Erosion Wear Response of Flyash-Glass Fiber-Polyester Composites:
A Study using Taguchi Experimental Design

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ABSTRACT: Glass fiber reinforced polyester (GFRP) composite finds widespread application in erosive environment due to its several advantages like high wear resistance, high strength-to-weight ratio and low cost. Flyash, an industrial waste, has tremendous potential to be utilized as filler material in polymer based composites because it is basically a mixture of oxide ceramics. This study makes use of a methodology based on Taguchi’s experimental design approach to characterize the erosion response of these composites. The procedure eliminates the need for repeated experiments and thus saves time, materials and cost. It identifies not only the significant control factors but also their interactions influencing the erosion rate predominantly. A mathematical model based on ductile/semi-ductile mode of erosion is proposed to gain insight on the wear mechanism. Determination of erosion rate using theoretical model confirms with experimental results and hence it can be used as a good predictive model. This model is validated by performing a confirmation experiment with an arbitrarily chosen set of factor combination. Finally, optimal factor settings for minimum wear rate under specified experimental conditions have been determined using genetic algorithm.

Keywords: Glass fiber, flyash, erosion rate, Taguchi method, genetic algorithm.

1.0 INTRODUCTION

Polymers are finding an ever-increasing application as structural materials in various components and engineering systems. The high specific strength and stiffness of polymers are primarily responsible for their popularity. However, the resistance of polymers to solid particle erosion has been found to be very poor [1]. In fact, it is two or three orders of magnitude lower than metallic materials [2]. One possible way to overcome such a shortcoming is to introduce a second phase in the polymer to form polymer matrix composites (PMCs). In order to obtain the desired material characteristics for a particular application, it is important to know how the changes in performance characteristics of composite occur with filler content under given loading conditions. The erosion wear behavior of fiber reinforced polymer (FRP) composite systems as a function of fiber content has been studied in the past [3, 4]. Miyazaki and Hamao [5] have examined the effect of fiber inclusion on the erosion behavior by comparing the erosion rate of an FRP with that of a neat resin, which is a matrix material of FRP. It has been observed that inclusion of brittle fibers in both thermosetting and thermoplastic matrices leads to compositions with higher erosion resistance. The most important factor for designing composites is the fiber/filler content since it controls the mechanical and thermo-mechanical properties. The solid particle erosion behavior of polymer composites as a function of fiber content has been studied to a limited extent [5, 6]. Tilly and Sage [7] have investigated the influence of impact velocity, impingement angle, particle size and weight of impacted abrasives on nylon, carbon-fiber-reinforced nylon, epoxy resin, polypropylene and glass-fiber-reinforced plastic. Impact velocity (v) happens to be an important test variable in erosion test and can easily over-shadow changes in other variables such as target material, impact angle etc [8]. Erosion rate (Er) depends on

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velocity by a power law, given as \( E_r = kv^n \), where \( k \) is a material constant. However, the exponent \( n \) is found to be material independent and is governed by test conditions including particle characteristics and the erosion test apparatus \([9, 10, 11]\). In addition to velocity, solid particle erosion is also governed by impact angle, particle size, particle shape and hardness \([12]\). The impact of above parameters has been studied independently keeping all parameters at fixed levels. Therefore, visualization of impact of various factors in an interacting environment really becomes difficult. To this end, an attempt has been made to analyze the impact of more than one parameter on solid particle erosion of PMCs because, in actual practice, the resultant erosion rate is the combined effect of impact of more than one interacting variables. An inexpensive and easy-to-operate experimental strategy based on Taguchi’s parameter design has been adopted to study effect of various parameters and their interactions. The experimental procedure has been successfully applied for parametric appraisal in wire electrical discharge machining (WEDM) process, drilling of metal matrix composites, and erosion behavior of metal matrix composites such as aluminium reinforced with red mud \([13-19]\).

The objective of the present investigation is to study the solid particle erosion characteristics of flyash filled glass fiber reinforced polyester composites under various experimental conditions. A mathematical model was proposed to determine erosion rate as a function of process variables so that results of theoretical and experimental data can be compared to gain insight into the wear mechanism.

2.0 MATHEMATICAL MODEL

Nomenclature:
- \( r \)  chord length of the indentation (m)
- \( d \)  erodent diameter (m)
- \( \delta \)  indentation depth (m)
- \( e_v \)  volumetric wear loss per particle impact (m³)
- \( E_v \)  total volumetric erosion wear rate (m³/sec)
- \( \alpha \)  angle of impingement (degree)
- \( v \)  impact velocity (m/sec)
- \( P \)  force on the indenter (N)
- \( H_v \)  hardness (N/m²)
- \( m \)  mass of single erodent particle (kg)
- \( M \)  mass flow rate of the erodent (kg/sec)
- \( N \)  number of impact per unit time (sec⁻¹)
- \( \rho_c \)  density of composite (kg/m³)
- \( \rho \)  density of erodent (kg/m³)
- \( \eta_{normal} \)  erosion efficiency with normal impact
- \( \eta \)  erosion efficiency
- \( E_r \)  erosion wear rate (kg/kg)

Solid particle erosion is a wear process in which the material is removed from a surface by the action of a high velocity stream of erodent particles entrained in a high velocity fluid stream. The particles strike against the surface and promote material loss. During the fluid stream, a particle carries momentum and kinetic energy which can be dissipated during impact due to its interaction with a target surface. As far as erosion study of polymer matrix composites is concerned, no specific model has been developed and thus the study of their erosion behavior has been mostly experimental. However, Mishra \([20]\) proposed a mathematical model for material removal rate in abrasive jet machining process in which the material is removed from the work piece in a similar fashion. This model assumes that the volume of material removed is the same as the volume of indentation caused by normal impact. This has a serious limitation as in a real erosion
process the volume of material removed is actually different from the indentation volume. Further, this model considers only the normal impact i.e. $\alpha = 90^\circ$ whereas in actual practice, particles may impinge on the surface at any angle ($0^\circ \leq \alpha \leq 90^\circ$). The proposed model addresses these shortcomings in an effective manner. It considers the real situation in which the volume of material removed by erosion is not same as the volume of material displaced and therefore, additional term “erosion efficiency ($\eta$)” is incorporated in the erosion wear rate formulae. Erosion efficiency ($\eta$) [21] has been defined as the fraction of the volume that is actually removed as erosion debris out of that which is displaced. In the case of a stream of particles impacting a surface normally (i.e. at $\alpha=90^\circ$), erosion efficiency ($\eta_{normal}$) defined by Sundararajan et. al [21] is given as

$$\eta = \frac{2ErHv}{\rho v^2}$$

If the impact of erodent at any angle $\alpha$ to the surface is to be considered, the actual erosion efficiency can be obtained by modifying Eq. (1) as

$$\eta = \frac{2ErHv}{\rho v^2 \sin^2 \alpha}$$

The model is based on the assumption that the kinetic energy of the impinging particles is utilized to cause micro-indentation in the composite material and the material loss is a measure of the indentation. The erosion is the result of cumulative damage of such non-interacting, single particle impacts. The model further assumes the erodent particles to be rigid, spherical bodies of diameter equal to the average grit size. It considers the ductile mode of erosion and assumes the volume of material lost in a single impact is less than the volume of indentation. The model is developed with the simplified approach of energy conservation which equals the erodent kinetic energy with the work done in creating the indentation.

The model for ductile mode erosion proceeds as follows.

From the geometry of Fig. 1, $r^2 = d \times \delta$

The volume of indentation = $\pi \delta^2 \left[ \frac{d}{2} - \frac{\delta}{3} \right]$  

So, the volumetric wear loss per particle impact is given by

$$e_v = \text{Volume of indentation} \times \eta = \eta \times \pi \delta^2 \left[ \frac{d}{2} - \frac{\delta}{3} \right]$$

and neglecting $\delta^3$ terms

$$e_v = \frac{\pi d \delta^3}{2} \times \eta$$
Considering N number of particle impacts per unit time, the volumetric erosion wear loss will be:

$$E_v = \frac{\pi d \delta^2}{2} N \eta$$  \hspace{1cm} (5)

Figure 1: Schematic of material removal mechanism in ductile mode erosion.

The impact velocity will have two components; one normal to the composite surface and one parallel to it. At zero impact angles, it is assumed that there is negligible wear because eroding particles do not practically impact the target surface [22]. Consequently, there will be no erosion due to the parallel component and the indentation is assumed to be caused entirely by the component normal to the composite surface (Fig. 2).

Figure 2: Resolution of impact velocity in normal and parallel directions
Now applying conservation of energy to the single impact erosion process, kinetic energy associated with the normal velocity component of a single erodent particle is equal to the work done in the indentation of composite. The energy of impact introduces a force $P$ on the indenter to cause the indentation in the composite. Thus,

$$\frac{1}{2}mv^2 \sin^2 \alpha = \frac{1}{2} P \delta$$  \hspace{1cm} (6)

So,

$$\frac{1}{2} (\pi d^3 \rho \frac{v^2}{6} \sin^2 \alpha) = \frac{1}{2} (\pi r^3 H_v) \delta$$  \hspace{1cm} (7)

On solving,

$$\delta^2 = \frac{\rho v^2 d^2 \sin^2 \alpha}{6H_v}$$  \hspace{1cm} (8)

The number of erodent particle impacting the target is estimated from the known value of erodent mass flow rate, $M$ as

$$N = \frac{M}{\pi d^3 \rho}$$  \hspace{1cm} (9)

Substituting the value of $\delta$ in Eq. (5)

$$E_v = \frac{v^2 \cdot \sin^2 \alpha \cdot \eta}{2H_v}$$

Erosion rate ($E_r$) defined as the ratio of mass lost due to erosion to the mass of erodent is now expressed as.

$$E_r = \frac{\rho_c \cdot \eta_v v^2 \cdot \sin^2 \alpha}{2H_v}$$  \hspace{1cm} (10)

Material removal by impact erosion wear involves complex mechanisms. A simplified theoretical model for such a process may appear inadequate unless its assessment against experimental results is made. So for the validation of the proposed model erosion tests on the composites were conducted at various operating conditions.

### 3.0 METHODS & MATERIALS

#### 3.1 Specimen Preparation

Composite slabs are prepared by conventional contact molding process using high strength E-glass fiber (360 roving taken from Saint Govion) as the reinforcement and flyash filled polyester resin as the matrix. E-glass fiber and polyester resin have modulus of 72.5 GPa and 3.25 GPa respectively and possess density of 2590 kg/m$^3$ and 1350 kg/m$^3$ respectively. Prior to mixing with polyester, raw flyash collected from the Captive Power Plant of NALCO (located at Angul, India) was sieved to obtain an average particle size in the range 80-100 µm. Chemical analysis of flyash suggests its composition as silicon oxide (48.3%), aluminium oxide (20.2%), iron oxide (6.4%) and titanium oxide (1.9%). Composites of three different compositions (0wt%, 10wt% and 20wt% flyash filling) with 50% glass fiber loading was made. Specimens of suitable dimensions are cut using a diamond cutter for physical characterization and erosion test.
3.2 Test apparatus

Fig. 3 shows the schematic diagram of erosion test rig confirming to ASTM G 76. The set up was capable of creating reproducible erosive situations for assessing erosion wear resistance of the prepared composite samples. The erosion tester consists of an air compressor, an air particle mixing chamber and accelerating chamber. Dry compressed air was mixed with the particles which are fed at constant rate from a sand flow control knob through a convergent brass nozzle of 3 mm internal diameter. These particles impact the specimen which can be held at different angles with respect to the direction of erodent flow using a swivel and an adjustable sample clip. The velocity of the eroding particles was determined using double disc method [23]. In the present study, dry silica sand (spherical) of different particle sizes (300 µm, 500 µm and 800 µm) was used as erodent. The samples were cleaned in acetone, dried and weighed to an accuracy of ± 0.1 mg accuracy using a precision electronic balance. It is then eroded in the test rig for 10 minutes and weighed again to determine the weight loss. The process was repeated till the erosion rate attains a constant value called steady state erosion rate.

![Figure 3. A schematic diagram of the erosion test rig.](image)

3.3 Test of microhardness, tensile strength, flexural strength, density and X-ray diffraction

Micro-hardness

Micro-hardness measurement was done using a Leitz micro-hardness tester. A diamond indenter, in the form of a right pyramid with a square base and an angle 136° between opposite faces, was forced into the material under a load F. The two diagonals X and Y of the indentation left on the surface of the material after removal of the load were measured and their arithmetic mean L is calculated. In the present study, the load considered F = 24.54 N and Vickers hardness number is calculated using the following equation.
\[ H_v = 0.1889 \frac{F}{L^2} \]  
\[ \text{and } L = \frac{X + Y}{2} \]

Where, \( F \) is the applied load (N), \( L \) is the diagonal of square impression (mm), \( X \) is the horizontal length (mm) and \( Y \) is the vertical length (mm).

**Tensile strength**

The tensile test was generally performed on flat specimens. The commonly used specimens for tensile test are the dog-bone type and the straight side type with end tabs. During the test a uniaxial load was applied through both the ends of the specimen. The ASTM standard test method for tensile properties of fiber resin composites has the designation D 3039-76. The length of the test section should be 200 mm. The tensile test is performed in the universal testing machine (UTM) Instron 1195 and results are analyzed to calculate the tensile strength of composite samples.

**Flexural strength**

The flexural strength (F.S.) of any composite specimen is determined using the following equation.

\[ F.S = \frac{3PL}{2bt^2} \]  
\[ \text{Where, } P \text{ maximum load} \]
\[ L \text{ span length of the sample} \]
\[ b \text{ the width of specimen, and} \]
\[ t \text{ the thickness of specimen.} \]

**Density**

The theoretical density of composite materials in terms of weight fraction can easily be obtained as for the following equations given by Agarwal and Broutman [24].

\[ \rho_{ct} = \frac{1}{(W_f / \rho_f) + (W_m / \rho_m)} \]  
\[ \text{Where, } W \text{ and } \rho \text{ represent the weight fraction and density respectively. The suffix} \]
\[ f, m \text{ and } ct \text{ stand for the fiber, matrix and the composite materials respectively.} \]
\[ \text{The composites under this investigation consists of three components namely matrix, fiber} \]
\[ \text{and particulate filler. Hence the modified form of the expression for the density of the} \]
\[ \text{composite can be written as} \]

\[ \rho_{ct} = \frac{1}{(W_f / \rho_f) + (W_m / \rho_m) + (W_p / \rho_p)} \]  
\[ \text{Where, the suffix ‘p’ indicates the particulate filler materials.} \]
The actual density ($\rho_{ct}$) of the composite, however, can be determined experimentally by simple water immersion technique. The volume fraction of voids ($V_v$) in the composites is calculated using the following equation:

$$V_v = \frac{\rho_{ct} - \rho_{ce}}{\rho_{ct}}$$  \hspace{1cm} (15)

**X-ray diffraction**

The X-ray diffraction patterns of the GF-polyester–flyash composites (20 wt% of fly ash) before and after erosion test were recorded on Philips X-ray diffractometer using CuKα radiation. The diffractograms were taken in terms of $2\theta$ in the range 10°–90°.

**3.4 Experimental Design**

Design of experiment is a powerful analysis tool for modeling and analyzing the influence of control factors on performance output. The most important stage in the design of experiment lies in the selection of the control factors. Therefore, a large number of factors were included so that non-significant variables can be identified at the earliest opportunity. Exhaustive literature review on erosion behavior of polymer composites revealed that parameters viz., impact velocity, impingement angle, fiber loading, filler content, erodent size and stand-off distance etc largely influence erosion rate of polymer composites [25, 26]. The impact of five such parameters were studied using $L_{27} (3^{13})$ orthogonal design. The operating conditions under which erosion tests were carried out are given in Table 1. The tests were conducted as per experimental design given in Table 2 at room temperature.

In Table 2, each column represents a test parameter whereas a row stands for a treatment or test condition which is nothing but combination of parameter levels. In conventional full factorial experiment design, it would require $3^5 = 243$ runs to study five parameters each at three levels whereas, Taguchi’s factorial experiment approach reduces it to 27 runs only offering a great advantage in terms of experimental time and cost. The experimental observations are further transformed into signal-to-noise (S/N) ratio. There are several S/N ratios available depending on the type of performance characteristics. The S/N ratio for minimum erosion rate can be expressed as “lower is better” characteristic, which is calculated as logarithmic transformation of loss function as shown below.

**Smaller is the better characteristic:**

$$\frac{S}{N} = -10 \log \left( \frac{1}{n} \sum y^2 \right)$$  \hspace{1cm} (16)

Where ‘n’ the number of observations, and $y$ the observed data. “lower is better (LB)” characteristic, with the above S/N ratio transformation, is suitable for minimization of erosion rate. The standard linear graph, as shown in Fig. 4, is used to assign the factors and interactions to various columns of the orthogonal array [27, 28]. Solid particle erosion is characterized by a large number of factors such as impact velocity, flyash percentage, stand-off distance (It is the distance between the nozzle tips to sample surface), impingement angle and erodent size.
Table 2 Orthogonal array for $L_{27}(3^{13})$ Taguchi Design

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Figure 4: Linear graph for $L_{27}$ array
The plan of the experiments is followed as per Taguchi orthogonal array design mentioned in Table 2 and Figure 2 as follows: the first column was assigned to impact velocity (A), the second column to flyash percentage (B), the fifth column to stand-off distance (C), the ninth column to impingement angle (D) and the tenth column to erodent size (E), the third and fourth column are assigned to \((A \times B)_1\) and \((A \times B)_2\) respectively to estimate interaction between impact velocity (A) and flyash percentage (B), the eight and eleventh column are assigned to \((A \times C)_1\) and \((A \times C)_2\) respectively to estimate interaction between the impact velocity (A) and stand-off distance (C) and the remaining columns are used to estimate experimental errors.

4.0 CONFIRMATION EXPERIMENT

The optimal combination of control factors has been determined in the previous analysis. However, the final step in any design of experiment approach is to predict and verify improvements in observed values through the use of the optimal combination level of control factors. The confirmation experiment is performed by conducting a new set of factor combination \(A_2B_3D_1E_3\) but factor C has been omitted because factor C and interaction \(A \times C\) have least effect on erosion rate as evident from Table 7. The estimated S/N ratio for erosion rate can be calculated with the help of following prediction equation:

\[
\hat{\eta}_1 = \bar{T} + (\bar{A}_2 - \bar{T}) + (\bar{B}_3 - \bar{T}) + [(\bar{A}_2 \bar{B}_3 - \bar{T}) - (\bar{A}_2 - \bar{T}) - (\bar{B}_3 - \bar{T})] + (\bar{D}_1 - \bar{T}) + (\bar{E}_3 - \bar{T})
\]  

(17)  

\[
\bar{\eta}_1 \quad \text{Predicted average}
\]

\[
\bar{T} \quad \text{Overall experimental average}
\]

\[
\bar{A}_2, \bar{B}_3, \bar{D}_1 \text{ and } \bar{E}_3 \quad \text{Mean response for factors and interactions at designated levels.}
\]

By combining like terms, the equation reduces to

\[
\bar{\eta}_1 = \bar{A}_2 \bar{B}_3 + \bar{D}_1 + \bar{E}_3 - 2\bar{T}
\]

(18)

A new combination of factor levels \(A_2, B_3, D_1\) and \(E_3\) is used to predict deposition rate through prediction equation and it is found to be \(\bar{\eta}_1 = -51.4966 \, \text{db} \).

Table 7: ANOVA table for erosion rate

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For each performance measure, an experiment is conducted for a different factors combination and compared with the result obtained from the predictive equation as shown in Table 8.

Table 8: Results of the confirmation experiments for Erosion rate

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<tr>
<th>Optimal control parameters</th>
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<td>Level</td>
<td>A B C D E</td>
<td>A B C D E</td>
</tr>
<tr>
<td>S/N ratio for Erosion rate (mg/kg)</td>
<td>-51.4966</td>
<td>-48.4788</td>
</tr>
</tbody>
</table>

The resulting model seems to be capable of predicting erosion rate to a reasonable accuracy. An error of 5.86% for the S/N ratio of erosion rate is observed. However, the error can be further reduced if the number of measurements is increased. This validates the development of the mathematical model for predicting the measures of performance based on knowledge of the input parameters.

5.0 FACTOR SETTINGS FOR MINIMUM EROSION RATE

In this study, an attempt is made to derive optimal settings of the control factors for minimization of erosion rate. The single-objective optimization requires quantitative determination of the relationship between erosion rates with combination of control factors. In order to express, erosion rate in terms of mathematical model in the following form is suggested.

\[
Er = K_0 + K_1 \times A + K_2 \times B + K_3 \times D + K_4 \times E + K_5 \times A \times B
\]  

(19)

Here, Er is the performance output terms and \( K_i \) (i = 0, 1, ..., 5) are the model constants. The constant are calculated using non-linear regression analysis with the help of SYSTAT 7 software and the following relations are obtained.

\[
Y = 0.522 - 0.060 \times A - 0.378 \times B + 0.234 \times D + 0.022 \times E + 0.581 \times A \times B
\]

\[
r^2 = 0.96
\]

(20)

The correctness of the calculated constants was confirmed as high correlation coefficients \( r^2 \) in the tune of 0.96 are obtained for Eq. (19) and therefore, the models are quite suitable to use for further analysis. Here, the resultant objective function to be maximized is given as:

Maximize \( Z = 1/f \)  

(21)

\( f \) Normalized function for erosion rate
Subjected to constraints:

\[ A_{\text{min}} \leq A \leq A_{\text{max}} \] (22)
\[ B_{\text{min}} \leq B \leq B_{\text{max}} \] (23)
\[ D_{\text{min}} \leq D \leq D_{\text{max}} \] (24)
\[ E_{\text{min}} \leq E \leq E_{\text{max}} \] (25)

The min and max in Eqs. 22-25 shows the lowest and highest control factors settings (control factors) used in this study (Table 9).

Table 9: Levels of the variables used in the experiment

<table>
<thead>
<tr>
<th>Control factor</th>
<th>Level</th>
<th>Level</th>
<th>Level</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
<td>II</td>
<td>III</td>
<td></td>
</tr>
<tr>
<td>A: Velocity of impact</td>
<td>32</td>
<td>45</td>
<td>58</td>
<td>m/sec</td>
</tr>
<tr>
<td>B: Flyash percentage</td>
<td>0</td>
<td>10</td>
<td>20</td>
<td>%</td>
</tr>
<tr>
<td>C: Stand off distance</td>
<td>120</td>
<td>180</td>
<td>240</td>
<td>mm</td>
</tr>
<tr>
<td>D: Impingement angle</td>
<td>30</td>
<td>60</td>
<td>90</td>
<td>degree</td>
</tr>
<tr>
<td>E: Erodent size</td>
<td>300</td>
<td>500</td>
<td>800</td>
<td>µm</td>
</tr>
</tbody>
</table>

Genetic algorithm (GA) is used to obtain the optimum value for single-objective outputs to optimize the single-objective function. The computational algorithm is implemented in Turbo C ++ and run on an IBM Pentium IV machine. Genetic algorithms (GAs) are mathematical optimization techniques that simulate a natural evolution process. They are based on the Darwinian Theory, in which the fittest species survives and propagate while the less successful tend to disappear. Genetic algorithm mainly depends on three types of operator’s viz., reproduction, crossover and mutation. Reproduction is accomplished by copying the best individuals from one generation to the next, what is often called an elitist strategy. The best solution is monotonically improving from one generation to the next. The selected parents are submitted to the crossover operator to produce one or two children. The crossover is carried out with an assigned probability, which is generally rather high. If a number randomly sampled is inferior to the probability, the crossover is performed. The genetic mutation introduces diversity in the population by an occasional random replacement of the individuals. The mutation is performed based on an assigned probability. A random number is used to determine if a new individual will be produced to substitute the one generated by crossover. The mutation procedure consists of replacing one of the decision variable values of an individual while keeping the remaining variables unchanged. The replaced variable is randomly chosen and its new value is calculated by randomly sampling within its specific range. In genetic optimization, population size, probability of crossover and mutation are set at 50, 75 %, and 5 % respectively for all the cases. Number of generation is varied till the output is converted. Table 10 shows the optimum conditions of the control factors with optimum performance output gives a better combination of set of input control factors. The pattern of convergence of performance output with number of generations is shown in Fig.15.
Table 10: Optimum conditions for performance output

<table>
<thead>
<tr>
<th>Control factors and Performance characteristics</th>
<th>Optimum conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>A: Impact velocity (m/sec)</td>
<td>32.31</td>
</tr>
<tr>
<td>B: Flyash percentage (%)</td>
<td>19.90</td>
</tr>
<tr>
<td>D: Impingement angle (degree)</td>
<td>47.16</td>
</tr>
<tr>
<td>E: Erodent size (µm)</td>
<td>518.40</td>
</tr>
<tr>
<td>Erosion rate (mg/kg)</td>
<td>239.8444</td>
</tr>
</tbody>
</table>

6.0 CONCLUSION

Based on the present study on solid particle erosion of flyash filled polyester-GF composite at various parameter settings of impact velocity, flyash percentage, stand-off distance, impingement angle and erodent size, conclusions can be drawn as follows.

1) Flyash, an industrial waste, can be used as a potential filler material in polyester matrix composites. Its compatibility with polyester resin is found to be better than that with epoxy. It has marginal effects on the mechanical properties of the composites but improves their erosion wear resistance.

2) Conservation of energy principle is applied to the multiple impact erosion process and consequently a mathematical model based on ductile mode erosion is developed. To overcome the shortcomings of existing theoretical models ‘erosion efficiency’ term has been introduced. It has been demonstrated that if supported by an appropriate magnitude of erosion efficiency, the model performs well for flyash filled polyester-GF composites for normal as well as oblique impact.

3) Solid particle erosion characteristics of these composites can be successfully analyzed using Taguchi experimental design scheme. Taguchi method provides a simple, systematic and efficient methodology for the optimization of the control factors. This approach not only needs engineering judgment but also requires a rigorous mathematical model to obtain optimal process settings.

4) The erosion efficiency ($\eta$), in general, characterizes the wear mechanism of composites. The flyash filled GF-polyester composites exhibit semiductile erosion response ($\eta =10-60\%$) for low impact velocities and ductile erosion response ($\eta < 10\%$) for relatively high impact velocity.

5) Factors like flyash percentage, impingement angle, erodent size and impact velocity in order of priority are significant to minimize the erosion rate. Although the effect of impact velocity is less compared to other factors, it cannot be ignored because it shows significant interaction with another factor i.e. the percentage of flyash in the composite.

6) Study of influence of impingement angle on erosion rate of the composites filled with different percentage of flyash reveals their semi-ductile nature with respect to erosion wear. The peak erosion rate is found to be occurring at 60° impingement angle under the various experimental conditions.

7) The rationale behind the use of genetic algorithm lies in the fact that genetic algorithm has the capability to find the global optimal parameter settings whereas the traditional optimization techniques are normally stuck up at the local optimum
values. The optimum settings are found at impact velocity = 32.31 m/sec, flyash percentage = 19.90%, impingement angle = 47.16°, erodent size = 518.40 µm, and resulting erosion rate = 239.8444 mg/kg as far as present experimental conditions are concerned.

8) In future, this study can be extended to polymer matrix composites using other filler materials and the resulting experimental findings can also be analyzed using different hybrid optimization techniques.

REFERENCES


