Subsurface colloids in groundwater contamination: a mathematical model

Tushar Kanti Sen a,∗, Sachin Shanbhag b, Kartic C. Khilar b

a Department of Chemical Engineering, National Institute of Technology, Rourkela 769008, Orissa, India
b Department of Chemical Engineering, Indian Institute of Technology, Powai, Mumbai 400076, India

Abstract

In this paper, a new role of subsurface colloidal fines on the transport of contaminants in groundwater flows has been presented. Here, an equilibrium three-phase model based on colloidal induced release, migration and finally capture of these colloidal fines at pore constrictions has been developed. This model is more realistic than the existing models in that the physics of release, migration and finally capture at pore constrictions leading to plugging phenomenon has been adequately incorporated. Presence of colloids can either facilitate or inhibit the spreading of contaminants depending on the values of the parameters, such as the inlet colloid concentration, the release coefficient and more significantly on the sensitivity of the permeability to the concentration of captured fines at the pore constrictions. For a range of conditions, plugging of the porous media occurs resulting in retardation of contaminant transport. The plugging-based retardation of contaminant transport can be used to develop a new containment technique.

Keywords: Colloidal fines; Contaminants; Plugging/entrapment; Groundwater

1. Introduction

The contamination of groundwater and soil due to mobilization and transport of organic, inorganic and biological pollutants from many sources such as spillage, land application and waste disposal practices is a well recognized problem. The presence and transport of contaminants constitute a potential threat to human health and ecosystems. Natural colloids, which are ubiquitous in groundwater system, are composed of inorganic and organic molecular constituents or microorganisms. Colloids are commonly defined as mean particles or macromolecules smaller than 1 μm in diameter but larger than 1 nm [1,2]. These colloidal particles having surface charge and are typically smaller than intergranular pores and fractures in rock and can migrate long distance with the groundwater. When these colloidal fines are transported over large distances, they can act as a carrier for contaminant, which are adsorbed on their surface and thereby enhance the spreading of pollutants in subsurface system. This contaminant transport has been known as “colloid-facilitated contaminant transport” in literature.

Experimentally as well as theoretically, it has been found that mobile colloidal fines can often carry contaminants such as radionuclides, transition metals and hydrophobic organic compounds adsorbed onto their surface and can, thus, significantly accelerate the transport of contaminants through porous media [1,3–19]. Kretzschmar et al. [1] mentioned in their review article that only a few field studies provide evidence for colloid-facilitated transport of contaminants. Previous studies concerning contaminant transport in subsurface environment have considered soil or groundwater as essentially a two-phase system where the contaminants could partition between the mobile aqueous phase and the immobile solid phase. Based on this concept, it has been predicted that many contaminants would be relatively immobile, as they are generally sparingly soluble and hence get adsorbed on the solid phase. But unexpected appearance of low solubility contaminants some distance away from known source or sooner than would be expected from their solubility, led to examination of the possible involvement of nonaqueous, mobile colloids in contaminant transport. Invoking colloids to explain such observations gave rise to the three-phase model of species transport, i.e. mobile liquid phase, mobile colloidal phase and the immobile solid phase. Few three-phase models describing coupled colloidal and contaminant transport have been reported so far.

∗ Corresponding author. Tel.: +91-661-2476638.
E-mail addresses: tksen@nitr.ren.nic.in, sentushar@rediffmail.com (T.K. Sen), kartic@che.iitb.ac.in (K.C. Khilar).
The release of colloidal fines from a pore surface is the first step in the phenomenon of migration of fines in sub-surface zone. In general, two major types of forces are responsible for the release of these colloidal fines: colloidal and hydrodynamic. The released colloidal fines while flowing with the liquid phase, can either re-adhere to the pore surface or flow without capture or get entrapped at the pore constrictions. The latter two occurrences are more common as the colloidal and hydrodynamical conditions that bring about their release are not likely to allow these particles to re-adhere back to the pore surface in the same conditions. It has been observed that the release of fines is threshold process that is, a minimum perturbation is required to detach the fines from the pore surface. This could be in terms of a critical hydrodynamics stress [20] or a critical salt concentration [2]. The release fines may get entrapped or plugged at the pore constrictions. Entrapment can occur in three forms, namely size exclusion, multiparticle bridging and surface deposition. Whether the particles get entrapped or not depends on the factors such as pore structure, relative sizes of fines and pore constrictions and concentration of fines [2,12]. Recently, it has been found experimentally that colloidal fines, kaolin retarded Ni$^{2+}$ transport at constant flow under plugging condition [12]. The existing reported models, however, do not adequately incorporate this phenomenon and therefore may fail to describe the process completely, i.e. for a wide range of conditions.

Moreover, the reported modeling studies, thus far, are based on the convective diffusion equations for the contaminant and for the colloidal fines, and the rate equations for the process of adsorption of contaminant. The mobilization/hydrodynamic release of indigenous colloidal fines and their possible capture at constriction sites are only considered in the reported model developed by Sen et al. [13].

In this paper, we presented a comprehensive mathematical model based on equilibrium adsorption of contaminants and on the colloidal induced release, migration and finally capture of colloidal fines at pore constrictions leading to plugging phenomenon. This model can be used to predict both facilitation and retardation of the transport of contaminants in presence of colloidal fines in underground flow depending on the flow and other conditions. Results of model simulations performed to investigate the impact of the model parameters on the spatial and temporal variations of contaminant concentrations and fluxes are discussed. It is shown that under a range of conditions, entrapment/plugging phenomenon can occur which leads to the retardation of contaminant transport. Based on this finding, a containment technology can be developed to check the spreading of groundwater and soil pollution as well as for an effective treatment and remediation.

### 2. Model development

This model has been formulated in a manner analogous to that of existing model such as the model developed by Sen et al. [13] and by Corapcioglu and Jiang [15]. Additional equations relating to colloid induced release and the

---

### Nomenclature

- $C_c$: aqueous contaminant concentration (kg/m$^3$)
- $C_{c0}$: aqueous contaminant concentration at the inlet (kg/m$^3$)
- $C_f$: concentration of suspended fines of (kg/m$^3$)
- $C_{f0}$: concentration suspended fines at the inlet (kg/m$^3$)
- $D$: dispersion coefficient of colloids (m$^2$/s)
- $D_0$: dispersion coefficient of contaminant (m$^2$/s)
- $k$: permeability (m$^2$)
- $K$: partition constant between fines-aqueous phases (m$^3$/kg)
- $K_{ma}$: partition constant between matrix-aqueous phases (m$^3$/kg)
- $L$: length (m)
- $q$: flow rate (m/s)
- $r_c$: rate of colloidal capture (kg/(m$^3$s))
- $r_e$: rate of colloidal release (kg/(m$^3$s))
- $t$: time (s)
- $x$: distance (m)
- $X_1$: mass fraction of contaminant on the fines (kg/kg)
- $X_m$: mass fraction of the contaminant on the solid matrix (kg/kg)

### Greek letters

- $\alpha$: colloid release rate constant (s$^{-1}$)
- $\beta$: colloid capture rate constant (s$^{-1}$)
- $\phi$: porosity of the matrix
- $\mu$: viscosity of aqueous phase (Pa s)
- $\rho_i$: density of fines (kg/m$^3$)
- $\rho_s$: density of solid matrix (kg/m$^3$)
- $\sigma_1$: concentration of fines on release sites (kg/m$^3$)
- $\sigma_2$: concentration of fines on capture sites (kg/m$^3$)
- $\sigma_{i0}$: initial concentration of fines on release sites (kg/m$^3$)
- $\sigma_{max}$: parameter defining the sensitivity of velocity changes (kg/m$^3$)

---

[8,10,11,13–15]. Most of these studies have assumed equi-librium interactions between the colloidal particles and the contaminant in the dissolved phase [10,13–15] These models predict a reduction of the effective retardation arising out of sorption on to the solid matrix depending on the concentra-tion of the colloidal fines and the partition coefficient for contaminant sorption on the colloidal fines.
variations in flow rate due to plugging/entrapment has been incorporated.

A cylindrical porous medium of length \( L \) in the direction of flow and of unit cross sectional area, having porosity \( \phi \) and initial permeability \( K_0 \) is considered. A set of unsteady state mass balance equations are derived based on the following assumptions:

- A single species of contaminant is considered.
- Contaminant is partitioning among the three phases, namely solid matrix, mobile colloidal fines and aqueous phase.
- The release of fines due to only colloidal forces, which is dependent on underground solution chemistry.
- The porosity remains virtually unaltered during plugging/entrapment phenomenon.
- The variation in concentrations and other quantities are significantly only in the major flow direction that is axial direction in this case.
- The release and capture sites are different.

An unsteady state mass balance equation for the colloidal fine particles in suspension can be written as

\[
\frac{\partial f_1}{\partial t} = D \frac{\partial^2 f_1}{\partial x^2} - q \frac{\partial f_1}{\partial x} + r_r - r_c
\]

(1)

where \( f_1 \) is the concentration of colloidal fines in suspension (kg/m³), \( D \) the hydrodynamic dispersion coefficient (m²/s), \( q \) the superficial flow flux (m/s), \( r_r \) the rate of colloidal induced release of fines (kg/(m³ s)), \( r_c \) the rate of capture of colloidal fines (kg/(m³ s)), \( \phi \) the porosity of the solid matrix, \( x \) the distance along flow (m), and \( t \) is time (s).

The particles are released from the surface of the body while they are entrapped at the constriction sites. The permeability reduces due to the entrapment at the pore constrictions. The rate equations for the release and capture of particles are given as follows:

For release:

\[
\frac{\partial \sigma_1}{\partial t} = -\alpha \sigma_1
\]

(2)

For capture:

\[
\frac{\partial \sigma_2}{\partial t} = \beta C_1
\]

(3)

where \( \sigma_1 \) is the concentration of fines at release sites (kg/m³) and \( \sigma_2 \) is the concentration of fines at capture sites (kg/m³). We assume a first order form for the rate equations

\[
r_r = \alpha \sigma_1
\]

(4)

\[
r_c = \beta C_1
\]

(5)

where \( \alpha \) is the release coefficient (s⁻¹) and \( \beta \) is the colloid capture coefficient (s⁻¹). It is pointed out that the equation for the rate of release of fines as given above would hold true as long as the salt concentration is smaller than the critical salt concentration [2]. Similarly, the equation for the capture of the fines would be applicable contingent on the fines concentration exceeding the critical particle concentration. However, in this simulation, these effects are not considered.

An overall contaminant balance on all the sites namely, the solid matrix, aqueous phase, captured colloids and free colloids yields

\[
\frac{\partial \rho_m X_m}{\partial t} + \phi \frac{\partial C_1}{\partial t} + \phi X_1 \frac{\partial C_f}{\partial t} + \frac{\partial r_r}{\partial t} - \frac{\partial r_c}{\partial t} = \phi D \frac{\partial^2 C_1}{\partial x^2} - q \frac{\partial C_1}{\partial x} - q \frac{\partial C_f}{\partial x}
\]

(6)

where \( \rho_m \) is the bulk density of the solid matrix, \( X_m \) the mass fraction of contaminant on solid matrix, \( X_1 \) the mass fraction of contaminant on colloids, \( C_f \) the contaminant concentration in aqueous phase (kg/m³) and \( D_f \) is the dispersion coefficient for dissolved contaminant (m²/s).

The low contaminant concentration generally encountered in groundwater permits us to assume linear partitioning between any of the solid phases and aqueous phase [21,22].

\[
X_m = K_m C_f
\]

(7)

\[
X_1 = K_1 C_c
\]

(8)

where \( K_m \) and \( K_1 \) are the partition coefficients for the matrix and colloidal phases, respectively. Literatures [22–24] reveal that \( K_1 \) is ordinarily several times larger than the corresponding values of \( K_m \). To model the variation of velocity in the porous media with changing concentration of the captured colloid, we use Darcy’s law for horizontal flow as follows:

\[
q = \frac{K}{\mu} \left( \frac{\Delta P}{L} \right)
\]

(9)

where \( K \) is the permeability of the medium (m²), \( \mu \) the viscosity of the fluid (Pa s) and \( \Delta P \) is the pressure difference across the medium.

To determine the decrease in permeability with entrapment, a relationship is required. No theoretical correlation has been reported between the permeability (\( K \)) and the amount of entrapment (\( \sigma_2 \)). An empirical expression [2], which has been used in our model, is as follows:

\[
K = K_0 \exp \left( -\frac{\sigma_2}{\sigma_{\text{max}}} \right)
\]

(10)

where \( K_0 \) is the initial permeability and \( \sigma_{\text{max}} \) is an adjustable parameter that characterizes the effectiveness of entrapment in reducing the permeability.

The overall permeability, \( K \) is computed by

\[
\frac{L}{K} = \int_0^L \frac{dx}{K(x)}
\]

(11)

The following initial and boundary conditions can be written for this geometry are

For colloids:

\[
q C_f|_r = q C_f|_l - D \frac{\partial C_f}{\partial x} |_l
\]

(12)
\[ C_f(x,0) = 0 \quad (13) \]
\[ \frac{\partial C_f}{\partial x} \bigg|_{x=L} = 0 \quad (14) \]
\[ \sigma_1(x,0) = \sigma_{10} \quad (15) \]
\[ \sigma_2(x,0) = 0 \quad (16) \]

For contaminants:
\[ q C_c(x,0) - q C_c(0) - D \frac{\partial C_c}{\partial x} \bigg|_{x=L} = 0 \quad (17) \]
\[ C_c(x,0) = 0 \quad (18) \]
\[ \frac{\partial C_c}{\partial x} \bigg|_{x=L} = 0 \quad (19) \]

Table 1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_{10} ) (g/m³)</td>
<td>50.0</td>
</tr>
<tr>
<td>( C_{20} ) (mg/m³)</td>
<td>1.0</td>
</tr>
<tr>
<td>( D ) (m²/s)</td>
<td>( 4 \times 10^{-6} )</td>
</tr>
<tr>
<td>( D_c ) (m²/s)</td>
<td>( 4 \times 10^{-7} )</td>
</tr>
<tr>
<td>( \alpha ) (s⁻¹)</td>
<td>( 1.0 \times 10^{-4} )</td>
</tr>
<tr>
<td>( \beta ) (s⁻¹)</td>
<td>( 1.0 \times 10^{-7} )</td>
</tr>
<tr>
<td>( \sigma_{10} ) (kg/m³)</td>
<td>1.0</td>
</tr>
<tr>
<td>( \sigma_{max} ) (kg/m³)</td>
<td>0.0015</td>
</tr>
<tr>
<td>( K_f ) (m³/kg)</td>
<td>5.0</td>
</tr>
<tr>
<td>( K_m ) (m³/m/kg)</td>
<td>1.0</td>
</tr>
<tr>
<td>#</td>
<td>0.4</td>
</tr>
<tr>
<td>( \mu_c ) (kg/m³)</td>
<td>2000</td>
</tr>
<tr>
<td>( \mu_L ) (kg/m³)</td>
<td>1500</td>
</tr>
</tbody>
</table>

Eqs. (1)–(19) provides a mathematical framework to determine various relevant quantities.

3. Results and discussion

For our simulations we consider a porous cylinder, across which a constant pressure difference \( \Delta P \) is applied. Through one end of the cylinder, colloids of concentration \( C_{f0} \) and contaminant of concentration \( C_{10} \) are injected. The temporal variation of the concentration of the colloids and the contaminant at the exit of the cylinder is monitored. Numerical solution to the set of partial differential equations has been obtained using a finite difference method. A fully implicit scheme employing a two-point forward difference approximation for time and space derivatives is used. In this method, the entire region of interest is divided into a grid of uniform resolution in each of the dimensions. The implicit formulations of partial differential equations along with boundary conditions led finally to a set of linear algebraic equations with a tridiagonal matrix, which was then solved by Gauss elimination solution algorithm at each time step.

Breakthrough curves are presented as plots of \( C_{ct} \) against time, where \( C_{ct} \) is the total mobile contaminant concentration (\( C_{ct} = C_c + X_f C_f \)). \( C_{ct} \) is chosen as the ordinate because it accounts not only for the contaminants in the aqueous phase, but also accounts for the contaminants on the mobile colloid phase. The model parameters used in the simulations have been obtained from the published works and are summarized in Table 1.

Simulations have been conducted in a variety of cases. Due to deficiency of constant pressure plugging-based

![Fig. 1. Contaminant flux vs. time plot for the case of without fines and no velocity change.](image1)

![Fig. 2. Contaminant flux vs. time plot for the case of with fines but no velocity change (colloid-facilitated contaminant transport).](image2)
contaminant transport experimental data, we have first compared our simulation results with the results of earlier modeling studies. With negligible decrease in flow rate due to plugging/entrapment, it is found that colloid-facilitated spreading of contaminants increases with increasing inlet colloidal concentrations and increasing partition coefficient of the contaminant between the colloids and aqueous phase. It decreases with increasing capture coefficient. These results are in good agreement with earlier studies [8,14,15] and are not presented here.

We shall present some typical results focused on the new role of the colloidal fines that is the retardation in the rate of contaminant transport. This retardation is a consequence of the reduction in velocity of flow, which in turn depends on the permeability reduction resulting from capture of fines at constriction sites.

3.1. Role of colloidal fines on contaminant transport

Figs. 1–3 present the temporal variation of the total contaminant flux issuing from the exit for three different cases. Fig. 1 represents the case in which no fines are present. A part of the contaminant sorbs on the solid matrix and the rest flows with the fluid. Fig. 2 indicates the effect of the presence of fines with variations in flow rate neglected. The parameter $\sigma_{\text{max}}$ is used to define the sensitivity in velocity changes to the concentration of the fines at capture sites. A value of $\sigma_{\text{max}} = 1.5 \text{ kg/m}^3$ is found to be large enough to adequately represent the case of no flow rate change. An increase in the steady state throughput is observed when compared to the case in which no fines were present. This represents the well documented case of colloid-facilitated transport of contaminants. Fig. 3 presents the effect of colloids coupled with the effect of velocity changes due to entrapped fines. Here, a smaller value of $\sigma_{\text{max}} (0.0015 \text{ kg/m}^3)$ is chosen to incorporate the velocity effects. It is observed that the magnitude of the flux goes through a maximum after which there is a significant decrease in total contaminant flux is obtained. Release of fine particles increases the concentration of particles thereby enhancing the rate of capture of particles. As a result, the permeability decreases causing rapid reduction in flow rate and decrease in total contaminant flux. This is can be called as colloid-retardation of contaminant transport. Therefore, depending on the values of the system...
parameters $C_0$, $\sigma_{\text{max}}$, $\sigma_1$, and $\alpha$, one can have either of colloid facilitation or retardation of contaminant transport.

### 3.2 Effect of partition coefficient on contaminant transport

An increase in the value of $K_m$ renders the aqueous phase relatively leaner with respect to the contaminant concentration, thus retarding the motion of the contaminant concentration front, which gives a delayed breakthrough as observed in Fig. 4. At small values of $K_m$, the association between the contaminant and the solid matrix is weak and the contaminant front travels almost independently of the colloid concentration. In such a case, the contaminant breakthrough curves are relatively unaffected by changes in $\alpha$ and $\beta$, since most of the contaminant is transported through the aqueous phase. Fig. 5 portrays the spatial total contaminant concentration profiles with $K_m$ as a parameter. At the inlet, the aqueous concentration of fines is maximum, once the temporary concentration effect of colloids from release sites wears out. Also the aqueous concentration of the contaminant is
maximum at the inlet. Therefore, the total contaminant concentration is maximum here. At high values of $K_m$, however, $C_{f0}$ depends on the relative rates of fines deposition and release. A considerably larger amount of contaminant sorbs onto the solid matrix and captured fines and thus gets trapped. It must also be noted that if the capture coefficient, $\beta$, were small then increased $K_m$ would significantly facilitate contaminant transport. Fig. 6 presents that the spatial distribution of velocity at different values of $K_m$ coincide. This is in tune with expected behavior of the system, since velocity changes are governed by the release, capture and sensitivity of the system to the concentration of entrapped fines.

3.3. Effect of rate of release of fines on contaminant transport

It can be inferred from Fig. 7 that the release rate, $\alpha$, affects the kinetics of the breakthrough without altering the steady state values attained. As $\alpha$ is increased the release of fines from release sites becomes faster. When the fines concentration front has not advanced far enough, the aqueous
concentration of fines downstream is primarily due to fines issuing from release sites. This local and temporary enhancement in the aqueous fines concentration results in greater amounts of fines being entrapped and thus a greater initial reduction in velocity (Fig. 8). After this significant initial fall in velocity most of the fines originating from the release sites inventory are flushed out and the small decrease in velocity can be attributed to the fines being fed at the inlet. The velocity decrease at smaller \( \alpha \) is slower, because of the absence of pockets of fines downstream. It has also been found that the effect of fines originating from the release sites inventory is smeared over a larger time interval, resulting in a larger slope compared to the case of larger \( \alpha \). This influence of \( \alpha \) on the spatial profiles of the total contaminant concentration are shown in Fig. 9. In the case of high \( \alpha \), the stock of fines at the release sites is exhausted before the contaminants get sorbed thereby depriving the system of any colloid facilitation due to fines at release sites.

Fig. 9. Spatial variation of the total contaminant concentration at 3000 min, for \( C_0 = 500 \) mg/m\(^3\) with \( \alpha \) as parameter.

Fig. 10. Breakthrough curves for total contaminant concentration with \( \beta \) as parameter at \( C_0 = 500 \) mg/m\(^3\).
3.4. Effect of capture coefficient on contaminant transport

When the value of the capture coefficient, \( \beta \), decreases, the amount of colloids captured decreases. Here, the presence of colloids in the system facilitates the transport of the contaminant (Fig. 10). When the value of \( \beta \) is small or when the changes in velocity are negligible, it is found that the breakthrough of the fines concentration front occurs much faster compared to the breakthrough of the contaminant concentration front, for the choice of parameter made. They thus serve as a device to transport the contaminant at much faster rates. The spatial total contaminant concentration profiles depicted in Fig. 11 indicate that at smaller capture rates, the amount of colloids with contaminant adsorbed on their surface deposited at the capture sites is smaller, thereby increasing the total contaminant concentration.

Fig. 12 portrays the effect of \( \beta \) on the hydraulic conductivity. At larger values of \( \beta \), the amount of colloids accumulated at the capture sites is larger when exposed to the same aqueous fines concentration, causing a commensurate
reduction in velocity. The unequal slopes of the two curves reaffirm the above conclusion.

4. Conclusions

- A one-dimensional three-phase transport model has been developed based on equilibrium adsorption of a contaminant species and on colloidal induced release, migration and capture of colloidal fines at pore constrictions. Therefore, this model offers a better description of the physics of this colloid-associated contaminant transport phenomenon.
- Presence of colloids has been shown to enhance or retard the extent of contamination depending crucially on the system parameters such as $C_{f0}$, $\sigma_{\text{max}}$, $\sigma_{10}$, and $\alpha$, particularly the sensitivity of the hydraulic conductivity to the concentration of captured fines.
- For a range of conditions, plugging of the porous media occurs resulting in retardation of contaminant transport in groundwater flows.
- To develop a containment technology utilizing this plugging phenomenon, more constant pressure experimental studies on colloid-associated contaminant transport are required.

References