Differential evolution computation applied to parameter estimation of induction motor

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Control of induction motor drive system requires an exact knowledge of its parameters. Efficient parameter estimation techniques are essential to obtain the parameters such as stator and rotor resistances, leakage and magnetizing inductances, because any mismatch between the actual and computed parameter values may lead to deterioration of control performance of the induction motor drive. In this paper, the differential evolution (DE) strategy - a global optimizer has been exploited for estimation of the above parameters of the induction motor. The main focus of the paper is on the application of the DE strategies to parameter estimation of an induction machine drive system based on the information of its input and output data, where input data comprises the stator voltages and the output data comprises the stator currents. Five different DE strategies were employed for implementing the induction motor parameter estimation schemes. Comparison of the results obtained through an extensive simulation studies on parameter estimations provide an idea how to choose an efficient estimator and to use them for efficiently control the drive.

Key words: differential evolution, induction motor, parameter estimation

1. Introduction

Modeling the dynamical properties of a system is an important step in analysis and design of a control systems. Modeling often results in a parametric model of the system which contains several unknown parameters. Experimental data are needed to estimate the unknown parameters.

A rich variety of estimation procedures were reported in literature for induction motor parameter estimation [1-3]. The simultaneous estimation of induction machine parameters and states are presented in [4-6]. The use of linear techniques based on the dynamic model of the induction motor is proposed in [7]. The use of ANNs and neuro-fuzzy methods for induction motor parameter estimation were proposed respectively in [8] and [9]. The extended Kalman filter has been employed to accomplish the joint estimation of the state variables and the machine parameters [10, 11]. The on-line tuning of the stator resistance, stator inductance, transient inductance, and rotor resistance has
been discussed in [12-14]. An interesting approach for tuning the rotor resistance is proposed in [15] based on model reference adaptive system schemes. All these investigations demonstrate that the performance of the drive can be improved through accurate estimation of the machine parameters. Generally, induction motor parameter estimation methods can be classified into five different categories, depending on the data availability and the way they are used.

The method of calculating parameters of the motor from the manufacture data of the motor requires a detailed knowledge of the machine’s construction, such as geometry and material parameters. It is the most accurate procedure, since it is closely related to the physical reality but is an expensive method since it is based on field calculation methods, such as the finite element method [16].

Parameter estimation based on steady-state motor models uses iterative solutions based on induction motor steady-state network equations [17-19]. This is the most common type of parameter estimation for system studies since the data needed for this is usually available.

The frequency-domain method of parameter estimation is basically a stand-still frequency response method that is based on measurements that are performed at standstill. The motor parameters are estimated from the resulting transfer function. This method cannot be used very often because of its high computational cost in the process of transformation from time domain to frequency domain. In fact, stand-still tests are not common in industry practice.

In the time-domain parameter estimation method, the time-domain motor measurements are performed and model parameters are adjusted to match the measurements. Since not all parameters can be observed using measurable quantities, the motor models need to be simplified [20]. The method is costly, and the required data is usually not available.

The real-time parameter estimation method is concerned with the tuning of the controllers of induction motor drive systems. This requires real-time parameter estimation techniques, using simplified induction motor models that are fast enough to continuously update the motor parameters and therefore prevent the detuning of induction machine controllers [21, 22].

The objective of the parameter estimation of induction motor is to determine a mathematical model of the motor with sufficient accuracy. To develop robust methods for parameter estimation, it is important to quantify the information content about machine parameters on measured signals. This is of particular importance when we restrict only to electrical terminal quantities, such as stator voltages and currents. Most of the existing parameter estimation methods such as Least Mean Square (LMS) and Recursive Least Square (RLS) methods use the regressor equation i.e.

\[ Y = X^T \theta + \varepsilon \]

where \( Y \) is the output vector, \( X \) is the regressor matrix, \( \theta \) is the parameters to be estimated and \( \varepsilon \) is the system noise. However, difficulties are encountered in the above regression
equation and in turn this method may be a viable choice for all situations in induction motor parameter estimation problems. Therefore, we explore an alternative way of solving the parameter estimation problem by using evolutionary method i.e. the DE which does not require the description of regression equation for parameter estimation.

The differential evolution [23, 24] is a population based stochastic optimization method that finds an increasing interest in the recent year for optimization techniques in the science and engineering communities due to its achievement of a global minimum. Therefore, it attracts the attention of the present work for neural network training. In this work a differential evolution has been applied as a global optimization method for feed-forward neural networks. In comparison to a gradient based methods such as gradient descent. Levenberg Marquardt algorithm differential evolution seems to provide advantage in terms of better optimal search.

The main contributions of this paper are as follows.

- Instead of being confronted with difficulties in finding expressions to represent the system by \( Y = X^T \theta + \varepsilon \), the DE method estimates the parameters directly.

- An extensive study on finding of an efficient DE strategy with a view of obtaining faster convergence for parameter estimation of induction motor has been pursued.

The rest paper is organized as follows. Section II includes a brief review on dynamics of induction motor. Section III gives an overview of the DE algorithm and its variants, In Section IV results are included to verify the effectiveness of the proposed DE based estimation method. Finally a brief conclusion is presented in section VI.

2. A brief review of induction motor dynamics

Although there are many models to describe induction motors, most of them highly complex and not suitable to be used in control. Also, since modern induction motor control is field oriented, d-q models will be analyzed. An excellent presentation on available model types can be found in [34]. The classical induction motor model (used in most control schemes) has identical d and q axis circuits. Since the classical model is a fourth order system with 6 elements of storage (inductances) the model can be reduced to a simpler model without any loss of information [34].

The following notations are used throughout the paper:

\( v_{ds}, v_{qs} \) – stator voltages in stationary reference frame,
\( i_{ds}, i_{qs} \) – stator currents in stationary reference frame,
\( \lambda_{dr}, \lambda_{qr} \) – rotor fluxes in stationary reference frame,
\( L_{ds}, L_{ms} \) – leakage and magnetizing inductance for stator,
\( L_{lr}, L_{mr} \) – leakage and magnetizing inductance for rotor,
\( R_s, R_r \) – stator and rotor resistances.
Fig. 1 shows the classical induction motor model (used in most control schemes) has identical d and q axis circuits. Fig. 2 shows the reduced induction motor model used in this work in stationary reference frame. Since the core loss resistance is much larger than the rotor resistance, it is neglected in this part of modeling. The following basic equations of induction machine can be derived

\[
\frac{d\lambda_{qr}}{dt} = n_p \omega_r \lambda_{dr} - \eta \lambda_{qr} + \eta L_m i_{qr} \\
\frac{d\lambda_{dr}}{dt} = -n_p \omega_r \lambda_{qr} - \eta \lambda_{dr} + \eta L_m i_{dr}
\]
\[
\frac{d i_{qs}}{dt} = -\beta n_p \omega_r \lambda_{dr} + \eta \beta \lambda_{qr} - \gamma_{qs} + \frac{1}{\sigma L_s} v_{qs}
\]
\[
\frac{d i_{ds}}{dt} = \beta n_p \omega_r \lambda_{qr} + \eta \beta \lambda_{dr} - \gamma_{ds} + \frac{1}{\sigma L_s} v_{ds}
\]

where:
\[
\eta = \frac{1}{T_R} = \frac{R_r}{L_m} - \text{is inverse of the rotor time constant},
\]
\[
\sigma = 1 - \frac{L_m}{L_l} - \text{leakage coefficient},
\]
\[
\beta = \frac{1}{L_l} - \text{inverse of leakage inductance},
\]
\[
\gamma = \frac{R_s + R_r}{L_l} - \text{inverse of stator time constant},
\]
\[
n_p - \text{number of poles pairs}.
\]

Equation (1)-(5) can be written in state variable form as
\[
\dot{X} = AX + BU
\]
and the output equation is
\[
Y = CX
\]
where
\[
A = \begin{bmatrix}
-R_s + R_r/L_l & 0 & \frac{R_r}{L_l} & -\omega_r/L_l \\
0 & -\frac{R_s + R_r}{L_l} & \omega_r/L_l & \frac{R_r}{L_l L_m} \\
-R_r & 0 & -\frac{R_s}{L_m} & \omega_r/L_m \\
0 & R_r & -\omega_r & \frac{R_s}{L_m}
\end{bmatrix},
\]
\[
B = \frac{1}{L_l} \begin{bmatrix}
1 \\
0 \\
0 \\
0
\end{bmatrix},
C = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0
\end{bmatrix},
\]
\[
U = \begin{bmatrix}
v_{ds} \\
v_{qs}
\end{bmatrix}.
\]

3. Problem formulation

The induction motor given in equations (7, 8) can be written in compact form as:
\[
\dot{X} = f(\theta, X, u), \quad y = g(X, \theta)
\]
where \( f \) and \( g \) are nonlinear functions

\[
\theta = \begin{bmatrix} R_s & R_r & L_1 & L_m \end{bmatrix}^T, \quad u = [v_1 \ v_2 \ v_3]^T
\]

\[
X = [i_{qs} \ i_{ds} \ \lambda_{qr} \ \lambda_{dr}]^T, \quad y = [i_1 \ i_2 \ i_3]^T
\]

In Fig.3, we present the parameter scheme of the induction motor drive system, where the optimization is to be performed using the DE algorithm.

Assumptions: In our model, we assume that the load torque \( T_L \) is zero, which eliminates the load torque as an additional input.

**4. A brief review on differential evolution considered for parameter estimation**

In a population of potential solutions to an optimization problem within an \( n \)-dimensional search space, a fixed number of vectors are randomly initialized, then evolved over time to explore the search space and to locate the minima of the objective function.

DE uses a greedy and less stochastic approach to problem solving than the other evolutionary algorithms. DE combines simple arithmetical operators with the classical operators of recombination, mutation and selection to evolve from a randomly generated starting population to a final solution. The fundamental idea behind DE is a scheme whereby it generates the trial parameter vectors. In each step, the DE mutates vectors by adding weighted, random vector differentials to them. If the cost of the trial vector
is better than that of the target, the target vector is replaced by the trial vector in the next generation. There are several variants of DE [25] which can be classified using the notation DE/x/y/z, where x specifies the vector to be mutated, y is the number of difference vectors used and z denotes the crossover scheme. x can be ’rand’ (randomly chosen population vector) or ’best’ (the best vector from the current population). y will be equal to 1, for one difference vector and will be equal to 2 for two difference vector. z is ’exp’ for exponential crossover and ’bin’ for binomial crossover. Using this notation, the basic DE-strategy can be written as follows.

- DE/best/1/exp
- DE/rand/1/exp
- DE/rand-to-best/1/exp
- DE/best/2/exp
- DE/rand/2/exp

Next we explain the working steps involved in employing a DE cycle as follows.

**Step 1: Parameter setup**
The user chooses the parameters of population size, the boundary constraints of optimization variables, the mutation factor \((F)\), the crossover rate \((CR)\), and the stopping criterion of maximum number of iterations (generations), \(G\).

**Step 2: Initialization of the population**
Set generation \(G = 0\). Initialize a population of \(i = 1, NP\) individuals (real-valued \(D\)-dimensional solution vectors) with random values generated according to a uniform probability distribution in the \(D\) dimensional problem space. These initial values are chosen randomly within user defined bounds.

**Step 3: Evaluation of the population**
Evaluate the fitness value of each individual of the population.

**Step 4: Mutation operation (or differential operation)**
Mutation is an operation that adds a vector differential to a population vector of individuals. For each target vector a mutant vector is produced using the following formula

\[
v_{i,G} = x_{r1,G} + F(x_{r2,G} - x_{r3,G})
\]  
(10)

where \(i, r1, r2, r3 \in \{1, 2, \ldots, NP\}\) are randomly chosen and must be different from each other. In equation (10), \(F\) is the mutation factor, which controls the amplification of the difference between two individuals so as to avoid search stagnation and is usually taken from the range \([0, 1]\).
Step 5: Recombination operation
Following the mutation operation, recombination is applied to the population. Recombination is employed to generate a trial vector by replacing certain parameters of the target vector with the corresponding parameters of a randomly generated donor (mutant) vector. There are two methods of recombination in DE, namely, binomial recombination and exponential recombination. In binomial recombination, a series of binomial experiments are conducted to determine which parent contributes which parameter to the offspring. Each experiment is mediated by a crossover constant, $CR$, ($0 \leq CR \leq 1$). Starting at a randomly selected parameter, the source of each parameter is determined by comparing $CR$ to a uniformly distributed random number from the interval $[0, 1)$. If the random number is greater than $CR$, the offspring gets its parameter from the target individual; otherwise, the parameter comes from the mutant individual. In exponential recombination, a single contiguous block of parameters of random size and location is copied from the mutant individual to a copy of the target individual to produce an offspring. A vector of solutions are selected randomly from the mutant individuals when $rand_j$, $(rand_j \in [0, 1]$ is a random number) is less than $CR$.

$$t_{j,i,G} = \begin{cases} v_{j,i,G} & \text{if } (rand_j \leq CR) \text{ or } j = j_{rand} \\ x_{j,i,G} & \text{otherwise} \end{cases}$$

$j = 1, 2, \cdots, D$, where $D$ is the number of parameters to be optimized.

Step 6: Selection operation
Selection is the procedure of producing better offspring. If the trial vector $t_{i,G}$ has an equal or lower value than that of its target vector, $x_{i,G}$ it replaces the target vector in the next generation; otherwise the target retains its place in the population for at least one more generation.

$$x_{i,G+1} = \begin{cases} t_{i,G} & \text{if } f(t_{i,G}) \leq f(x_{i,G}) \\ x_{i,G} & \text{otherwise} \end{cases}$$

Once new population is installed, the process of mutation, recombination and selection is replaced until the optimum is located, or a specified termination criterion is satisfied, e.g., the number of generations reaches a preset maximum $G_{\text{max}}$.

At each generation, new vectors are generated by the combination of vectors randomly chosen from the current population (mutation). The upcoming vectors are then mixed with a predetermined target vector. This operation is called recombination and produces the trial vector. Finally, the trial vector is accepted for the next generation if it yields a reduction in the value of the objective function. This last operator is referred to as a selection.

Fig.4 shows a two dimensional objective function that illustrates the different vectors, on which differential evolution is applied. It shows the process of generating trial vector for the scheme explained in equation (10).

Fig.5 provides a simple pseudo-code for the implementation of a differential evolution strategy.
Figure 4. Two dimensional objective functions.

```plaintext
While (convergence criterion not yet met)
{
    //Xt defines a vector of the current vector population
    //Vi defines a vector of the new vector population
    for (i=0; i<Np; i++)
    {
        r1 = rand (NP); //select a random index from 1, 2, Np
        r2 = rand (NP); //select a random index from 1, 2, Np
        r3 = rand (NP); //select a random index from 1, 2, Np
        Ui = Xt + F*(Xt.Xt);
        if (f(Ui) <= f(Xt))
        {
            Vi = Ui;
        }
        else
        {
            Vi = Xi;
        }
    }
}//end while
```

Figure 5. Pseudo code for Differential Evolution Algorithm.

5. Results and discussion

Simulation Setup
The parameter estimation schemes as described in Section 4 have been applied to the induction motor by using the input-output data i.e. the stator voltage (transformed d-q
axis, equation) and the stator current (d-q transformed, equation) to estimate motor resistance and inductance.

Table 1. Parameters of the induction motor drive

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage</td>
<td>220 V</td>
</tr>
<tr>
<td>Power</td>
<td>5 Hp</td>
</tr>
<tr>
<td>Frequency</td>
<td>50 Hz</td>
</tr>
<tr>
<td>Stator resistance</td>
<td>0.39 W</td>
</tr>
<tr>
<td>Rotor resistance</td>
<td>0.22 W</td>
</tr>
<tr>
<td>Leakage inductance</td>
<td>0.006 H</td>
</tr>
<tr>
<td>Magnetizing inductance</td>
<td>0.068 H</td>
</tr>
<tr>
<td>RPM</td>
<td>1750 rpm</td>
</tr>
</tbody>
</table>

All the five variants of the DE schemes (described in Section 4) for identifying the motor parameters, $R_S$, $R_r$, $L_l$, and $L_m$ have been implemented using the common set-up for DE given in Table 2.

Table 2. Parameters for DE simulation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total sampling number, $T$</td>
<td>500</td>
</tr>
<tr>
<td>Population size, $S$</td>
<td>20</td>
</tr>
<tr>
<td>Upper and lower bound of stator resistance</td>
<td>[0, 1]</td>
</tr>
<tr>
<td>Upper and lower bound of rotor resistance</td>
<td>[0, 1]</td>
</tr>
<tr>
<td>Upper and lower bound of leakage inductance</td>
<td>[0, 1]</td>
</tr>
<tr>
<td>Upper and lower bound of magnetizing inductance</td>
<td>[0, 1]</td>
</tr>
<tr>
<td>Mutation constant factor, $F$</td>
<td>0.6</td>
</tr>
<tr>
<td>Cross over constant, $CR$</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Estimation of stator resistance

Figures 6-10 present the estimation results obtained with using five different DE strategies as mentioned earlier. The actual and the estimated stator resistances are shown in Fig. 6, Fig. 7, Fig.8, Fig.9 and Fig.10 for Strategy 1; Strategy 2; Strategy 3; Strategy 4; and Strategy 5 respectively. In all the cases, the estimated value of the stator resistance approaches to its actual value (0.39W), but the time of convergence was different for these five strategies.
Differential Evolution Computation Applied to Estimation of Induction Motor

Figure 6. Stator resistance estimation performance by Strategy-1.

Figure 7. Stator resistance estimation performance by Strategy-2.

Figure 8. Stator resistance estimation performance by Strategy-3.
Estimation of rotor resistance
Figures 11-15 show rotor estimation performances obtained with five different strategies. In all the cases, the value of the estimated rotor resistance approaches to the actual rotor resistance, 0.22 W but with different time of convergence as shown in Table 3.

Estimation of leakage inductance
Figures 16-20 show the estimated and actual leakage inductances for all the five variants of DE discussed previously.

Estimation of magnetizing inductance
Figures 21-25 show the estimated and actual leakage inductance curves for five DE strategies considered as above.
Figure 11. Rotor resistance estimation performance by Strategy-1.

Figure 12. Rotor resistance estimation performance by Strategy-2.

Figure 13. Rotor resistance estimation performance by Strategy-3.
Figure 14. Rotor resistance estimation performance by Strategy-4.

Figure 15. Rotor resistance estimation performance by Strategy-5.

Figure 16. Estimation performance for leakage inductance by Strategy-1.
Figure 17. Estimation performance for leakage inductance by Strategy-2.

Figure 18. Estimation performance for leakage inductance by Strategy-3.

Figure 19. Estimation performance for leakage inductance by Strategy-4.
Figure 20. Estimation performance for leakage inductance by Strategy-5.

Figure 21. Estimation performance for magnetizing inductance by Strategy-1.

Figure 22. Estimation performance for magnetizing inductance by Strategy-2.
Figure 23. Estimation performance for magnetizing inductance by Strategy-3.

Figure 24. Estimation performance for magnetizing inductance by Strategy-4.

Figure 25. Estimation performance for magnetizing inductance by Strategy-5.
Comparison of Squared Errors with Different DE Strategies

In Figures 26-30, we present squared error versus time curves for the five DE variants considered. It is clear from Figs. 26-30 that out of the five schemes, i.e. DE/best/1/exp scheme has the lowest MSE and achieves faster parameter convergence. Hence this scheme is considered as the best amongst the five DE schemes for estimating the motor parameters.

Table 3 shows the comparison of the performance of all the five strategies of differential evolution considered in this paper in terms of time of convergence, number of function evaluation and no of iterations. The maximum number of iteration taken was 150 where as different strategies reached to particular stopping criteria at different iterations. The first strategy i.e. DE/best/1/exp converged at the minimum number of iteration and function evaluation.
Figure 28. Squared error Strategy-3.

Figure 29. Squared error Strategy-4.

Figure 30. Squared error Strategy-5.
### Table 3. Comparison of performance of five strategies

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Time of convergence (in seconds)</th>
<th>No of iteration</th>
<th>No of function evaluation</th>
</tr>
</thead>
<tbody>
<tr>
<td>DE/best/1/exp</td>
<td>117.408</td>
<td>80</td>
<td>1600</td>
</tr>
<tr>
<td>DE/rand/1/exp</td>
<td>317.826</td>
<td>140</td>
<td>2800</td>
</tr>
<tr>
<td>DE/rand-to-best/1/exp</td>
<td>197.091</td>
<td>87</td>
<td>1740</td>
</tr>
<tr>
<td>DE/best/2/exp</td>
<td>324.045</td>
<td>140</td>
<td>2800</td>
</tr>
<tr>
<td>DE/rand/2/exp</td>
<td>330.37</td>
<td>150</td>
<td>3000</td>
</tr>
</tbody>
</table>

### 6. Conclusion

The main purpose of this work was to determine the suitability of the evolutionary computation (EC) approach to overcome the difficulties encountered in employing the conventional parameter estimation techniques such as LMS and RLS (see the description made in the introduction section). The success of DE on this problem certainly underlines that it is a promising approach to estimate the parameters of the induction motor drive using real motor data. The approach can be easily extended to similar problems.

In the paper, we have demonstrated the application of the differential evolution algorithm for efficiently solving the identification problem of an induction motor. We have considered five different DE formulations towards estimating the parameters i.e. stator and rotor resistances, leakage inductance and magnetizing inductance of the Induction Motor Drive System. The problem of expressing the induction motor parameter estimation system by the regressor equation $Y = X^T\theta + \varepsilon$ has been rightly resolved by using the DE method and estimate of parameters obtained directly.

From the results presented in Section 5 it is pertinent that for a given induction motor, the unknown parameters can be successively evolved accurately using the DE approach proposed in the paper. After having studied the performances of the five different DE variants we conclude that the strategy, **DE/best/1/exp** gives the better result in terms of faster convergence time and accuracy in estimating parameters.

### References


