A comparative study on different power system frequency estimation techniques

B. Subudhi* and P.K. Ray

Department of Electrical Engineering,
National Institute of Technology,
Rourkela 769008, India
Email: bidyadharnitrkl@gmail.com
Email: pravat_ray@rediffmail.com
*Corresponding author

S.R. Mohanty

Department of Electrical Engineering,
National Institute of Technology,
Allahabad, India
Email: soumyaigit@rediffmail.com

A.M. Panda

Department of Electrical Engineering,
Indira Gandhi Institute of Technology,
Sarang 759146, India
Email: amp_igit@yahoo.com

Abstract: This paper presents the estimation of frequency which is an important power system parameter by Extended Least Square (ELS) technique. The above technique is validated by comparing its performance with the existing techniques such as Kalman Filter (KF) and Least Mean Square (LMS) technique, etc. Using different simulation studies with signals having different signal to noise ratio values and with step change in frequency, it is observed that ELS technique outperforms over LMS and KF methods on power system frequency estimation. Initialisation of covariance matrix in KF method and complicacy due to incorporation of correlation matrix in LMS algorithm affect their convergence. But ELS algorithm becomes very simple and attractive due to the absence of covariance and correlation matrix.

Keywords: ELS technique; extended least square technique; KF; Kalman filter; LMS technique; least mean square technique; covariance matrix; correlation matrix; system structure matrix; observation vector; frequency estimation; power system parameters.

Reference to this paper should be made as follows: Subudhi, B., Ray, P.K., Mohanty, S.R. and Panda, A.M. (2009) 'A comparative study on different power system frequency estimation techniques', *Int. J. Automation and Control*, Vol. 3, Nos. 2/3, pp.202–215.

Biographical notes: B. Subudhi currently is a Professor at the Department of Electrical Engineering, NIT, Rourkela, India and Coordinator of the Centre for Excellence, Industrial Electronics & Robotics. He (SM'08 IEEE) has received his bachelor degree in electrical engineering from Regional Engineering College, Rourkela (presently NIT, Rourkela), India, MTech in control and instrumentation from Indian Institute of Technology, Delhi, in 1994 and PhD degree in control system engineering from University of Sheffield, UK, in 2003. His research interests include system identification, intelligent control, control of mobile and flexible robot manipulators, estimation of signals and systems. He is a Senior Member of IEEE. He is serving as a Technical Committee Member, IEEE Intelligent Control Society.

P.K. Ray is working as Lecturer at the Department of Electrical Engineering, Indira Gandhi Institute of Technology, Sarang, Orissa, India. Currently, he is pursuing his PhD from NIT, Rourkela, India. He received his BE (Electrical) degree from I.G.I.T Sarang in 2000 and ME (Electrical) degree from Bengal Engineering and Science University, Howrah, in 2003. He has more than 6 years of experience in teaching and research. His research interests include system identification techniques and advanced signal processing.

S.R. Mohanty is working as a Lecturer at the Department of Electrical Engineering, Motilal Nehru National Institute of Technology, Allahabad, India. He received his BE (Electrical) degree from Utkal University, Bhubaneswar, in 1997 and ME (Electrical) degree from UCE Burla in 1999. He obtained his PhD degree in Electrical Engineering from IIT Kharagpur in 2007. He has more than 6 years of experience in teaching and research. His research interests include advanced signal processing application to power system protection.

A.M. Panda is a Professor and Head in the Department of Electrical Engineering at the Indira Gandhi Institute of Technology, Sarang, Orissa, India. He received his BE (Electrical) degree from UCE Burla in 1977 and ME (Electrical) degree from IIT Roorkee in 1984. He obtained his PhD degree in electrical engineering from Utkal University, Bhubaneswar, in 1996. He has guided four PhD students and has more than 20 years of experience in teaching and research. His research interests include power system operation control, robotics and artificial intelligence.

1 Introduction

For an improved power quality, fast and accurate estimation of supply frequency, voltage and its variation in real-time is essential in a complex power system. Variations in system frequency from its normal value indicate the occurrence of a corrective action for its restoration to its original value. In this context, a large number of numerical methods are available for frequency estimation from the digitised samples of the system voltage. Conventionally, it is assumed that the power system voltage waveform is purely sinusoidal and therefore the time between two zero crossings is an indication of system frequency. Discrete Fourier transforms, least squares technique (Sachdev and Giray, 1985; Soderstrom and Stocia, 1989; Jiang and Zhang, 2004; Mishra, 2005), finite impulse response filter (Sidhu, 1999), orthogonal filter (Sidhu and Sachdev, 1998), recursive Newton-type algorithm (Terzija et al., 1994), algebraic derivative approach (Trapero et al., 2008), Adaptive notch filters (Ljung and Soderstrom, 1983; Mendel, 1987; Soderstrom and Stocia, 1989; Jung, 1999; Mojiri et al., 2007), etc. are some of the popular signal processing techniques used for frequency measurements of power system signals. A large number of numerical techniques and their practical implementations are found in the literature, but these approaches suffer from inaccuracies due to the presence of noise and harmonics and other system changing conditions such as change in fault inception angle, change in fault resistance, etc. with amplitude, phase and frequency of a signal buried with noise and harmonics. It may be noted that the Extended Least Square (ELS) technique has attracted widespread attention due to its fast and accurate estimations. In view of addressing the difficulties of the existing methods (such as DFT, LS, Recursive Newton-type, Adaptive Notch Filter, etc.) and to achieve fast and accurate estimation of nominal and off-nominal power system frequency, an ELS technique (Abu Al-Feilat et al., 1994; Bettayeb and Qidwai, 1998) has been proposed for power system frequency estimation problems. Estimation of power system frequency at different Signal to Noise Ratio (SNR) values is carried out using three different estimation techniques. The results on the cases of jump in frequency and mean square error performance both in presence and absence of noise are also discussed.

2 Frequency estimation algorithms

2.1 A review on Kalman filtering application to frequency estimation

Kalman Filter (KF) (Dash et al., 1999; Dash et al., 2000; Kumar et al., 2006; Huang et al., 2008; Pigazo and Moreno, 2008) is a stochastic state estimator for parameter estimation. From the discrete values of the three-phase voltage signal of a power system, a complex voltage vector is formed using the well-known $\alpha\beta$ transformation (Dash et al., 1999; Pradhan et al., 2005). A nonlinear state space formulation is then obtained for this complex signal and KF approach is used to compute the true state of the model. As frequency is modelled as a state, the estimation of the state vector yields the unknown power system frequency. The discrete representation of three phase voltages of a power system can be expressed as follows:

$$\begin{aligned} V_a(k) &= V_m \cos(\omega k \Delta T + \phi) + \varepsilon_a(k) \\ V_b(k) &= V_m \cos(\omega k \Delta T + \phi - \frac{2\pi}{3}) + \varepsilon_b(k) \\ V_c(k) &= V_m \cos(\omega k \Delta T + \phi + \frac{2\pi}{3}) + \varepsilon_c(k) \end{aligned} \tag{1}$$

where V_a, V_b and V_c are three-phase voltage signals. V_m is the amplitude of the signal, ω is the angular frequency, $\varepsilon_a(k), \varepsilon_b(k), \varepsilon_c(k)$ are the noise terms, ΔT is the sampling interval, k is the sampling instant, ϕ is the phase of fundamental component. The complex form of signal derived from the three phase voltages is obtained by transform as mentioned below:

$$V_\alpha(k) = \sqrt{\frac{2}{3}}(V_a(k) - 0.5V_b(k) - 0.5V_c(k))$$
$$V_\beta(k) = \sqrt{\frac{2}{3}}(0.866V_b(k) - 0.866V_c(k)) \tag{2}$$

A complex voltage can be obtained from above equation (2) as follows:

$$\begin{aligned} V(k) &= V_\alpha(k) + jV_\beta(k) \\ &= Ae^{j(\omega k\Delta T+\phi)} + \eta(k) \end{aligned} \tag{3}$$

where A is the amplitude of the signal and η_k is the noise component.

The observation signal V_k can be modelled in a state space form as:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & x_1(k) \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} \tag{4}$$

$$y(k) = V(k) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \eta(k) \tag{5}$$

where the states x_1 and x_2 are as follows:

$$x_1(k) = e^{j\omega\Delta T} = \cos k\omega\Delta T + j\sin k\omega\Delta T \tag{6}$$

$$x_2(k) = Ae^{j(\omega k\Delta T+\phi)} \tag{7}$$

The above linear filter is equivalent to the nonlinear one

$$\begin{aligned} x(k+1) &= F(x(k)) \\ y(k) &= Hx(k) + \eta(k) \end{aligned} \tag{8}$$

Comparing equations (5) and (8):

$$H = \begin{bmatrix} 0 & 1 \end{bmatrix} \tag{9}$$

Applying KF to the system in (8):

$$K(k) = \hat{P}(k/k-1)H^T(H\hat{P}(k/k-1)H^T + Q)^{-1} \tag{10}$$

where K is the Kalman gain, H is the observation vector, P is the covariance matrix and Q is the noise covariance of the signal.

So the covariance matrix is related with Kalman gain with the following equation:

$$\hat{P}(k/k) = \hat{P}(k/k-1) - K(k)H\,\hat{P}(k/k-1) \tag{11}$$

So the updated estimated state is related with previous state with the following equation:

$$\hat{x}(k/k) = \hat{x}(k/k-1) + K(k)(V(k) - H\,\hat{x}(k/k-1) \tag{12}$$

After the convergence of state vector is attained, the frequency is calculated from equation (6) as follows:

$$\hat{f} = \frac{1}{2\pi\Delta T}(\sin^{-1}\text{Im}(\hat{x}_1)) \tag{13}$$

where $\hat{f}$ is the estimated frequency of the signal, Im ( ) stands for the imaginary part of a quantity.

2.2 Least mean square algorithm

To enhance the convergence characteristics of a power system signal, Least Mean Square (LMS) algorithm is adopted where the formulated structure looks very simple and this algorithm is found to be accurate one under various systems changing condition to estimate correct measure of frequency. The complex voltage signal as expressed in Subsection 2.1 is given by:

$$V_k = V_{\alpha k} + jV_{\beta k} \tag{14}$$

The voltage can be modelled as

$$V_k = Ae^{j(\omega k\Delta T + \varphi)} + \varepsilon_k \tag{15}$$

$$\hat{V}_k = V_{k-1}e^{j\omega k\Delta T} \tag{16}$$

This model is utilised in the proposed frequency estimation algorithm and the scheme that describes the estimation process.

The error signal in this case is as follows:

$$e_k = V_k - \hat{V}_k \tag{17}$$

where V_k is the estimated value of voltage at the kth instant. So equation (15) becomes:

$$\hat{V}_k = W_k\hat{V}_{k-1}$$

where $W_k = e^{j\hat{\omega}_k\Delta T}$ denotes the weight of the voltage signal, $\hat{\omega}$ is the estimated angular frequency. The significance of the above model is that the input vector contains one element only and so also the weight vector. The complex LMS algorithm is applied to estimate the state. The algorithm minimises the square of the error recursively by altering the complex weight vector W_k at each sampling instant using equation (18) given below:

$$W_k = W_{k-1} + \mu_k e_k \hat{V}_k^* \tag{18}$$

where * represents the complex conjugate of a variable. The step size μ_k is varied for better convergence of the LMS algorithm in the presence of noise. For complex states, the equations are modified as follows:

$$\mu_{k+1} = \lambda\mu_k + \gamma R_k R_k^* \tag{19}$$

where R_k represents the autocorrelation of e_k and e_{k-1} and R_k^* represents the complex conjugate of R_k. It is computed as:

$$R_k = \rho R_{k-1} + (1-\rho) e_k e_{k-1} \tag{20}$$

where ρ is an exponential weighting parameter and $0 < \rho < 1$, and $\lambda(0 < \lambda < 1)$ and $\gamma > 0$ control the convergence time. μ_{n-1} is set to $\mu_{\max}$ or $\mu_{\min}$ when it falls below or above the lower and upper boundaries, respectively. These values are chosen based on signal statistics. At each sampling interval, the frequency is calculated as follows:

$$f_k = \frac{1}{2\pi\Delta T} \sin^{-1}\left[\operatorname{Im}(W_k)\right] \tag{21}$$

where Im ( ) stands for the imaginary part of a quantity.

2.3 *Extended least square algorithm*

Let a signal buried with noise is represented by the following structure:

$$z(k) = A_1 \sin(\omega_0 k + \phi_1) + \varepsilon(k) \tag{22}$$

Normally, SNR in a power system can be taken from 30 dB to 40 dB. So for the purpose of estimation

$$z(k) = \left[\sin\omega_0 k \quad \cos\omega_0 k\right]\left[\alpha \quad \beta\right]^T + \varepsilon(k) \tag{23}$$

or in the standard form

$$z(k) = \phi(k)\theta + \varepsilon(k) \tag{24}$$

Here, $z(k)$ represents noisy measurement

$\varepsilon(k)$ shows the noise term

$\phi(k)$ is the system structure matrix

$\theta(k)$ is the vector of unknown parameter

$P(k)$ is the covariance of the parameter vector.

The estimated value for the required parameter can be expressed by equation (20) as follows:

$$P(k+1) = P(k) + [\phi^T(k)\phi(k)]^{-1} \tag{25}$$

$$\hat{\theta}(k+1) = \hat{\theta}(k) + P(k+1)\phi^T(k)[z(k+1) - \phi(k)\hat{\theta}(k)] \tag{26}$$

α and β are the parameters to be estimated. These are given by the equations (22) and (23), respectively as following:

$$\alpha = A_1 \cos\phi_1 \tag{27}$$

$$\beta = A_1 \sin\phi_1 \tag{28}$$

The actual required parameters are A_1 and ϕ_1, which can be expressed in terms of α and β as follows:

$$A_1 = \sqrt{\alpha^2 + \beta^2} \quad (29)$$

$$\phi_1 = \tan^{-1}\frac{\beta}{\alpha} \quad (30)$$

Once the estimate of amplitude and phase is done, it is required to estimate the frequency. This can be evaluated from the noisy measurement $z(k)$ as follows:

$$f_0 = \frac{1}{2\pi k}[\sin^{-1}(\frac{z(k)-\varepsilon(k)}{A_1}) - \phi_1] \quad (31)$$

3 Results and discussions

A synthetic signal of 1 pu amplitude, 50 Hz frequency and 0.5 pu phase angle is generated in MATLAB platform. Then the algorithms such as KF, LMS and ELS have been implemented with a sampling interval of 1 millisecond. A three-phase signal with 1 pu amplitude in each phase is also generated in MATLAB platform. From the complex signal, a two-phase signal is generated by $\alpha\beta$ transformation. The initial covariance matrix is taken as ρI, where I is the identity matrix and $\rho > 1$. Here the observation vector H is taken as $\begin{bmatrix} 0 & 1 \end{bmatrix}$. The SNR is taken as 30 dB. The frequency estimation can be done with the steps illustrated in the KF algorithm. Similarly for LMS algorithm the complex signal is also generated as that of the method adopted in KF. The complex weight matrix is updated with the right choice of step size (μ = 0.18) and correlation matrix (R = 0). From the complex weight matrix, the estimation of frequency is made using equation (21). In case of ELS technique, the signal is expressed in parametric form as given by the equation (23). Estimation of parameter is carried out by using equation (26). The sampling time is taken as 1 millisecond. The estimation of frequency has been accomplished in three steps. The first two steps as described in mathematical formulation are made to estimate amplitude and phase, followed by estimation of frequency.

Next, we will present the simulation results obtained using three different algorithms, namely KF, LMS and ELS.

A three-phase signal of 1 pu amplitude is generated in MATLAB and is shown in Figure 1. Estimation of amplitude for signal at SNR of 30 dB has been accomplished using KF and ELS method. From the KF algorithm, it is found that the estimated amplitude oscillates up to 20 samples and 1 pu for the rest of the samples, but in case of ELS algorithm the estimated amplitude is 1 pu for the entire samples which is shown in Figure 2.

Figure 1 Three-phase signal

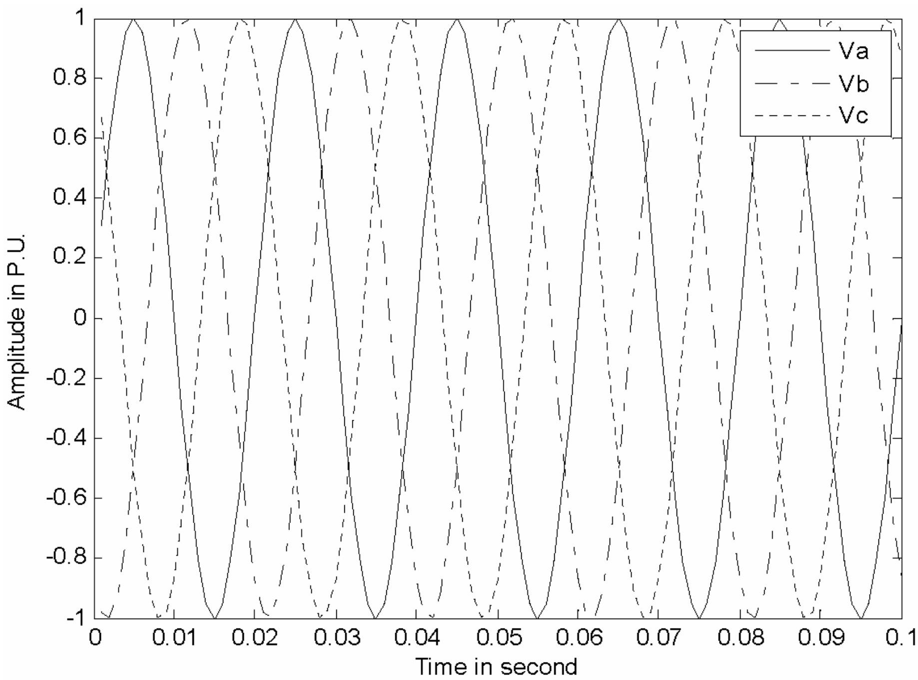
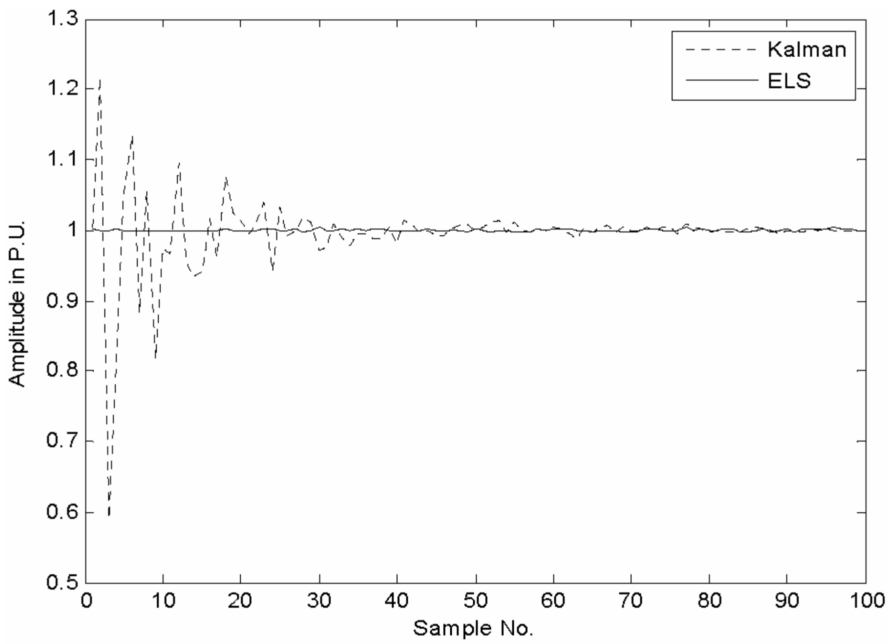


Figure 2 Comparison of estimation of amplitude at SNR (30 dB)



Figures 3 and 4 show a comparison of estimation of power system frequency using three techniques at SNR of 10 and 20 dB, respectively. From these figures, it is concluded that as SNR value goes on increasing estimation becomes more and more accurate for the case of three techniques but estimation using ELS technique outperforms over LMS and KF.

Figure 3 Comparison of estimation of power system frequency at SNR (10 dB)

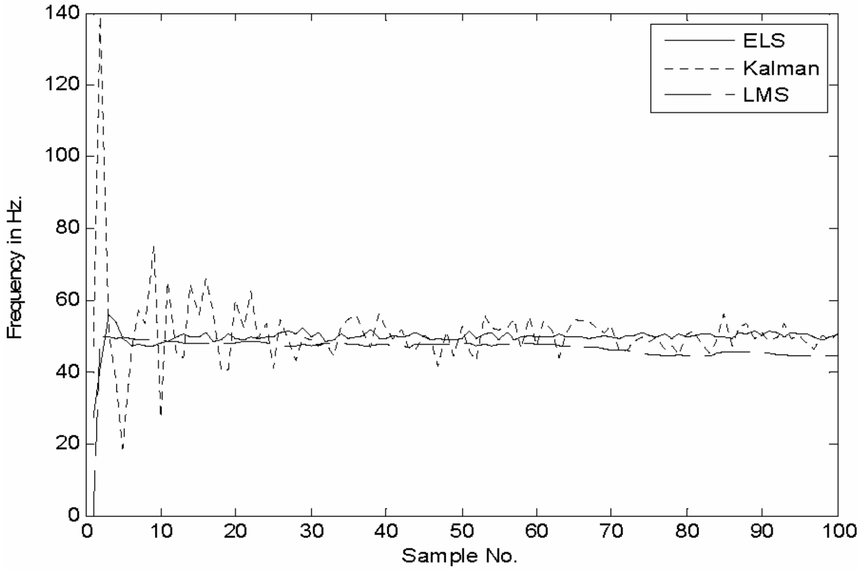
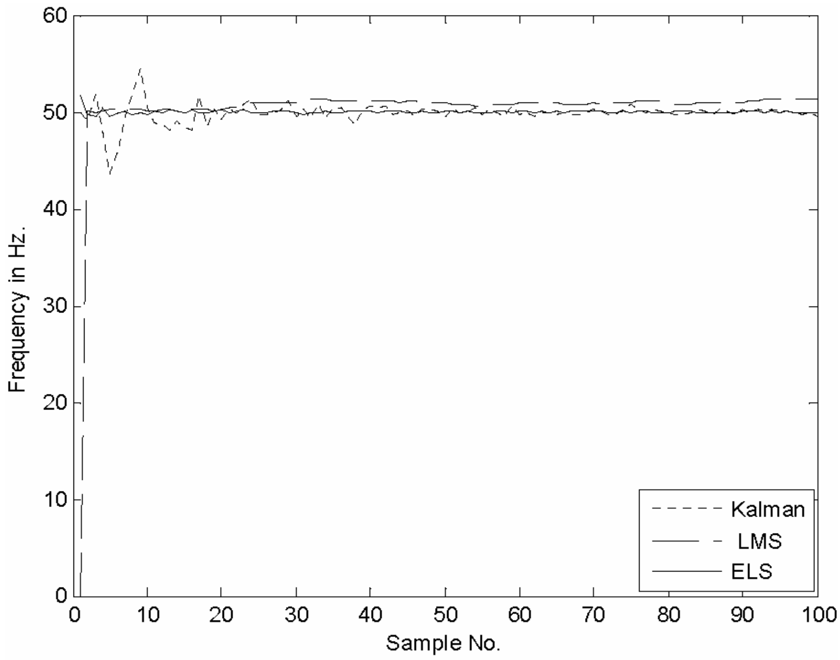


Figure 4 Comparison of estimation of power system frequency at SNR (20 dB)



Estimation of frequency by different algorithms is superimposed in Figure 5 with SNR of 30 dB. From Figure 5, it is clear that KF approach exhibits more oscillation and does not settle around 50 Hz. At the same time, LMS approach exhibits oscillations in the few initial samples and finally settles around 50 Hz. But ELS algorithm exhibits fewer oscillations and also settles around 50 Hz. So the estimation of frequency by ELS algorithm is preferred.

Figure 5 Comparison of estimation of power system frequency at SNR (30 dB)

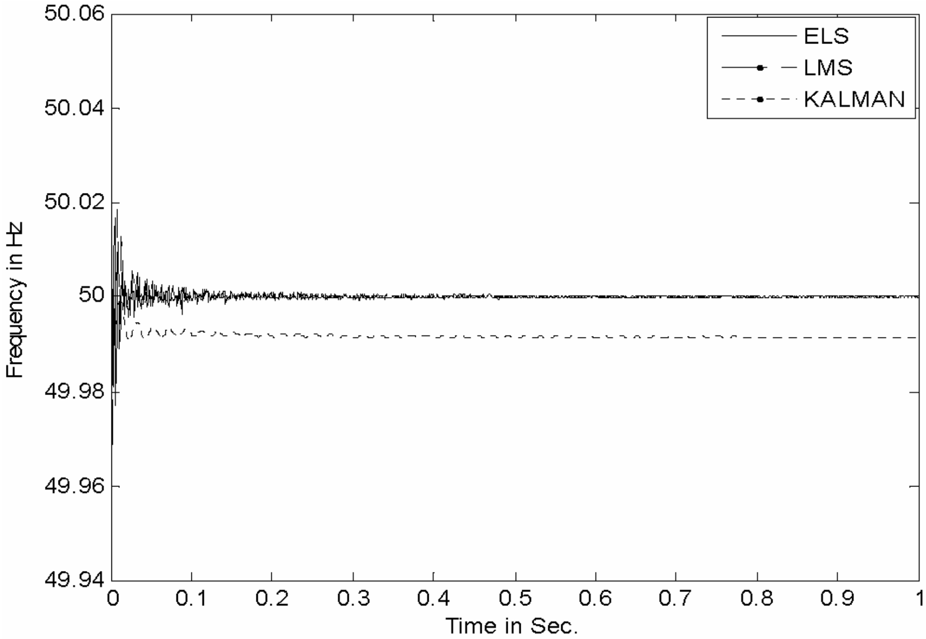


Figure 6 shows the estimation of power system frequency with a jump in frequency at 50–60 Hz at 50th sample of the signal by using three different methods. In case of estimation using KF, there is more oscillation in the first fifth samples after that it settles around 50 Hz till 50th sample and around 60 Hz till the last sample. LMS algorithm estimates frequency accurately in case of jump, but in few initial samples its estimation of frequency value rises to 50 Hz. But in the case of ELS algorithm, its estimation is around 50 Hz starting from the initial sample to 50th sample and 60 Hz till the last sample.

Estimation error is the difference between actual frequency and estimated frequency. Estimation error by different algorithms is carried out in Figure 7 at SNR of 30 dB. In case of KF, the estimation error is almost constant and comes near about 0.0152 Hz. In LMS algorithm, the estimation error is more for few initial samples and it is found to be 0.0086 Hz for rest of the samples. But for the ELS algorithm, the estimation error is reduced for the total simulation time and found as 0.0001 Hz which is definitely superior as compared to previous methods.

Figure 6 Jump in frequency with signal at SNR (30 dB)

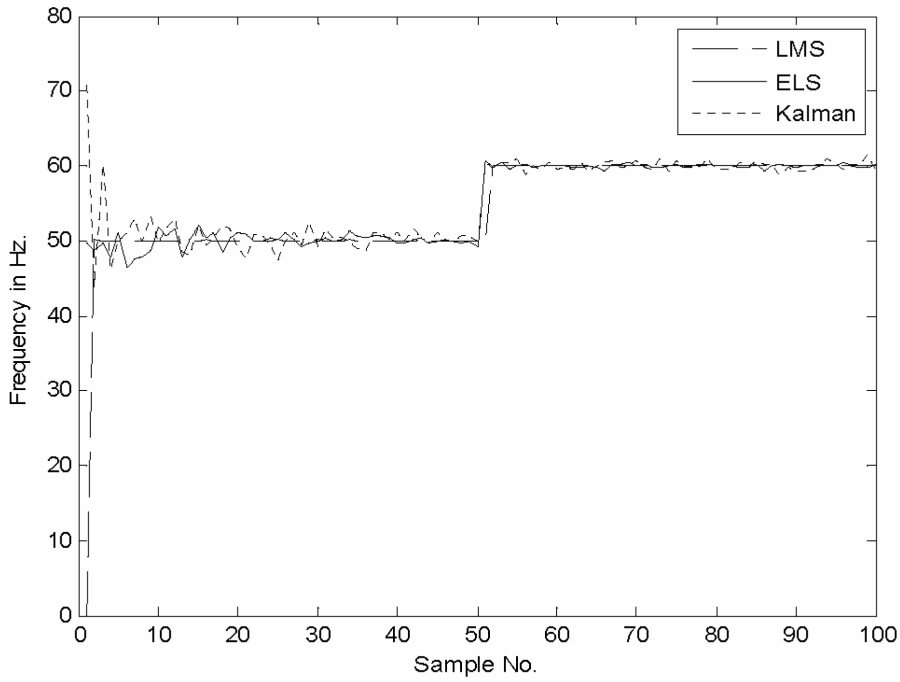
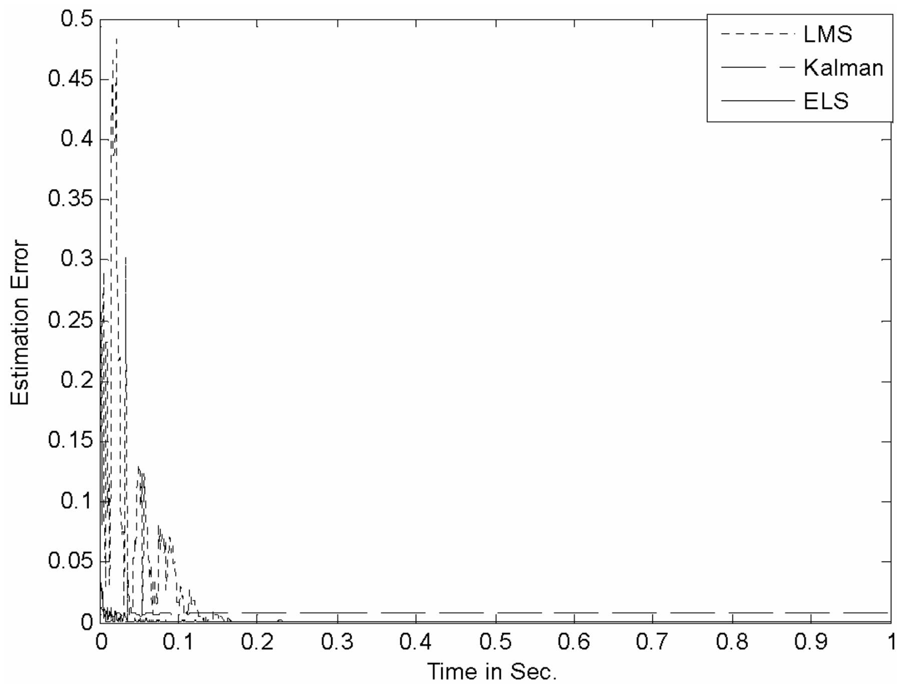


Figure 7 Comparison of estimation error



Figures 8 and 9 show mean squared error in the estimation of frequency of signal with and without noise, respectively. It is found from the figures that maximum MSE in case of KF without noise is of the order of 10^{-6}, whereas the same with noise is of the order of 10^{0}. For the case of LMS algorithm, it has MSE value of 2.5 for few initial samples with noise after that it converges to zero, whereas the same without noise converges to zero over the samples. But ELS has consistent performance both for with and without noise cases whose MSE curve converges to zero over the samples.

Figure 8 Comparison of MSE performance in the estimation of frequency at SNR (30 dB)

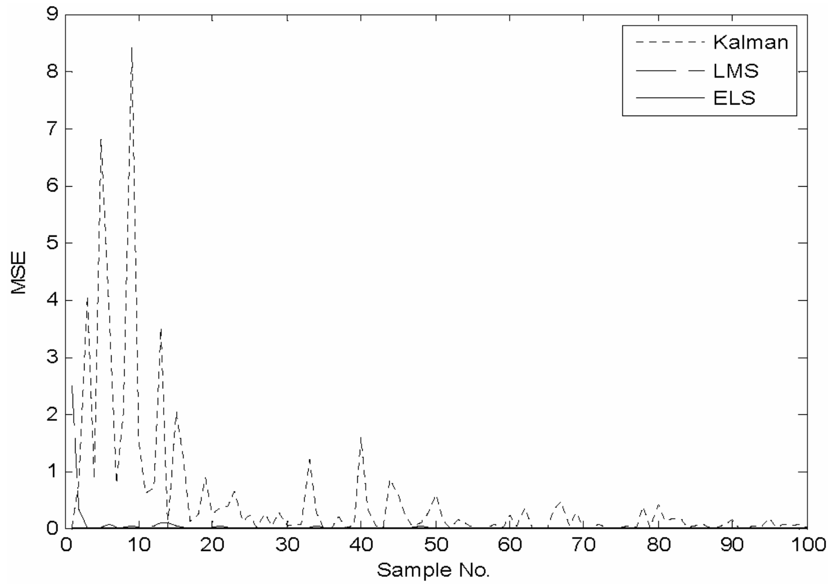
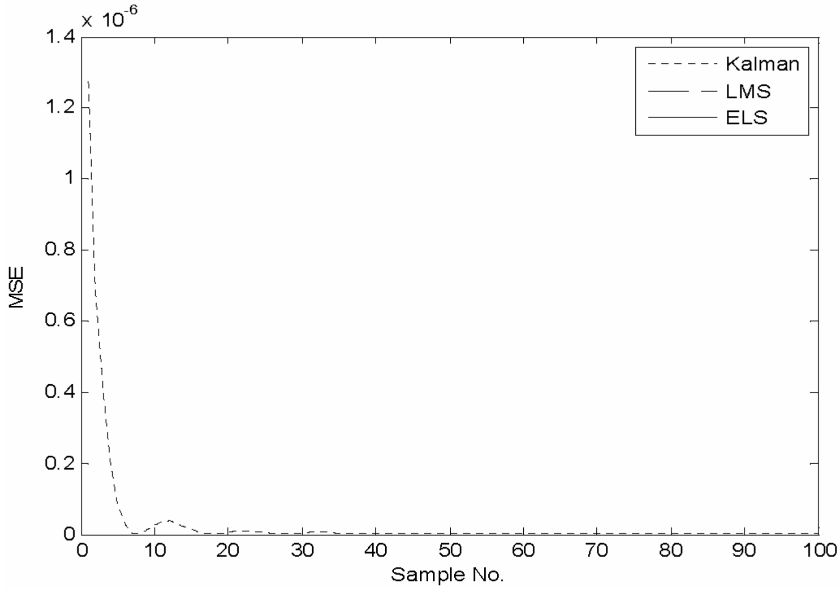


Figure 9 Comparison of MSE performance in the estimation of frequency without noise



From Table 1, the computational time for estimation of frequency by ELS algorithm is 0.2340 seconds, which is less in comparison to other two methods. The estimation error (RMS) in the case of ELS algorithm is also less in comparison to other two methods.

Table 1 Comparative assessment of methods

Method	*Estimation error (RMS)*	*Computational time (seconds)*
KF	0.0152	1.1870
LMS	0.0086	0.3440
ELS	0.0001	0.2340

4 Conclusions

This paper presents the estimation of frequency of a signal by various estimation techniques. However, choice of the covariance matrix is very crucial at the initial instant for KF algorithm. Improper choice of covariance matrix leads to more computational time with more estimation error. LMS algorithm seems to be very complex due to the implementation of correlation matrix and proper choice of step size. But at the same time, ELS algorithm is very simple by representation of the parametric form of the signal. With the one-step computation, the amplitude and phase of the signal are determined followed by the estimation of frequency. The computational time in the case of ELS is less due to the simplicity of the algorithm and estimation error is also less. Validation of the ELS algorithm can be done in MATLAB platform with various system changing conditions and all possible types of faults. Real-time implementation of the algorithm can be realised by a DSP processor.

References

Abu Al-Feilat, E.A., El-Amin, I. and Bettayeb, M. (1994) 'Power system harmonic estimation: a comparative study', *Electric Power System Research*, Vol. 29, pp.91–97.

Bettayeb, M. and Qidwai, U. (1998) 'Recursive estimation of power system harmonics', *Electric Power System Research*, Vol. 47, pp.143–152.

Dash, P.K., Pradhan, A.K. and Panda, G. (1999) 'Frequency estimation of distorted power system signals using extended complex Kalman filter', *IEEE Transactions on Power Delivery*, Vol. 14, No. 3, pp.761–766.

Dash, P.K., Pradhan, A.K., Panda, G., Jena, R.K. and Panda, S.K. (2000) 'On line tracking of varying harmonics using an integrated complex Kalman filter and Fourier linear combiner', *IEEE Conference on Power Engineering Society*, Vol. 3, pp.1575–1580.

Huang, C-H., Lee, C-H., Shih, K-J. and Wang, Y-J. (2008) 'Frequency estimation of distorted power system signals using a robust algorithm', *IEEE Transactions on Power Delivery*, Vol. 23, No. 1, pp.41–51.

Jiang, J. and Zhang, Y. (2004) 'A revisit to block and recursive least squares for parameter estimation', *Computers and Electrical Engineering*, Vol. 30, pp.403–416.

Jung, L. (1999) *System Identification: Theory for the User*, 2nd ed., PTR Prentice-Hall, Upper Saddle River, NJ.

Kumar, A., Das, B. and Sharma, J. (2006) 'Robust dynamic state estimation of power system harmonics', *Electrical Power & Energy Systems*, Vol. 28, pp.65–74.

Ljung, L. and Soderstrom, T. (1983) *Theory and Practice of Recursive Identification*, MIT Press, Cambridge, MA.

Mendel, J.M. (1987) *Lessons in Digital Estimation Theory*, Prentice-Hall, Englewood Cliffs, NJ.

Mishra, S. (2005) 'A hybrid least square-fuzzy bacterial foraging strategy for harmonic estimation', *IEEE Transactions on Evolutionary Computation*, Vol. 9, No. 1, pp.61–73.

Mojiri, M., Karimi-Ghartemani, M. and Bakhshai, A. (2007) 'Estimation of power system frequency using an adaptive notch filter', *IEEE Transactions on Instrumentation and Measurement*, Vol. 56, No. 6, pp.2470–2477.

Pigazo, A. and Moreno, V.M. (2008) '3ϕ-3ω signal model for power system harmonics and unbalance identification using Kalman filtering', *IEEE Transactions on Power Delivery*, Vol. 23, No. 2, pp.1260–1261.

Pradhan, A.K., Routray, A. and Basak, A. (2005) 'Power system frequency estimation using least mean square technique', *IEEE Transactions on Power Delivery*, Vol. 20, No. 3, pp.1812–1816.

Sachdev, M.S. and Giray, M.M. (1985) 'A least square technique for determining power system frequency', *IEEE Transactions on Power Apparatus and Systems*, Vol. 104, No. 2, pp.437–443.

Sidhu, T.S. (1999) 'Accurate measurement of power system frequency using a digital signal processing technique', *IEEE Transactions on Instrumentation and Measurement*, Vol. 48, No. 1, pp.75–81.

Sidhu, T.S. and Sachdev, M.S. (1998) 'An iterative technique for fast and accurate measurement of power system frequency', *IEEE Transactions on Power Delivery*, Vol. 13, No. 1, pp.109–115.

Soderstrom, T. and Stocia, P. (1989) *System Identification*, Prentice-Hall, Englewood Cliffs, NJ.

Terzija, V.V., Djuric, M.B. and Kovacevic, B.D. (1994) 'Voltage phasor and local system frequency estimation using Newton-type algorithms', *IEEE Transactions on Power Delivery*, Vol. 4, No. 3, pp.1368–1374.

Trapero, R., Sira-Ramirez, H. and Feliu Batlle, V. (2008) 'On the algebraic identification of the frequencies, amplitudes and phases of two sinusoidal signals from their noisy sum', *International Journal of Control*, Vol. 81, No. 3, pp.507–518.