial\theta_1}{\partial Y}\bigg|_{Y=-1/4} \qquad (30)$$

$$Nu_{12} = \frac{h_{12}\,D_h}{k} = -\frac{1}{\theta_1^*}\frac{\partial\theta_1}{\partial Y}\bigg|_{Y=1/4} \qquad (31)$$

Similarly, Nu_{21} and Nu_{22}, the Nusselt numbers for the case $A \to \infty$, depicted in Fig. 3 are given by,

$$Nu_{21} = \frac{h_{21}\,D_h}{k} = \frac{1}{\theta_2^*}\frac{\partial\theta_2}{\partial Y}\bigg|_{Y=-1/4} = -\frac{1}{\theta_1^*}\frac{\partial\theta_1}{\partial Y}\bigg|_{Y=1/4} \qquad (32)$$

$$Nu_{22} = \frac{h_{22}\,D_h}{k} = -\frac{1}{\theta_2^*}\frac{\partial\theta_2}{\partial Y}\bigg|_{Y=1/4} = \frac{1}{\theta_1^*}\frac{\partial\theta_1}{\partial Y}\bigg|_{Y=-1/4} \qquad (33)$$

Last equalities in Eqs. (32) and (33) follow from Eqs. (11) and (12) and Eq. (28). Noting the interrelationship between $\theta_1(X, Y)$, $\theta_2(X, Y)$ and $\theta(X, Y)$ given by Eq. (16) and recalling Eqs. (6) and (7), the Nusselt numbers at $Y = -1/4$ and $1/4$, Nu_{1x} and Nu_{2x} (i.e., the Nusselt number values for a desired A) can be expressed in terms of Nu_{11}, Nu_{21} and Nu_{12}, Nu_{22} as

$$Nu_{1x} = \frac{Nu_{11}\left(\theta^* + 1\right)}{\left(\theta^* + 1\right)(A + 1) - 2A} + \frac{Nu_{21}\left(\theta^* - 1\right)A}{\left(\theta^* + 1\right)(A + 1) - 2A} \qquad (34)$$

$$Nu_{2x} = \frac{Nu_{12}\left(\theta^* - 1\right)}{\left(\theta^* + 1\right)(A + 1) - 2} + \frac{Nu_{22}\left(\theta^* + 1\right)A}{\left(\theta^* + 1\right)(A + 1) - 2} \qquad (35)$$

It follows from Eqs. (30) to (33),

$$Nu_{11} = Nu_{22} \qquad (36)$$

$$Nu_{12} = Nu_{21} \qquad (37)$$

Equations (34) and (35) simplify to,

$$Nu_{1x} = \frac{Nu_{11}\left(\theta^* + 1\right)}{\left(\theta^* + 1\right)(A + 1) - 2A} + \frac{Nu_{12}\left(\theta^* - 1\right)A}{\left(\theta^* + 1\right)(A + 1) - 2A} \qquad (38)$$

$$Nu_{2x} = \frac{Nu_{12}\left(\theta^* - 1\right)}{\left(\theta^* + 1\right)(A + 1) - 2} + \frac{Nu_{11}\left(\theta^* + 1\right)A}{\left(\theta^* + 1\right)(A + 1) - 2} \qquad (39)$$

Thus, Nu_{1x} and Nu_{2x} for a desired A can be calculated if Nu_{11} and Nu_{12}, the Nusselt number values at the wall $Y = -1/4$ and at the wall $Y = 1/4$ for $A = 0$ and the bulk mean temperature θ^* which is independent of A are known. Though, Nu_{1x} as calculated from Eq. (38) becomes equal to Nu_{2x} obtained from Eq. (39) for $A = 1$, the relations do not yield the Nu_x values for $A = 1$.

VALIDATION OF SUPERPOSITION RELATIONS

Numerical results for the simple case of fully developed velocity profile and developing temperature neglecting axial conduction for $-1 < A < 1$ values have been obtained employing the Successive Accelerated Replacement (SAR) scheme (Satyamurty [9], Marpu and Satyamurty [10], Ramjee Repaka and Satyamurty, [11]). The details are also available in Ramjee Repaka [7]. The non-dimensional governing equations and the boundary conditions for the system depicted in Fig. 1 with the assumptions mentioned are as following.

$$U\left(\frac{\partial \theta}{\partial X^*}\right) = \left(\frac{\partial^2 \theta}{\partial Y^2}\right) \quad (40)$$

U, the fully developed velocity in Eq. (40) is given by

$$U = (u/U_{avg}) = (3/2)\ \{1-(4Y)^2\} \quad (41)$$

The relevant thermophysical properties are, ρ, the density, C_p, the specific heat, k, the thermal conductivity and α (= $k/\rho C_p$) is the thermal diffusivity. X^* in Eq. (40) is the normalized non-dimensional axial distance given by,

$$X^* = X/Pe \quad (42)$$

Pe the Peclet number in Eq. (42) is defined by,

$$Pe = (U_{avg}\, D_h)/\alpha = Re\, Pr \quad (43)$$

Where Re, the Reynolds number and Pr, the Prandtl number are defined by,

$$Re = (\rho\, U_{avg}\, D_h)/\mu,\ Pr = (\mu\, C_p)/k \quad (44)$$

Solutions to Eq. (40) are sought subject to the following boundary conditions.

$$\left.\begin{aligned} &\theta = 1 \quad \text{at } X^* = 0 \text{ for } -1/4 \le Y \le +1/4 \\ &\theta = \theta_{w1} = -(1 - A)/(1 + A) \text{ at } Y = -1/4 \text{ for all } X^* > 0 \\ &\theta = \theta_{w2} = (1 - A)/(1 + A) \text{ at } Y = 1/4 \text{ for all } X^* > 0 \end{aligned}\right\} \quad (45)$$

Non-Dimensional Temperature

In order to validate the superposition relation for $\theta(X, Y)$, Eq. (20), a normalized difference $\Delta\theta_n$, in the θ values obtained numerically and from the superposition relation has been defined as following.

$$\Delta\theta_n = \frac{(\theta_{\text{sup}} - \theta_{\text{num}})}{\theta_{\text{sup}}} \quad (46)$$

θ_{sup} and θ_{num} in Eq. (46) are the values of θ obtained from the superposition relation and from numerical solution respectively. A plot of $\Delta\theta_n$ vs. θ_{sup} is shown in Fig. 5. The normalized difference is < 0.0005, implying less than 0.05 %, that too at values of θ close to zero.

Nusselt Number

From the numerical solutions to Eq. (40) obtained employing the SAR [7, 9, 10, 11] scheme, local Nusselt number values corresponding to $A = 0$, Nu_{11} and Nu_{12}, needed to apply the superposition relations {Eqs. (38) and (39)}, have been calculated. Variation of Nu_{11} and Nu_{12} with X^* is shown in Fig. 6 (a) and (b) respectively. For large X^*, both Nu_{11} and $Nu_{12} \rightarrow 4$. The values of Nu_{11} and Nu_{12} for all X^* obtained by the present numerical scheme agree well with the depictions available in Mitrovic et al. [3].

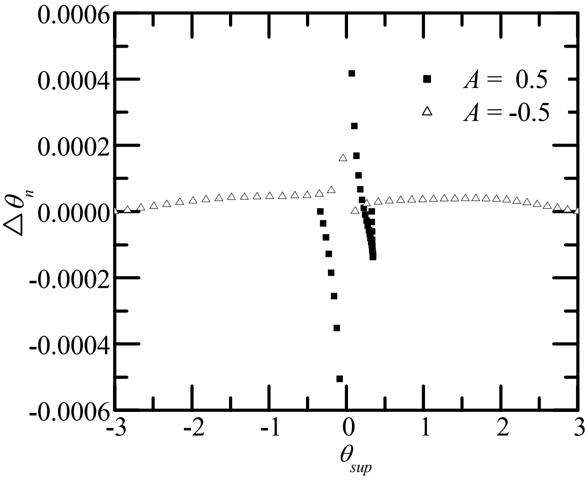
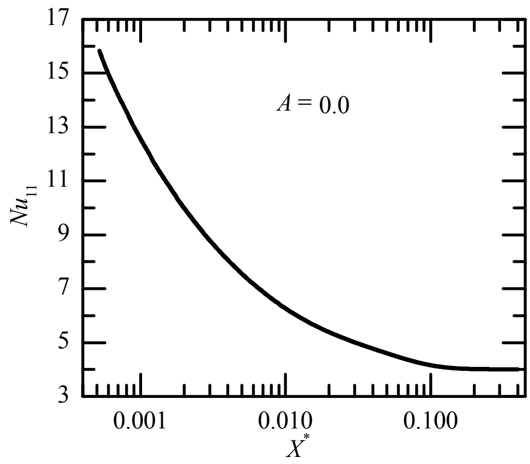
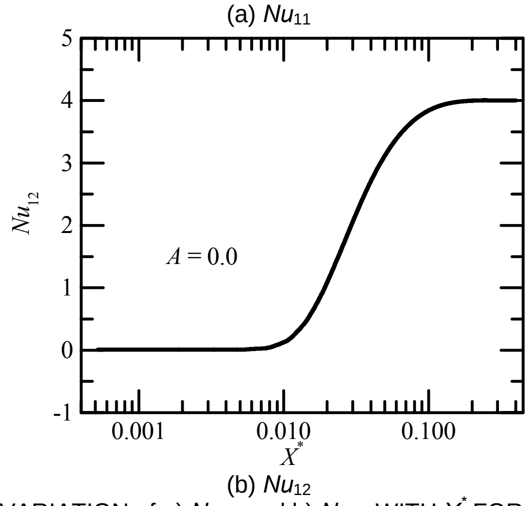


FIG. 5 NORMALIZED DIFFERENCE BETWEEN THE θ VALUES OBTAINED NUMERICALLY AND BY SUPERPOSITION



(a) Nu_{11}



(b) Nu_{12}

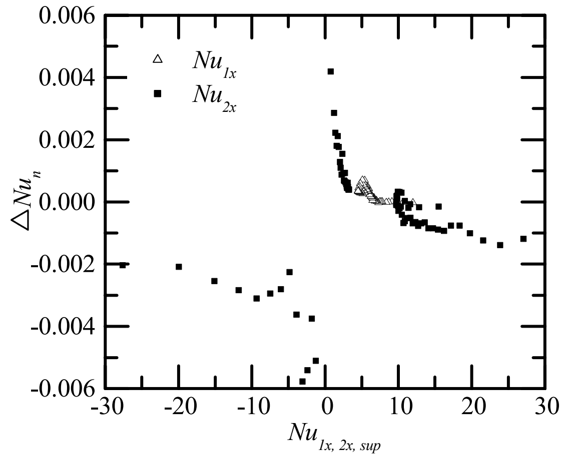
Fig. 6 VARIATION of a) Nu_{11} and b) Nu_{12} WITH X^* FOR $A = 0$

Values of Nu_{1x} and Nu_{2x} have been calculated for different values of A using the superposition relations given by Eqs. (38) and (39) employing the numerically obtained values of Nu_{11} and Nu_{12}. In order to validate the superposition relations for Nu_{1x} and Nu_{2x} a normalized difference ΔNu_n, has been defined as following.

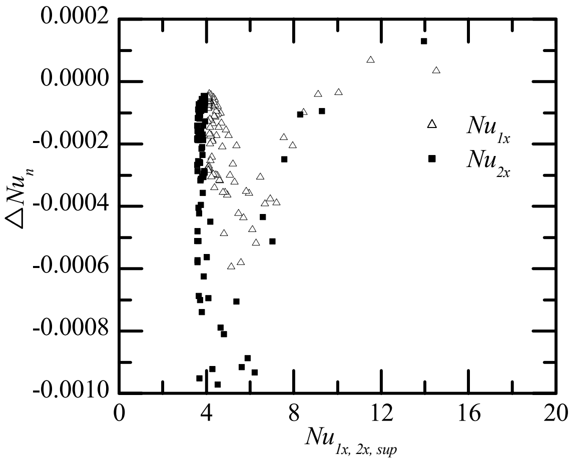
$$\Delta Nu_n = \frac{(Nu_{1x,2x,\text{sup}} - Nu_{1x,2x,\text{num}})}{Nu_{1x,2x,\text{sup}}} \quad (47)$$

The additional subscript $_{sup}$ and $_{num}$ for Nu_{1x} and Nu_{2x} indicate the values obtained from superposition relations {Eqs. (38) and (39)} and from numerical solutions. A plot of ΔNu_n vs. $Nu_{1x,\,2x,\,sup}$ is shown in Fig. 7 (a) for A = 0.5 and in Fig. 7 (b) for A = -0.5. The normalized difference ΔNu_n < 0.006 for A = 0.5 and, ΔNu_n < 0.001 for A = -0.5 which correspond to 0.6 % and 0.1 % respectively. Also, Nu_{2x} values have been limited to 30, even though Nu_{2x} displays an unbounded swing {see, Hatton and Turton [2] or Mitrovic et al. [3]} for A > 0. Relatively larger ΔNu_n for A = 0.5 is due to this unbounded swing where Nu_{2x} passes through zero.

A note on the Nusselt number values for A = 1 (the case of symmetric heating) is to be made. For A = 1, the fully developed value of 7.54 (see, Shah and London [1]) continues to be 7.54, for further larger X^*, though $\theta \to 0$ in the limit. A comparison of Nu_x values obtained numerically and from superposition relations is shown in Table 1. It is interesting to note that the superposition relations predict the Nu_x values for A = 1 also, upto X^* = 0.1, which is far beyond the fully developed X^* = 0.04, see, Shah and London [1]. For X^* > 0.1, Nu_x predicted using Eqs. (38) and (39) vary between 7.54 and 8, which is essentially due to Nu_{11} and Nu_{12} being not exactly equal to 4. Even a small difference from 4 in the numerically obtained values of Nu_{11} and Nu_{12}, (e.g., at X^* = 0.2, Nu_{11} = 4.0165, Nu_{12} = 3.9988 and θ^* = 0.00229) leads to Nu_x = 7.861 and not 4. Whereas, If $Nu_{11} = Nu_{12}$ = 4 are used, $Nu_{1x} = Nu_{2x}$ = 4 for any A.



(a) *A = 0.5*

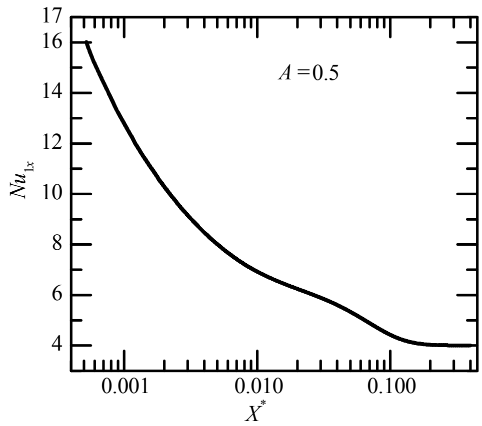


(b) *A = - 0.5*

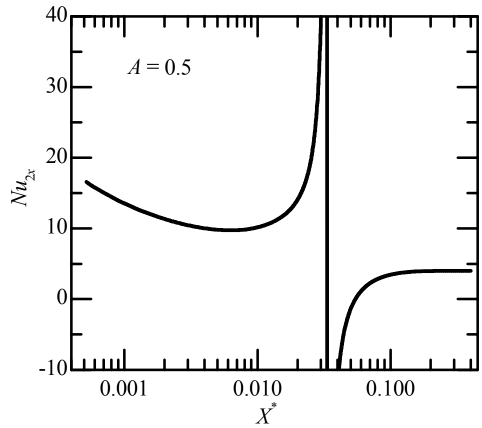
Fig. 7 NORMALIZED DIFFERENCE BETWEEN THE NUSSELT NUMBER VALUES OBTAINED NUMERICALLY AND BY SUPERPOSITION FOR (a) *A = 0.5 AND (*b) *A = - 0.5*

Table 1 NUSSELT NUMBER VALUES FOR *A* = 1, OBTAINED NUMERICALLY AND BY SUPERPOSITION

X^*	0.001	0.005	0.01	0.025	0.04	0.1	0.20
$Nu_{x,\,num}$	12.877	8.538	7.739	7.536	7.53	7.53	7.53
$Nu_{x,\,sup}$	12.877	8.539	7.739	7.536	7.53	7.54	7.86



(a) Nu_{1x}



(b) Nu_{2x}

Fig. 8 VARIATION OF (a) Nu_{1X} AND (b) Nu_{2X} WITH X^* AS OBTAINED BY SUPERPOSITION RELATIONS

Plots of Nu_{1x} and Nu_{2x} vs. X^* for A = 0.5 calculated from the superposition relations, Eqs. (38) and (39) are given in Fig. 8 (a) and (b). Superposition relation for Nu_{2x} {Eq. (39)} captures the feature of unbounded swing and adiabatic points (see, Mitrovic et al. [3]} correctly. It can be seen from Fig. 8 (b) that, the unbounded in Nu_{2x} variation, a consequence of the definition of the heat transfer coefficient, is due to the bulk mean temperature, T_b tends to T_{w2} which makes $h_2 \to \pm \infty$, and hence Nu_{2x}.

NUSSELT NUMBER VALUES FOR *A* = –1

Relations given by Eqs. (38) and (39) do not pose any difficulty in evaluating Nu_{1x} and Nu_{2x} for A = –1, though the boundary conditions given by Eq. (45) on θ for A = –1, pose some problem in obtaining the numerical solutions. Nu_{1x} and Nu_{2x} for A = –1 have been calculated using Eqs. (38) and (39) employing the values of Nusselt numbers for A = 0. It can be seen from Eqs. (38) and (39) that,

$$Nu_{1x} = Nu_{2x} \quad \text{for } A = -1 \tag{48}$$

It may be recalled that $Nu_{1x} = Nu_{2x}$ for A = 1 also. It is to be however noted that this equality is not to be misunderstood as, Nu_{1x} or Nu_{2x} for A = 1, are respectively equal to Nu_{1x} or

Nu_{2x} for $A = -1$. Variation of Nu_{1x} and Nu_{2x} with X^* is shown in Fig. 9 for $A = -1$.

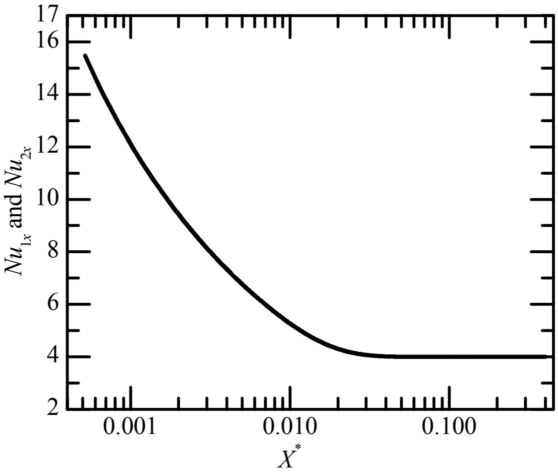


FIG. 9 Nu_{1x} AND Nu_{2x} VARIATIONS WITH X^* FOR $A = -1$

It can be seen from Fig. 9 that, $Nu_{1x} = Nu_{2x} \to 4$ for large values of X^* for $A = -1$ also, as has been the case in general for $A \neq 1$. Further, $Nu_{1x} = Nu_{2x}$ for $A = -1$ just as when $A = 1$. The difference between the two cases is that, when $A = 1$, heat is transferred to the fluid from both the walls or from fluid to both the walls equal in magnitude, whereas, when $A = -1$, heat is transferred from one wall to the fluid and from the fluid to the other wall equal in magnitude. Thus, there is no net heat transfer when $A = -1$.

Values of θ^*, Nu_{11} and Nu_{12} which are needed in calculating the Nusselt numbers from the superposition relations, Eqs. (38) and (39) are given in Table 2 for ready reference.

Table 2 VALUES OF NON-DIMENSIONAL BULK MEAN TEMPERATURE, θ^* AND, Nu_{11} AND Nu_{12}.

X^*	0.001	0.005	0.01	0.025	0.05	0.075	0.1	0.14	0.20
θ^*	0.930	0.794	0.676	0.429	0.202	0.095	0.045	0.013	0.002
Nu_{11}	12.391	7.565	6.261	5.169	4.602	4.314	4.161	4.055	4.016
Nu_{12}	0.044	0.007	0.131	1.615	3.122	3.636	3.841	3.958	3.998

CONCLUSIONS

Nusselt number values for a specified asymmetry in wall temperatures can be calculated from the values for the boundary condition of the first kind (see, Shah and London [1], p. 33), using the superposition relations developed in the present article. The expressions have been validated by comparing with the numerically obtained values employing a simple model. However, the relations can be expected to be of general validity as long as geometric and flow symmetry are preserved and thermophysical properties are constant.

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